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ORIGINAL PAPER

Identifying a new paradigm for assessing irrigation system performance

Luciano Mateos

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Abstract There are many definitions of irrigation system efficiency that are applied over a range of scales. Many traditional definitions considered only the water diverted as the water volume of concern. Considering also the water consumed in defining effective irrigation efficiency is a shift from the classical definition of system efficiency. In this paper, equations are derived for calculating the following system performance measures: the irrigation consumptive use coefficient, irrigation system efficiency, irrigation water and soil salinities, relative yield, and productivity of consumed, diverted and beneficially used water. The expressions are based on quite general assumptions and are valid for systems with a single water source and layouts composed of (or simplified to) irrigation units arranged in a row. The aim of these expressions is to illustrate how system performance is affected by the reuse of water which depends on the system's hydraulic connections and the irrigation unit performance. Illustrations of the model are provided for systems in series and in parallel. Testing and refinement by removing some of the general assumptions underlying the model will be needed to develop practical applications that can be more confidently applied for comparison and improvement of irrigation systems.

List of symbols and acronyms

ρ_1	fraction of the non-consumed water that is reincorporated into the line source (main channel)
ρ_2	fraction of the non-consumed water that circulates to the next unit downstream in the row
ρ_b	fraction of non-consumed water that is both reused and beneficial
B_o	decrease in relative yield per unit of increase in soil salinity above CS_{th}
CC	line source (main channel) water salinity
CD	drainage water salinity
CI	irrigation water salinity
CS	soil salinity
CS_{th}	threshold of soil salinity above which yield is reduced
CU	consumptive use
f	fraction of non-consumed water that is beneficial
F	water flow in the line source (main channel)
g	fraction of non-consumed water that is lost as a result of percolation
h	fraction of consumed water that is beneficial
I	irrigation water
IB	irrigation water that is beneficially used
ICUC	irrigation consumptive use coefficient
IE	irrigation efficiency
j	order of the irrigation unit in the system
k	fraction of the main channel water flow derived to irrigation unit 1
LF	leaching fraction
n	number of irrigation units in the irrigation system
RY	relative yield
WP2	water productivity per unit of water diverted
WP3	water productivity per unit of irrigation water consumed
WP4	water productivity per unit of irrigation water beneficially used
Y_{max}	maximum yield achievable in the environment under consideration

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L. Mateos (✉)
Instituto de Agricultura Sostenible, CSIC,
Apdo 4084, 14080 Cordoba, Spain
e-mail: ag1mainl@uco.es

Subscripts

1, 2, …, j	the order of the irrigation unit in the system
n	the number of irrigation units in the irrigation system
p	parallel arrangement of the irrigation units
s	series arrangement of the irrigation units
u	irrigation unit

Introduction

The most straight forward design of an irrigation system is that of a single source, delivering water through a series of discharge points in an irrigation area. Excess water from channel overflow and drainage from fields is collected and conveyed away in a drainage network not connected to the supply. The supply of water is used to grow crops and logically the effectiveness of the delivery system can be assessed as the ratio of water volume actually used to grow the crops relative to the volume of water at the start of the supply. This is the conceptual construct applied by Israelsen (1950) who defined irrigation efficiency (IE). This classical concept of efficiency reflects a strong engineering approach. However, irrigation systems are rarely as simple as that described above with many recent ones built with distribution and drainage networks that are coupled but unconnected and still others where various levels of connection exist either deliberately or inadvertently. In this last case, drainage, water reuse becomes an intrinsic part of the irrigation system. It was with this more complex system in mind that Jensen (1993) (cited by Burt et al. 1997) proposed changing the name of this ratio to irrigation consumptive use coefficient (ICUC). The term irrigation efficiency was reserved for the same ratio but using all the beneficial uses of the diverted water as the numerator rather than just consumptive use (Burt et al. 1997).

In more complex irrigation systems where reuse is significant, the traditional concept of efficiency may be misleading since reuse water can be beneficially used with no long-term detrimental effects on productivity. To acknowledge this, Jensen et al. (1980) introduced the concept of effective IE (which deducts the amount of water that will be reused from the water loss) and Willardson et al. (1994) proposed replacing the term efficiency by the term fraction (for e.g. the consumed fraction) to eliminate the word and the concept of efficiency altogether. The International Water Management Institute adopted these updated concepts as the cornerstone of what they called the IWMI water resources paradigm (Perry 1999).

With these more recent considerations, a change in IE at one scale will have different effects on the whole system depending on its hydraulic layout. Burt et al. (1997) stressed the validity of their definitions of irrigation performance measures at all scales of analysis. Seckler et al. (2003) and Jensen (2007) reviewed the concept of IE to place it within the context of integrated water resources management. However, additional formulations are still necessary to analyse how the configuration (or changes in the configuration) of the system would affect global IE and how interventions at smaller scales (i.e. improvement of water application in the irrigation units) would influence IE, questions for which water resource managers and policy makers surely need answers.

Keller et al. (1996) provided one quantitative method to illustrate the difference between the classical and effective IE in a series of irrigation cycles. Solomon and Davidoff (1999) developed analytical expressions relating unit and sub-unit performance in irrigation systems arranged in parallel, incorporating the possibility of circulating water between consecutive sub-units. Mateos et al. (2000), using a similar approach to that of Keller et al. (1996) and Solomon and Davidoff (1999), showed how the ICUC changes with the number and arrangement of irrigation units in a system. Furthermore, Mateos et al. (2000) illustrated how performance changes at the scale of the individual irrigation unit affect whole system performance and, more importantly, they addressed the effect of water reuse and system arrangement on the degradation of the quality of irrigation water.

As irrigation water use is subject to greater scrutiny, there is need for more consistent and robust analysis methods to enable comparison and identify improvement opportunities including the assessment of water productivity. There are several ways of expressing water productivity, some of which can be related to indicators of technical performance (Playán and Mateos 2006). It is therefore possible to develop a sound theoretical foundation for comprehensively assessing irrigation system performance. In this study, I extend the analytical methods for assessing irrigation performance that were developed by Keller et al. (1996), Solomon and Davidoff (1999) and Mateos et al. (2000) to typical arrangements of irrigation systems fed by rivers or artificial channels. The analysis addresses not only IE (and ICUC) but also the degradation of the quality of irrigation water and water productivity.

