



## How to escape from the trap of the warranted rate of growth

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# **How to escape from the trap of the warranted rate of growth**

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## **Abstract**

Post-Keynesian models consider growth to be demand-led – a logical consequence of Keynes's principle of effective demand. After Harrod (1939) they try to unearth the hidden variables that might allow the adaptation of the warranted rate, determined from the supply side, to autonomous demand-growth expectations. The purpose of this paper is to show that an investment function based on a flexible accelerator and integrated in a supermultiplier is able to shape the warranted rate in consonance with the autonomous trend. Hopefully it will build bridges with other postKeynesian strands that have so far dismissed the supermultiplier solution.

*Key words:* Growth, multiplier-accelerator models, postKeynesian economics, Kaleckian economics, Sraffian economics.

JEL classification: E11, E12, E22

## 1. Introduction

After *The General Theory*, Keynes' disciples tried to dynamize the principle of effective demand in order to prove that, in capitalist economies, growth is demand-led (Keynes, 1936). The actual rate of growth is supposed to adjust to the expected rate of growth of aggregate demand. But, are there no supply restrictions? And, what happens if the autonomous trend fails to meet such restrictions?

(Harrod, 1939) shows the existence of a potential rate of growth determined by technology and saving propensity:  $g^* = s/k^*$ , where  $k^*$  is the optimal capital/output ratio and  $s$  is the average propensity to save. Both parameters are supposed to evolve smoothly. Harrod labels this rate "warranted" because businessmen can be sure of selling their capacity output, year after year, if the expected demand growth ( $g_d$ ) coincides with the potential or "warranted" one ( $g^*$ ). He warns, however, that his multiplier-accelerator model is extremely unstable. A small deviation of  $g_d$  from  $g^*$  can make the system explode or implode –the genesis of the "knife edge" image which is associated with Harrod's model.

Keynes' disciples in Cambridge (the first post-Keynesian group) find the solution on the distribution side (Kaldor, 1955-56, 1957, 1961; Robinson, 1956, 1962). A higher rate of (Robinson, 1962) growth can be approached by re-distributing income from wages to profits, to which a higher propensity to save is associated. This is the first hidden link in Harrod's growth theory. In recent time, some Marxian economists have been exploring new distributive variables suitable for this purpose (Duménil & Lévy, 1999; Skott, 2010).

(Skott, 2010) Kaleckian economists point to the rate of capacity utilization that is implicit in the current capital/output ratio:  $k_t = k^*/u_t$ . This idea, already present in the first studies by (Kalecki, 1971), has been emphasized by the neoKaleckian economists ((Lavoie, 2010), (Hein, Lavoie, & Treek, 2011), (Lavoie, 2013b)). They contend that the rate of capacity is the hidden link that allows the supply forces to accommodate to any change in demand growth, both in the short and the long period. Only recently neoKaleckians have recognized the possibility of considering normal capacity utilization as a long-period equilibrium condition ((Allain, 2013) (Lavoie, 2013a)).

Sraffian economists could not be indifferent to the debate, given their interest in long-period theories that refer to normal capacity utilization. (Cesaratto, 2012) distinguishes between core and periphery Sraffians. The first group, based in Rome ((Garegnani, 1992); (Palumbo & Trezzini, 2003)) has given up its attempt to build a long-period theory of output on the same footing as the Sraffian theory of value and distribution (Sraffa, 1960). In the second group, Cesaratto identifies three authors that use a multiplier-accelerator model ("supermultiplier", for short): ((Serrano, 1995); (Bortis, 1997); (Dejuán, 2005))<sup>1</sup>. For them, the warranted rate becomes endogenous; it adapts to the autonomous trend via changes in output composition. The  $s$  in the numerator of the warranted rate would be replaced by  $i_t = (1 - c - z_t)$ , where  $i_t$  and  $z_t$  are, respectively, the shares in output of expansionary investment and (proper) autonomous demand. Output composition would be the hidden link in growth theory that we shall lay bare in due course.