Simplified irrigation system layouts

The arrangement of the units in an irrigation system is determined by the hierarchical branched layout of the distribution network. A drainage network with a mirror image structure of the supply system can collect return flows from the irrigation units with the possibility that some return flow can be reused. The analysis of such systems may require ad hoc water and salt balance models (Mateos et al. 2000). However, the layout of a scheme, a water supply basin or large project can often be simplified by aggregating units. The new, aggregated units can then be conceptually arranged as a single row. This is the case for a series of irrigation cycles in a closed system (Keller et al. 1996; Mateos et al. 2000). In such a system, all of the drainage water from one use (cycle) becomes available for reuse in the next downstream unit (Fig. 1a). The process may continue until the initial water supply is totally consumed. A consequence of this arrangement is that the area of irrigation units must decrease, as one progresses downstream, at a rate determined by the ICUC of each unit. Herein, such systems are referred to as being 'in series'.

More typical is an arrangement in which a single, primary water source (surface reservoir, snow pack or aquifer) supplies a line source (river or canal) feeding irrigation units located along with it. Irrigation schemes in a large irrigation project served by a unique conveyance canal are typical of this type of arrangement. The irrigation schemes along the Nile River in Africa, the Colorado River in North America or the Murray River in Australia are examples of arrangements that can be simplified to single-source single-row layouts. In the case of a main canal supplying various irrigation schemes, the drainage water from the irrigation units typically returns to the river from which the canal derived. In basin-scale systems, the river acts as both the line-source of irrigation water and the collector of return flows. Intermediate situations—part of the drainage returning to the line source and part flowing to a separate collector—may be found as well. The different situations are represented in Fig. 1b. The fractions ρ_1, ρ_2 and $1 - \rho_1 - \rho_2$ are, respectively, the proportion of the non-consumed water that is reincorporated into the line source, circulated to the next unit downstream in the row, or not reused. Solomon and Davidoff (1999) provided expressions for global ICUC and IE for the particular case in which $\rho_1 = 0$. Mateos et al. (2000) analysed the global ICUC and water-quality degradation in systems with $\rho_1 = 0$ and either $\rho_2 = 0$ or $1 - \rho_1 - \rho_2 = 0$. In contrast to the term 'in series' used for the closed systems described above, these systems are named herein systems 'in parallel' because water flows along parallel paths, through the irrigation units, from the supply channel to the common drainage collector.

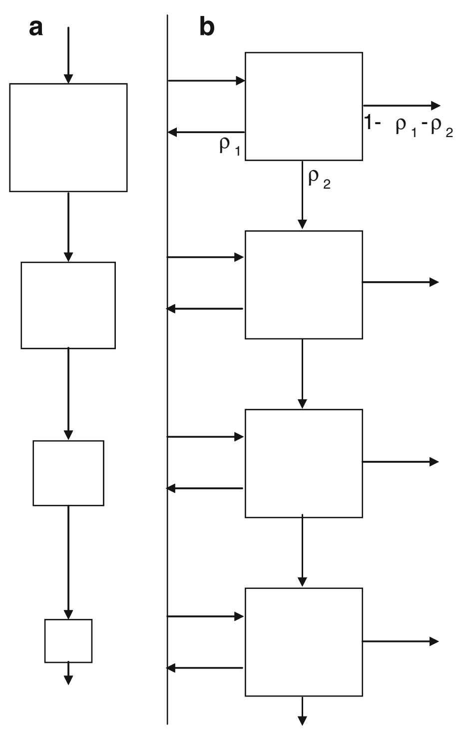

Fig. 1 Arrangement of irrigation units in systems in series (**a**) and in parallel (**b**). The fractions ρ_1, ρ_2 and $1 - \rho_1 - \rho_2$ (indicated only next to the first unit) are the proportion of the non-consumed water that is reincorporated into the line source, circulated to the next unit downstream, or not reused, respectively

Expressions for the performance assessment of irrigation systems

The water used in an irrigation unit can be classed as consumed or non-consumed (Fig. 2). The consumed water may be beneficial, e.g. transpiration (fraction h) or non-beneficial, e.g. soil evaporation ($1 - h$). Similarly, the non-

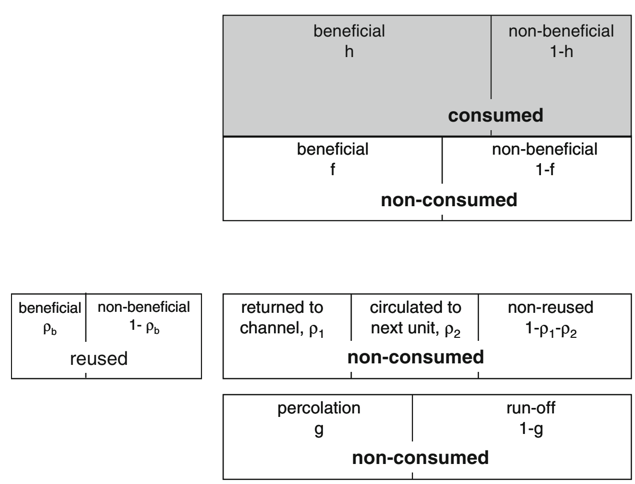

Fig. 2 Fractions of the consumed and non-consumed water in an irrigation unit. ρ_1, fraction of the non-consumed water that is returned to the source channel. ρ_2, fraction of the non-consumed water that is circulated to the next irrigation unit downstream. h, fraction of consumed water that is beneficial. ρ_b, fraction of non-consumed water that is both reused and beneficial. f, fraction of non-consumed water that is beneficial. g, fraction of non-consumed water that is lost as a result of percolation

consumed water may be beneficial, e.g. leaching requirement (fraction f) or non-beneficial, e.g. deep drainage ($1 - f$). For instance, the leaching requirement can be classified as non-consumed beneficial water.

Alternatively, the non-consumed water may be divided into three fractions defined in the previous section: ρ_1, ρ_2, and $1 - \rho_1 - \rho_2$ (Fig. 2). Furthermore, the reused water may have a beneficial (fraction ρ_b) and a non-beneficial ($1 - \rho_b$) component; and non-consumed water may be removed from the system either by deep drainage (fraction g) or as surface runoff ($1 - g$).

The derivation of the expressions for the performance assessment of irrigation systems is presented in Appendix. The resulting formulae and their underlying assumptions are presented below.

Irrigation consumptive use coefficient

The ICUC is the fraction of the irrigation water destined for consumptive use (Burt et al. 1997). For simplicity, in the successive expressions it will be assumed that ICUC of the irrigation unit (ICUC_u), irrigation unit denoted by subscript u, is constant for all units in the system. Then, the ICUC of a system composed of n units in series is:

$$\mathrm{ICUC}_{n,s} = 1 - (1 - \mathrm{ICUC}_u)^n \tag{1}$$

If the units are in parallel:

$$\mathrm{ICUC}_{n,p} = \frac{n\mathrm{ICUC}_u}{n - (n-1)(1 - \mathrm{ICUC}_u)(\rho_1 + \rho_2)}. \tag{2}$$

In the parallel system, it is assumed that not only ICUC_u is constant across the system, but also the unit irrigation water (I_u) and the unit consumptive use are constant (CU_u). Therefore, the unit irrigation water results from adding water from the main channel to the return flows coming from the upstream irrigation unit (fraction ρ_2), till obtaining an amount of water I_u.

Irrigation efficiency

The IE is the fraction of the irrigation water that is beneficially used (Burt et al. 1997). The IE of a system composed of n units in series is:

$$\mathrm{IE}_{n,s} = h[1 - (1 - \mathrm{ICUC}_u)^n] + f(1 - \mathrm{ICUC}_u)^n. \tag{3}$$

If the units are in parallel and IE_u is the IE of a single unit, then the IE of the whole system is:

$$\mathrm{IE}_{n,p} = \frac{n\mathrm{IE}_u - (n-1)\rho_b(1 - \mathrm{ICUC}_u)}{n - (n-1)(1 - \mathrm{ICUC}_u)(\rho_1 + \rho_2)}. \tag{4}$$

Irrigation water salinity

In the series system, the irrigation water for a unit is the return flow from the upstream unit. Thus, the evapo-concentration of the irrigation water of unit j determines the salinity of the irrigation water for unit $j + 1$. Assuming a steady state regime, no dissolution/addition/extraction of soil salts and denoting irrigation salinity by CI, for a generic irrigation unit, j, in a system in series, the irrigation water salinity relative to the primary source salinity is:

$$\frac{\mathrm{CI}_{j,s}}{\mathrm{CI}_1} = \left(\frac{1}{1 - \mathrm{ICUC}_u}\right)^{j-1} \tag{5}$$

With the same assumptions, if the units are in parallel the irrigation water salinity at unit j is:

$$\mathrm{CI}_j = [1 - (1 - \mathrm{ICUC}_u)\rho_2]\mathrm{CC}_j + \rho_2\mathrm{CI}_{j-1} \tag{6}$$

where CC_j is the salinity of the water in the source channel.

If $\rho_1 = 0$ in the parallel system, then the ratio $\frac{\mathrm{CI}_{j,p}}{\mathrm{CI}_1}$ becomes independent of CI_1:

$$\frac{\mathrm{CI}_{j,p}}{\mathrm{CI}_1} = \rho_2^{j-1} + \left(\frac{1 - \rho_2^{j-1}}{1 - \rho_2}\right)[1 - (1 - \mathrm{ICUC}_u)\rho_2]. \tag{7}$$

In practice, steady state regime, no dissolution/addition/extraction of soil salts is a major assumption. Therefore, expressions (5), (6) and (7) may be used only for illustrating how irrigation water salinity could evolve with the water reuse and the arrangement of the irrigation units, but not for practical applications.

Soil salinity

The assumptions for calculating soil salinity (CS) were: (1) the salinity of the soil solution is the mean of irrigation and drainage (CD) water salinities, (2) drainage occurs when soil water content is greater than field capacity and (3) soil water content at field capacity is half the soil water content at saturation.

Based on these three assumptions, the soil salinity in a generic irrigation unit, j, in a system in series is:

$$\frac{\mathrm{CS}_{j,s}}{\mathrm{CI}_1} = \frac{2 - \mathrm{ICUC}_u}{4(1 - \mathrm{ICUC}_u)^j} \tag{8}$$

If the system is in parallel,

$$\frac{\mathrm{CS}_{j,p}}{\mathrm{CI}_1} = \left[\rho_2^{j-1} + \left(\frac{1 - \rho_2^{j-1}}{1 - \rho_2}\right)[1 - (1 - \mathrm{ICUC}_u)\rho_2]\right] \times \left[\frac{1}{2} + \frac{\mathrm{ICUC}_u}{4g(1 - \mathrm{ICUC}_u)}\right]. \tag{9}$$

Irrigation water productivity

Irrigation water productivity of the whole irrigation system per unit of water diverted, if the system is in series, is:

$$\mathrm{WP2}_{n,s} = \frac{Y_{\max} \sum_{j=1}^{n} (1 - \mathrm{ICUC}_u)^{j-1} \mathrm{RY}_j}{100 I_u} \quad (10)$$

and, if the system is in parallel:

$$\mathrm{WP2}_{n,p} = \frac{Y_{\max} \sum_{j=1}^{n} \mathrm{RY}_j}{100 I_u [n - (n-1)(1 - \mathrm{ICUC}_u)(\rho_1 + \rho_2)]} \quad (11)$$

where $Y_{\max}$ is the maximum yield achievable in the environment under consideration and RY_j is the yield in irrigation unit j relative to $Y_{\max}$.

Water productivity per unit of irrigation water consumed is:

$$\mathrm{WP3}_{n,s} = \frac{Y_{\max} \sum_{j=1}^{n} (1 - \mathrm{ICUC}_u)^{j-1} \mathrm{RY}_j}{100 I_u \mathrm{ICUC}_n} \quad (12)$$

if the system is in series, and

$$\mathrm{WP3}_{n,p} = \frac{Y_{\max} \sum_{j=1}^{n} \mathrm{RY}_j}{100 I_u [n - (n-1)(1 - \mathrm{ICUC}_u)(\rho_1 + \rho_2)] \mathrm{ICUC}_n} \quad (13)$$

if the system is in parallel.

Finally, water productivity per unit of irrigation water beneficially used is:

$$\mathrm{WP4}_{n,s} = \frac{Y_{\max} \sum_{j=1}^{n} (1 - \mathrm{ICUC}_u)^{j-1} \mathrm{RY}_j}{100 I_u \mathrm{IE}_n} \quad (14)$$

in a system in series, and

$$WP4_{n,p} = \frac{Y_{\max} \sum_{j=1}^{n} \mathrm{RY}_j}{100 I_u [n - (n-1)(1 - \mathrm{ICUC}_u)(\rho_1 + \rho_2)] \mathrm{IE}_n} \quad (15)$$

in a system in parallel.