The goals of this paper are: (1) To advance in the understanding of the supermultiplier model. For this purpose we present an open economy driven by exports and simulate an increase in the rate of growth of exports ("the autonomous trend", so to speak)<sup>2</sup>. (2) To reinforce the hypothesis of "demand-led growth" by showing the stability of the supermultiplier solution (*Section 2*). (3) To clarify the differences with other postKeynesian solutions and build appropriate bridges among them (*Section 3*).

## **2. The supermultiplier solution to demand-led growth.**

### **2.1. Hypothesis of a Classical-Keynesian model**

The economy we are going to analyze has been growing until the base year along its fully adjusted path of growth. This means that all the relevant variables are growing at

<sup>1</sup> (Hicks, 1950) was the first to use the supermultiplier, but in a context alien to our current concerns.

<sup>2</sup> The driver of the economy in a supermultiplier model can be capitalist consumption (Serrano, 1995), modernization investment that transforms capacity without increasing it (Dejuán, 2005), residential investment (Dejuán, 2013), real public expenditure (Allain, 2013) or any combination of these variables plus exports (Dejuán, 2013b) (N. d. Freitas & Dweck, 2013). To simplify the exposition all our comments we refer to an upward shift of the autonomous trend. *Mutatis mutandis*, the reader can apply the solutions to a downward shift.

the potential or warranted rate, the one that results from investing, year after year, the savings corresponding to capacity income, after subtracting the part that finances exports and other elements of autonomous demand.

Our economy consists of three vertically integrated sectors that produce three different commodities for final demand (plus the intermediate goods required in each production process): (1) Consumption goods purchased by households; (2) Investment goods purchased by firms willing to expand capacity; (3) Goods for exports.

To avoid the problems derived from a change in the composition of output, we assume that the three industries share the same technology reflected in the optimal labor coefficient ( $l^*$ ) and the optimal capital coefficient ( $k^*$ ). Technology is assumed to be constant in the period under analysis. Also distribution, here represented by the real wage ( $w$ ) and the normal rate of profit ( $r^*$ ) corresponding to the exogenous technology and the real wage.

To adjust production to demand changes, installed capacity (fixed capital) can be operated more or less hours a day. The ratio between the actual numbers of hours per day and the optimal number measures the actual rate of capacity utilization ( $u_t$ ). There are obvious limits. For sure, equipment cannot be used more than 24 hours a day. Engineers will set the technical limit some hours below ( $u^{\wedge}$ ). Economist will refer to the normal or optimal threshold ( $u^*$  that we normalize at 1). It is associated to the maximum rate of profit free from the risk of losing customers when capacity is at  $u^{\wedge}$  and firms experience a new burst of demand.  $u^*$  plays the role of a gravity center because firms are interested in reverting to it to maximize profits in the long run (Kurz, 1986)<sup>3</sup>.

According to the Keynesian–Kaleckian principle of effective demand, the equilibrium level of output in a given period ( $Y_t$ ) is supposed to adjust to the expected aggregate

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<sup>3</sup> Note the difference in the treatment of labour and capital. We distinguish between the normal “capital / output” ratio ( $k^*$ ) and actual one: ( $k_t = k^* \cdot u_t$ ). After a shock, firms try to recover normal capacity in order to maximize long term profits. On the contrary, we cannot speak of a natural or normal rate of employment at which the economy converges. In a Keynesian model, full employment is not an equilibrium condition either in the short or in the long period.

demand ( $D_t$ ) that is made up of induced consumption ( $C_t$ ), expansionary investment ( $I_t$ ) and autonomous demand ( $Z_t = X_t - M_t$ , where  $X$  stands for exports and  $M$  for imports)

$$[1] \quad Y_t = D_t = C_t + I_t + Z_t$$

*Induced final consumption* is a proportion of the disposable income of households. If, for simplicity, we assume the extreme classical hypothesis (wages, and only wages, are consumed) the aggregate propensity to consume ( $c$ ) can be identified with the share of wages in income ( $\omega = w \cdot l$ ). The link between demand and distribution becomes apparent in equation [3].