Relative yield was computed based on the model proposed by Maas and Hoffman (1977) to estimate the yield of crops under soil salinity conditions. This model assumes that yield is not affected by soil salinity below a crop-specific threshold. Above this threshold, relative yield declines with soil salinity linearly.

In reality, Mass and Hoffmans's model is subject to multiple exceptions, transpiration is also influenced by salinity stress, and irrigation uniformity has an effect on the relationship between yield decline and soil salinity. None of these effects were considered in the derivation of the water productivity expressions, therefore the use of Eqs. 10–15 must take into account the practical implications of this limitation.

Illustration of irrigation system performances

The expressions above were used to illustrate the effects on system performance of two types of interventions: the improvement of the unit irrigation performance and the reuse of the return flows, both considered for series and parallel systems. The first type of intervention takes place at unit level, typically through public subsidies or low interest loans that allow upgrading or changing the on-field irrigation systems. The improvement of the unit irrigation system was simulated by an increase of ICUC_u from 0.6 to 0.8 [assuming that this intervention did not affect either the paths or fractions (ρ_1, ρ_2) of water circulation among units]. Regarding the second type of intervention, the reuse of return flows may occur naturally or it may require some kind of construction or earth work. In the case of parallel systems, the reuse of return flows was simulated, firstly, as the circulation of varying fractions of non-consumed water from each irrigation unit to its downstream unit (ρ_2 equal to 0, 0.5 or 1), and, secondly, reincorporating a fraction of the return flow to the main channel ($\rho_1 = 0.5$).

Each scenario was examined with one to eight cycles. The performance of each scenario is presented on the basis of three indicators: IE, irrigation water salinity relative to the initial salinity and WP2. The parameters assumed for the exercise are presented in Table 1. Crop-related values are typical of cotton. For alternative analysis, an electronic spreadsheet containing the equations presented in this paper is available on request.

Irrigation efficiency

Irrigation efficiency of the whole system (IE_n) increased with the number of reuse cycles up to a plateau that was highest for the series system, followed by the parallel system with $\rho_1 = 0$ and $\rho_2 = 1$, and by the parallel system with either ρ_1 or ρ_2 equal to 0.5 (Fig. 3a). Reaching the plateau required a number of reuse cycles that was smaller as the plateau became higher. Obviously, IE_n was independent of the number of cycles in the systems without reuse (one single cycle or $\rho_1 = \rho_2 = 0$).

In the systems without reuse, the improvement of unit performance from $\mathrm{ICUC}_u = 0.6$ to 0.8 increased IE_n in the same magnitude as IE_u. The same improvement of the unit performance had an insignificant effect on IE_n ($\Delta\mathrm{IE}_n$ less than 0.04) when the number of cycles was equal to or greater than three in the series system or equal to or greater than seven in the parallel system with $\rho_1 = 0$ and $\rho_2 = 1$ (Fig. 3b). However, if the reuse fractions ρ_1 or ρ_2 were 0.5, at least 65% of the IE_u increase was transformed into an IE_n increase.

Table 1 Parameter values chosen for the simulation of the selected scenarios

Parameter	Symbol	Unit	Value
Fraction of channel flow derived at the head	k	–	0.1
Flow in channel at the entrance	F_1	$m^3\ s^{-1}$	0.5
Salinity of irrigation water at the entrance to the system	CI_1	$g\ L^{-1}$	1
Fraction of return flow comprising drainage water	g	–	0.5
Soil salinity threshold	CS_{th}	$g\ L^{-1}$	4.9
Slope of the relative yield decline with increasing salinity	B_o	$\%\ g^{-1}\ L$	3.85
Maximum yield	Y_{max}	$kg\ ha^{-1}$	6,000
Irrigation supply at the head irrigation unit	I_1	$m^3\ ha^{-1}$	8,750

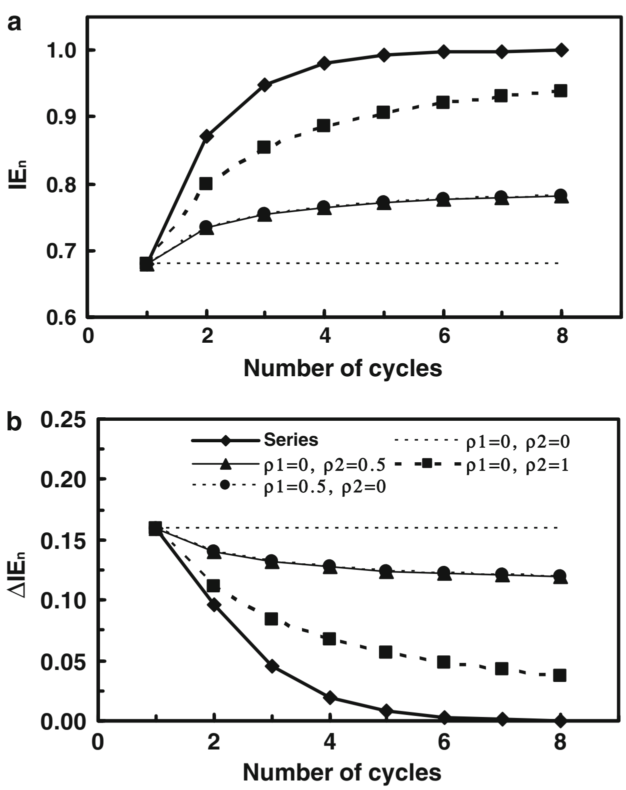

Fig. 3 Irrigation efficiency (IE_n) of the systems with unit irrigation consumptive use coefficient ($ICUC_u$) equal to 0.6 and varying number of reuse cycles (**a**) and increment in IE_n result of the improvement of unit performance from $ICUC_u = 0.6$ to 0.8 (**b**). The different *lines* and *symbols* represent series arrangement and parallel arrangements result of four combinations of reuse fractions: ($\rho_1 = 0,\ \rho_2 = 0$), ($\rho_1 = 0,\ \rho_2 = 0.5$), ($\rho_1 = 0,\ \rho_2 = 1$) or ($\rho_1 = 0.5,\ \rho_2 = 0$)

Irrigation water salinity

Irrigation water salinity relative to the initial water salinity (CI_j/CI_1) increased dramatically with the number of cycles in the series system whereas in the parallel system it was not affected by the number of cycles when ρ_1 and $\rho_2 = 0$ (Fig. 4a). Between these two extremes, CI_j/CI_1 increased linearly when $\rho_2 = 1$ but less markedly with a reuse fraction of 0.5 (either ρ_1 or ρ_2). The degradation of the water quality with reuse fraction 0.5 differed according to the type of reuse, despite the identical IE_n (Fig. 3a). When ρ_1 was the fraction equal to 0.5, the first cycles increased the water salinity at a greater rate than successive cycles. The opposite occurred when ρ_2 was the fraction equal to 0.5.

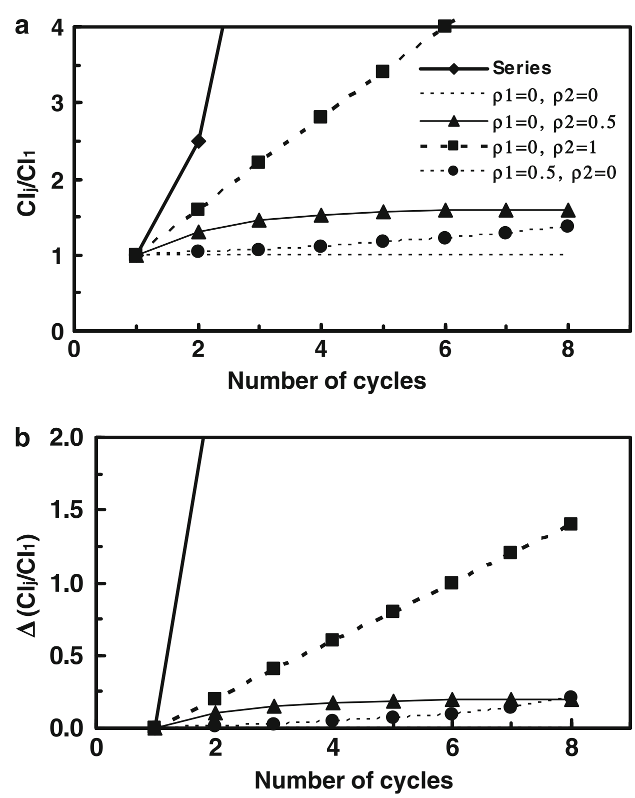

Fig. 4 Salinity of the irrigation water after a number of cycles in relation to the initial salinity of the irrigation water (CI_j/CI_1) of systems with unit irrigation consumptive use coefficient ($ICUC_u$) equal to 0.6 and varying number of reuse cycles (**a**); and increment of (CI_j/CI_1) result of the improvement of unit performance from $ICUC_u = 0.6$ to 0.8 (**b**). The different *lines* and *symbols* represent series arrangement and parallel arrangements result of four combinations of reuse fractions: ($\rho_1 = 0,\ \rho_2 = 0$), ($\rho_1 = 0,\ \rho_2 = 0.5$), ($\rho_1 = 0,\ \rho_2 = 1$) or ($\rho_1 = 0.5,\ \rho_2 = 0$)

The improvement of unit performance from $\mathrm{ICUC}_u = 0.6$ to 0.8 had a negative effect on the quality of the irrigation water (Fig. 4b). This effect was remarkable in the series system; for example, the improvement resulted in an increment of $\mathrm{CI}_j/\mathrm{CI}_1$ $[\Delta(\mathrm{CI}_j/\mathrm{CI}_1)]$ equal to 2.5 when water had been reused two times only. In the parallel system with $\rho_1 = 0$ and $\rho_2 = 1$, this increment was also significant, although not as large as in the series system (0.2 for two reuse cycles). Finally, $\Delta(\mathrm{CI}_j/\mathrm{CI}_1)$ was small but appreciable in the parallel system with either ρ_1 or ρ_2 equal to 0.5 and it was nil when there was no reuse (Fig. 4b).

Irrigation water productivity

If there was no reuse in the system, the improvement of ICUC_u resulted in an increase of the productivity of diverted water (WP2) from 0.51 to 0.69 kg ha^{-1} mm^{-1} ($\Delta\mathrm{WP2} = 0.18$ kg ha^{-1} mm^{-1}) (Fig. 5). For the series system, the impact on WP2 was appreciable only if there were less than three cycles. With a reuse fraction of 0.5, this impact was notable even with a large number of cycles ($\Delta\mathrm{WP2} = 0.13$ kg ha^{-1} mm^{-1}).

Perhaps, the most interesting case was that of the parallel system with $\rho_1 = 0$ and $\rho_2 = 1$. For $\mathrm{ICUC}_u = 0.6$, WP2 always increased with the number of cycles considered (Fig. 5a), whereas for $\mathrm{ICUC}_u = 0.8$ it increased up to three cycles but decreased for more cycles (Fig. 5b). Therefore, after four cycles ΔWP2 became negative (Fig. 5c). The reason for this contrasting behaviour was that the irrigation water salinity when $\mathrm{ICUC}_u = 0.8$ degraded to the point that yield declined after three reuses. However, the leaching fraction resulting from $\mathrm{ICUC}_u = 0.6$ was sufficient to keep the soil salinity below the yield decline threshold up to seven cycles.