$$[2] \quad C_t = c \cdot Y_t$$

$$[3] \quad c = \omega = w \cdot l$$

*Autonomous demand* includes expenditure that is independent of current income and does not increase productive capacity. In this paper we have identified it with exports. We take as given the level of exports at the initial period ( $X_o$ ), and its expected rate of growth (the *autonomous trend*,  $g_x$ ).

$$[4] \quad X_t = X_o (1 + g_x)^t$$

To obtain the macroeconomic equilibrium we should subtract imports, i.e. the portion of consumption and capital goods purchased in the rest of the world. The import propensity ( $m$ ) is considered a fixed share of current income.

$$[5] \quad M_t = m \cdot Y_t$$

*Expansionary investment* is the corner stone of any growth model and deserves special attention. Firms try to increase their productive capacity in order to match, in the most efficient way, the expected permanent increases in demand. This is the acceleration principle whose main ingredients are the optimal capital/output ratio ( $k^*$ ) and the expected growth of demand ( $g_d$ ).

Long-term demand expectations ( $g_d$ ) is the true independent variable of the Keynesian system (John Eatwell, 1983). Keynes emphasized the psychological factors influencing entrepreneurs' growth expectations. However, since the new productive capacity is

supposed to last many years (even decades), firms are bound to look also at the objective factors encapsulated in the autonomous trend ( $g_x$ ) that flow as a constant stream of demand. The Schumpeterian treatment of long waves of prosperity and depression has a say here (Schumpeter, 1912/1934). If  $g_x > g_d$  firms will be compelled to overuse capacity first; then to accelerate investment to recover the normal position. If overutilization persists despite the expansion of capacity entrepreneurs will be invited to revise demand expectations upwards. In this way the expected growth of demand adjusts to the autonomous trend as (Allain, 2013) has aptly shown.

The accelerator computes expansionary investment by the difference between the required capital in the next and following periods ( $KR_{t+1}$ ), and the current installed capital at period  $t$  ( $KI_t$ ). It is a "prospective and flexible accelerator" in the sense that it is forward looking but does not forget past disequilibria in order to clear them out. Expansionary investment may be computed in the following way<sup>4</sup>:

$$[6] \quad I_t = KR_{t+1} - KI_t = k^* \cdot g_d \cdot D_t + KJ_t$$

$KJ_t = KR_t - KI_t$  are the shortages of capacity ("excesses", if negative). If the economy is advancing along its fully adjusted path of growth,  $KJ_t=0$  and  $D_t=Y_t$ . Then we write the pure accelerator:

$$[7] \quad I_t = k^* \cdot g^* \cdot Y_t$$

Here  $g^*$  stands for the warranted rate of growth at which all the relevant variables are supposed to grow along a fully adjusted path of growth. Dividing by income we get the equilibrium share of investment corresponding to the warranted, potential or steady path of growth  $g^*$ :

$$[8] \quad i^* = k^* \cdot g^*$$

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<sup>4</sup> The intermediate steps are:  $I_t = k^* \cdot D_t \cdot (1+g_d) - KI_t = k^* \cdot g_d \cdot D_t + [k^* \cdot D_t - KI_t]$ .

To describe the performance of actual economies (at the “unit level”) we should replace the warranted rate of growth by the expected growth of demand and add the investment necessary to adjust capacity to the new path of growth<sup>5</sup>:

$$[9] \quad i'_t = k^* \cdot g_{d,t} + u''_t$$

$u''=u_t-1$  stands for the deviations of the utilization rate from this normal level that had been set at 1.