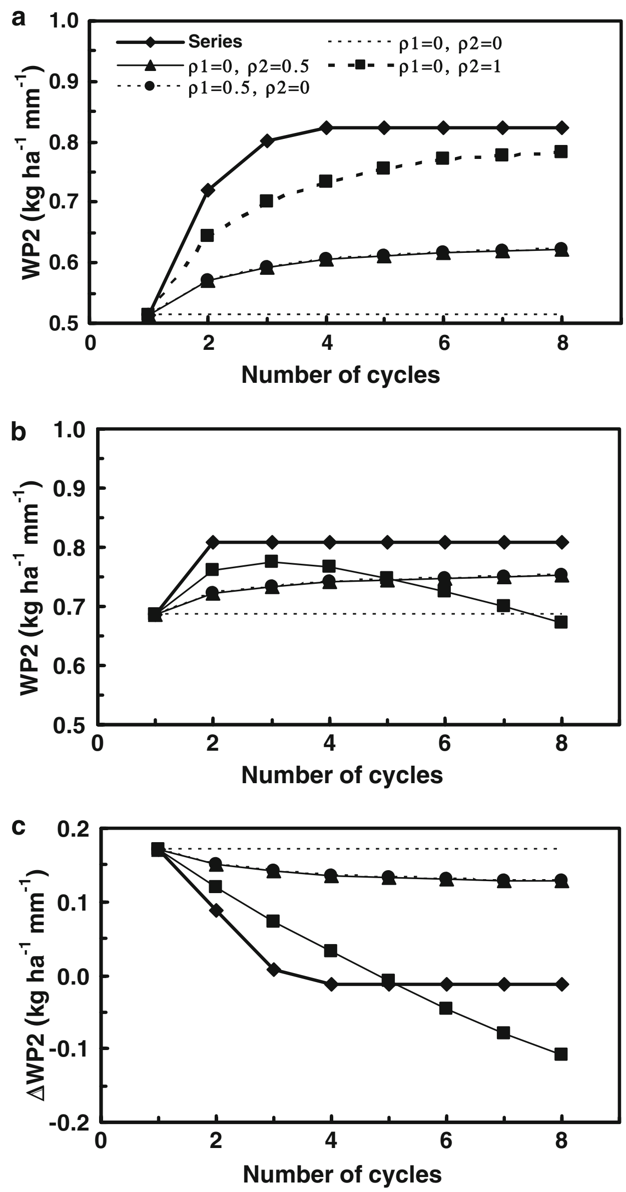

Fig. 5 Water productivity of diverted water (WP2) of systems with varying number of reuse cycles, unit irrigation consumptive use coefficient (ICUC_u) equal to 0.6 (**a**) or 0.8 (**b**), and increment in WP2 result of the improvement of unit performance from $\mathrm{ICUC}_u = 0.6$ to 0.8 (**b**). The different *lines* and *symbols* represent series arrangement and parallel arrangements result of four combinations of reuse fractions: ($\rho_1 = 0$, $\rho_2 = 0$), ($\rho_1 = 0$, $\rho_2 = 0.5$), ($\rho_1 = 0$, $\rho_2 = 1$) or ($\rho_1 = 0.5$, $\rho_2 = 0$)

Conclusions

Despite recent criticism, the concept and definition of classical IE is still valid if properly used in the context of multiple irrigation units. In fact, it remains adequate for analysing the performance of systems composed of multiple irrigation units arranged in different configurations.

The first step for irrigation performance assessment must be a clear definition of the boundaries of the system of interest. Interventions or irrigation practices that improve system performance at the scale of a given domain may have little or no impact on irrigation performance at other scales. For instance, a basin-scale programme aimed at increasing global ICUC or IE should assign funds to increase on-farm ICUC and IE only if ρ_1 and ρ_2 are small or if the number of reuse cycles is small. In contrast, if the reuse is significant, an increase in on-farm ICUC or IE will have little effect on global ICUC or IE.

The expressions developed in this paper for calculating irrigation performance indicators in systems arranged in series and in parallel with a single source and various opportunities for water reuse, constitute a conceptual framework for judging, planning and management policies under the new paradigm of integrated water resources. Decisions about new irrigation developments or about rehabilitation and modernisation of existing irrigation schemes need to be based on this kind of analysis. Moreover, these expressions may be the basis for more complex models applicable to the performance assessment of any kind of irrigation system. Formulae for analysing irrigation schemes fed by a ground water aquifer have not been included in this study but their derivation should not be difficult.

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Appendix

Irrigation consumptive use coefficient

The ICUC is the fraction of the irrigation water (I) destined for consumptive use (CU) (Burt et al. 1997). For the irrigation unit (denoted by subscript u):

$$\mathrm{ICUC}_u = \frac{\mathrm{CU}_u}{I_u} \tag{16}$$

For simplicity, in the successive derivations it will be assumed that I_u, CU_u and therefore ICUC_u are constant for all units in the parallel system and that ICUC_u is constant in the series system.

For a system composed of n (1, 2, …) units (number of units indicated by the first subscript) arranged in series (series denoted by the second subscript, s):

$$\begin{aligned}\mathrm{ICUC}_{n,s} &= \frac{\sum_{i=1}^{n}\mathrm{CU}_i}{I_1} = \frac{I_1\mathrm{ICUC}_u + I_1(1-\mathrm{ICUC}_u)\mathrm{ICUC}_u + \cdots + I_1(1-\mathrm{ICUC}_u)^{n-1}\mathrm{ICUC}_u}{I_1}\\ &= \mathrm{ICUC}_u\frac{1-(1-\mathrm{ICUC}_u)^n}{1-(1-\mathrm{ICUC}_u)} = 1-(1-\mathrm{ICUC}_u)^n\end{aligned} \tag{17}$$

When the units are in parallel (parallel denoted by the second subscript, p), if $n = 1$:

$$\mathrm{ICUC}_{1,p} = \mathrm{ICUC}_u \tag{18}$$

If $n = 2$:

$$\begin{aligned}\mathrm{ICUC}_{2,p} &= \frac{2\mathrm{CU}_u}{I_u + [I_u - (1-\mathrm{ICUC}_u)(\rho_1+\rho_2)I_u]}\\ &= \frac{2I_u\mathrm{ICUC}_u}{2I_u - (1-\mathrm{ICUC}_u)(\rho_1+\rho_2)I_u}\\ &= \frac{2\mathrm{ICUC}_u}{2 - 1(1-\mathrm{ICUC}_u)(\rho_1+\rho_2)}\end{aligned} \tag{19}$$

For n units:

$$\mathrm{ICUC}_n^p = \frac{n\mathrm{ICUC}_u}{n-(n-1)(1-\mathrm{ICUC}_u)(\rho_1+\rho_2)} \tag{20}$$

In the parallel system, the unit irrigation water results from adding water from the main channel to the return flow coming from the upstream irrigation unit (fraction ρ_2), till obtaining an amount of water I_u.

Irrigation efficiency

The IE is the fraction of the irrigation water that is beneficially used (IB) (Burt et al. 1997). For the irrigation unit (irrigation unit denoted by subscript u):

$$\mathrm{IE}_u = \frac{\mathrm{IB}_u}{I_u} \tag{21}$$

For a system composed of n units in series:

$$\begin{aligned}\mathrm{IE}_{n,s} &= \frac{h\sum_{i=1}^{n}\mathrm{CU}_i + I_1(1-\mathrm{ICUC}_u)^n f}{\mathrm{I}_1}\\ &= \frac{hI_1[1-(1-\mathrm{ICUC}_u)^n] + I_1(1-\mathrm{ICUC}_u)^n f}{\mathrm{I}_1}\\ &= h[1-(1-\mathrm{ICUC}_u)^n] + f(1-\mathrm{ICUC}_u)^n\end{aligned} \tag{22}$$

If the units are in parallel and $n = 1$:

$$\mathrm{IE}_{1,p} = \mathrm{IE}_u \tag{23}$$

For $n = 2$:

$$\begin{aligned}\mathrm{IE}_{2,p} &= \frac{I_u\mathrm{IE}_u + \mathrm{I}_u\mathrm{IE}_u - \rho_b I_u(1-\mathrm{ICUC}_u)}{I_u + [I_u - (1-\mathrm{ICUC}_u)(\rho_1+\rho_2)\mathrm{I}_u]}\\ &= \frac{2\mathrm{IE}_u - 1\rho_b(1-\mathrm{ICUC}_u)}{2-1(1-\mathrm{ICUC}_u)(\rho_1+\rho_2)}\end{aligned} \tag{24}$$

For n units in parallel:

$$\mathrm{IE}_{n,p} = \frac{n\mathrm{IE}_u - (n-1)\rho_b(1-\mathrm{ICUC}_u)}{n-(n-1)(1-\mathrm{ICUC}_u)(\rho_1+\rho_2)}. \tag{25}$$

Equations 24 and 25 contain the assumption that if water is considered beneficial once, even if not consumed, it cannot be considered beneficial again. Otherwise, IE could take values greater than unity.

Irrigation water salinity

We assumed a steady state regime and no internal input from soil salts, i.e. the salts leaving an irrigation unit are those that entered it.

In a series system, the irrigation water for a unit is the return flow from the upstream unit. Thus, the evapo-concentration of the irrigation water of unit j determines the salinity of the irrigation water for unit $j+1$. Denoting irrigation salinity by CI, using subscript $(1, 2,\ldots, j,\ldots, n)$ to indicate the order of the irrigation unit in the system layout and a second subscript, s, to denote arrangement in series, for a generic irrigation unit, j:

$$\frac{\mathrm{CI}_{j,s}}{\mathrm{CI}_1} = \left(\frac{1}{1-\mathrm{ICUC}_u}\right)^{j-1}. \tag{26}$$

For calculating the same ratio in a system in parallel (arrangement in parallel denoted by subscript p), first we need to calculate the channel flow (F). If we express I_u as a fraction (k) of the head flow (F_1), then the flow in the channel upstream irrigation unit j is:

$$\begin{aligned} F_j &= F_1 - (j-1)I_u + (j-1)(1-\mathrm{ICUC}_u)\rho_1 I_u \\ &\quad + (j-2)(1-\mathrm{ICUC}_u)\rho_2 I_u \\ &= F_1[1-(j-1)k + (j-1)(1-\mathrm{ICUC}_u)\rho_1 k \\ &\quad + (j-2)(1-\mathrm{ICUC}_u)\rho_2 k]. \end{aligned} \tag{27}$$

The salinity of the water in the channel (CC) at the entrance to irrigation unit j is:

$$\begin{aligned} \mathrm{CC}_j &= \frac{\{F_{j-1} - [I_u - I_u(1-\mathrm{ICUC}_u)\rho_2]\}\mathrm{CC}_{j-1} + I_u(1-\mathrm{ICUC}_u)\rho_1\frac{\mathrm{CI}_{j-1}}{1-\mathrm{ICUC}_u}}{F_j} \\ &= \frac{1}{F_j}\{F_{j-1} - F_1 k[1-(1-\mathrm{ICUC}_u)\rho_2]\}\mathrm{CC}_{j-1} + \frac{1}{F_j}\{F_1 k\rho_1\}\mathrm{CI}_{j-1} \end{aligned} \tag{28}$$

where CI_{j-1} is the salinity of the irrigation water at unit $j-1$. The salinity of the irrigation water at unit j is:

$$\begin{aligned} \mathrm{CI}_j &= \frac{[I_u - I_u(1-\mathrm{ICUC}_u)\rho_2]\mathrm{CC}_j + I_u(1-\mathrm{ICUC}_u)\rho_2\frac{\mathrm{CI}_{j-1}}{1-\mathrm{ICUC}_u}}{I_u} \\ &= [1-(1-\mathrm{ICUC}_u)\rho_2]\mathrm{CC}_j + \rho_2\mathrm{CI}_{j-1} \end{aligned} \tag{29}$$

If $\rho_1 = 0$, then the ratio $\frac{\mathrm{CI}_{j,p}}{\mathrm{CI}_1}$ becomes independent of CI_1 and F_1:

$$\frac{\mathrm{CI}_1}{\mathrm{CI}_1} = 1. \tag{30}$$

$$\begin{aligned} \frac{\mathrm{CI}_{2,p}}{\mathrm{CI}_1} &= \frac{1}{\mathrm{CI}_1}\Bigg[(1-\mathrm{ICUC}_u)\rho_2\frac{\mathrm{CI}_1}{(1-\mathrm{ICUC}_u)} \\ &\qquad + [1-(1-\mathrm{ICUC}_u)\rho_2]\mathrm{CI}_1\Bigg] \\ &= \rho_2 + [1-(1-\mathrm{ICUC}_u)\rho_2]. \end{aligned} \tag{31}$$

$$\begin{aligned} \frac{\mathrm{CI}_{3,p}}{\mathrm{CI}_1} &= \frac{1}{\mathrm{CI}_1}\Bigg[(1-\mathrm{ICUC}_u)\rho_2\frac{\mathrm{CI}_2}{(1-\mathrm{ICUC}_u)} \\ &\quad + [1-(1-\mathrm{ICUC}_u)\rho_2]\mathrm{CI}_1\Bigg] \\ &= \frac{1}{\mathrm{CI}_1}[\rho_2\mathrm{CI}_1\{\rho_2 + [1-(1-\mathrm{ICUC}_u)\rho_2]\} \\ &\quad + [1-(1-\mathrm{ICUC}_u)\rho_2]\mathrm{CI}_1] \\ &= \rho_2^2 + \rho_2[1-(1-\mathrm{ICUC}_u)\rho_2] \\ &\quad + [1-(1-\mathrm{ICUC}_u)\rho_2] = \rho_2^2 \\ &\quad + (\rho_2+1)[1-(1-\mathrm{ICUC}_u)\rho_2]. \end{aligned} \tag{32}$$

$$\begin{aligned} \frac{\mathrm{CI}_{j,p}}{\mathrm{CI}_1} &= \rho_2^{j-1} + \left(\rho_2^{j-2} + \rho_2^{j-3} + \cdots + \rho_2 + 1\right) \\ &\quad \times [1-(1-\mathrm{ICUC}_u)\rho_2] = \rho_2^{j-1} + \left(\frac{1-\rho_2^{j-1}}{1-\rho_2}\right) \\ &\quad \times [1-(1-\mathrm{ICUC}_u)\rho_2]. \end{aligned} \tag{33}$$

Soil salinity

The assumptions for calculating soil salinity (CS) are: (1) the salinity of the soil solution is the mean of the irrigation and drainage (CD) water salinities, (2) drainage occurs when soil water content is greater than field capacity and (3) soil water content at field capacity is half the soil water content at saturation.

Then, for a system in series:

$$\begin{aligned} \frac{\mathrm{CS}_{j,s}}{\mathrm{CI}_1} &= \frac{1}{2}\frac{1}{2}\left[\left(\frac{1}{1-\mathrm{ICUC}_u}\right)^{j-1} + \left(\frac{1}{1-\mathrm{ICUC}_u}\right)^{j}\right] \\ &= \frac{2-\mathrm{ICUC}_u}{4(1-\mathrm{ICUC}_u)^j}. \end{aligned} \tag{34}$$

If the system is in parallel, first, we define the 'leaching fraction' (LF), assuming a steady state regime, as:

$$\mathrm{LF} = \frac{g(1-\mathrm{ICUC}_u)}{\mathrm{ICUC}_u + g(1-\mathrm{ICUC}_u)} = \frac{\mathrm{CI}_{j,p}}{\mathrm{CD}_{j,p}} \quad (35)$$

where g is the fraction of non-consumed water that goes to percolation. Therefore,

$$\frac{\mathrm{CS}_{j,p}}{\mathrm{CI}_{j,p}} = \frac{1}{\mathrm{CI}_{j,p}}\frac{1}{2}\left(\frac{\mathrm{CI}_{j,p}+\mathrm{CI}_{j,p}/\mathrm{LF}}{2}\right) = \frac{\mathrm{LF}+1}{4\mathrm{LF}} = \frac{2g(1-\mathrm{ICUC}_u)+\mathrm{ICUC}_u}{4g(1-\mathrm{ICUC}_u)} = \frac{1}{2} + \frac{\mathrm{ICUC}_u}{4g(1-\mathrm{ICUC}_u)}. \quad (36)$$

Finally, we can relate soil salinity to the irrigation water salinity at the entrance of the system by multiplying numerator and denominator by CI_1 and by making use of the unit's irrigation water salinity relative to the initial irrigation water salinity derived in the previous section:

$$\frac{\mathrm{CS}_{j,p}}{\mathrm{CI}_1} = \frac{\mathrm{CI}_{j,p}}{\mathrm{CI}_1}\left[\frac{1}{2} + \frac{\mathrm{ICUC}_u}{4g(1-\mathrm{ICUC}_u)}\right] = \left[\rho_2^{j-1} + \left(\frac{1-\rho_2^{j-1}}{1-\rho_2}\right)[1-(1-\mathrm{ICUC}_u)\rho_2]\right] \times \left[\frac{1}{2} + \frac{\mathrm{ICUC}_u}{4g(1-\mathrm{ICUC}_u)}\right]. \quad (37)$$

Relative yield

Maas and Hoffman (1977) proposed a model to estimate relative yield (RY, %) of crops limited by soil salinity. For an irrigation unit j:

$$\mathrm{RY}_j = 100, \text{ if } \mathrm{CS}_j \le \mathrm{CS}_{\mathrm{th}} \quad (38)$$

$$\mathrm{RY}_j = \max\left[100 - B_{\mathrm{o}}(\mathrm{CS}_j - \mathrm{CS}_{\mathrm{th}}); 0\right], \text{ if } \mathrm{CS}_j > \mathrm{CS}_{\mathrm{th}} \quad (39)$$

where $\mathrm{CS}_{\mathrm{th}}$ is the threshold of soil salinity above which yield is reduced and B_{o} is the decrease in relative yield per unit of increase in soil salinity above the threshold. CS_j may be calculated from Eq. 34 or 37 if CI_1 is known.

Irrigation water productivity

Here we consider three definitions of irrigation water productivity: production per unit of water diverted (WP2), production per unit of irrigation water consumed (WP3) and production per unit of irrigation water beneficially used (WP4):

$$\mathrm{WP2}_{n,s} = \frac{Y_{\max}\sum_{j=1}^{n}(1-\mathrm{ICUC}_u)^{j-1}\mathrm{RY}_j}{100 I_u} \quad (40)$$

$$\mathrm{WP2}_{n,p} = \frac{Y_{\max}\sum_{j=1}^{n}\mathrm{RY}_j}{100 I_u[n-(n-1)(1-\mathrm{ICUC}_u)(\rho_1+\rho_2)]} \quad (41)$$

$$\mathrm{WP3}_{n,s} = \frac{Y_{\max}\sum_{j=1}^{n}(1-\mathrm{ICUC}_u)^{j-1}\mathrm{RY}_j}{100 I_u \mathrm{ICUC}_n} \quad (42)$$

$$\mathrm{WP3}_{n,p} = \frac{Y_{\max}\sum_{j=1}^{n}\mathrm{RY}_j}{100 I_u[n-(n-1)(1-\mathrm{ICUC}_u)(\rho_1+\rho_2)]\mathrm{ICUC}_n} \quad (43)$$

$$\mathrm{WP4}_{n,s} = \frac{Y_{\max}\sum_{j=1}^{n}(1-\mathrm{ICUC}_u)^{j-1}\mathrm{RY}_j}{100 I_u \mathrm{IE}_n} \quad (44)$$

$$\mathrm{WP4}_{n,p} = \frac{Y_{\max}\sum_{j=1}^{n}\mathrm{RY}_j}{100 I_u[n-(n-1)(1-\mathrm{ICUC}_u)(\rho_1+\rho_2)]\mathrm{IE}_n} \quad (45)$$

where $Y_{\max}$ is the maximum yield achievable in the environment under consideration.

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