## 2.2. Long period equilibrium and the traverse towards a new autonomous trend.

Let us start by analyzing an economy where all variables grow at the warranted rate. The equilibrium level of output in  $t$  will be given by the following expression that deploys [1].

$$[10a] \quad Y_t = c \cdot Y_t + k^* \cdot g^* \cdot Y_t + (X_t - m \cdot Y_t)$$

$$[10b] \quad Y_t = \frac{1}{1-c+m-k^* \cdot g^*} \cdot X_o (1 + g_x)^t = \mu^* \cdot X_t$$

$\mu^*$  is the supermultiplier that accounts for the dragging effects on consumption goods ( $c-m$ ) and on equipment ( $k^* \cdot g^*$ ).

If the autonomous trend is truly autonomous, we can expect a change from time to time. What will happen then? Let us imagine a twin economy where the autonomous trend shifts to  $g'_x$  higher than  $g_x$ . (Variables with a dash will refer to the second economy whose autonomous trend has risen).

In our first approximation we shall assume that the change is known by businessmen who import the extra equipment ( $KJ_t$ ) required to enter into the new growth path. Probably a part of it will not be imported but taken from accumulated inventories. And only a part of them will be reproduced in the next period. The extra equipment produced will be, therefore,  $(a \cdot KJ_t)$  where  $0 < a < 1$ . The production process will generate the typical multiplier effects on consumption. Not acceleration effects on investment since this is a transient demand (a once and for all adjustment).

<sup>5</sup> [6] is divided by current income. Remember that  $KJ_t = KR_t - KI_t$ . Dividing by  $Y_t$  we get:  $j_t = k^* - k_t$ . And dividing by  $k_t$  we get:  $j_t = u_t - 1 = u''$ .

As a second (more realistic) approximation, let us consider that firms adapt their demand expectations little by little looking at the orders they receive at a rhythm  $g'_x$  and at the disequilibria in capacity utilization that do not disappear despite the increases in capacity.

After the acceleration of the autonomous trend, the level of output can be computed by the following expression where  $\mu'^*$  is the supermultiplier corresponding to  $g'_{dt}$ ,  $\mu$  is the traditional income multiplier, and  $0 < a < 1$  is a parameter that informs about the extra capacity obtained out of inventories that needs to be reproduced next year.

$$[11] \quad Y'_t = \left\{ \frac{1}{1-c+m-k^*.g'_{dt}} \cdot X_o (1 + g'_x)^t \right\} + \left[ \frac{1}{1-c+m} (a \cdot KJ_{(t-1)}) \right] \\ = \{\mu'^* \cdot X_t\} + [\mu \cdot (a \cdot KT_{(t-1)})]$$

The economy is out of its long period dynamic equilibrium but adjusting to it. When  $g_x > g_d$ , firms are forced to overuse capacity in order to avoid losing customers. Overutilization has two effects that reinforce each other. (1) It increases investment above the level suggested by the pure acceleration mechanism. The investment share of output will increase at the expense of the autonomous share ( $(i > I; z' < z)$ ). (2) If overutilization persists despite the increase in capacity, firms are invited to revise upwards their growth expectations until  $g_d = g'_x$ .

When the extra-capacity has been installed and demand expectations coincide with the new autonomous trend the economy will resume the new trend of growth that can be tracked entirely by the supermultiplier. The right-hand term of the previous expressions disappears since  $KT=0$ . After  $n$  periods output can be computed by the following equation.

$$[12] \quad Y_{(t+n)} = \frac{1}{1-c+m-k^*.g^*} \cdot X_{(t+n)} = \mu^{**} \cdot X_{(t+n)}$$

Notice that in the new equilibrium, the following variables keep constant: technology ( $k^*$  and  $u^*$ ), distribution ( $w$  and  $r^*$ ,  $\omega$  and  $\beta$ ) and expenditure patterns ( $c_w=1$ ,  $c=\omega$ ,  $m$ ). On the contrary, the dash on  $x$ ,  $i$ ,  $g^*$  and  $\mu$  reminds us about the dependence of these variables on the autonomous trend.

### 2.3. Stability conditions and limits to the autonomous trend.

Long-period equilibrium requires that the autonomous trend coincides with the warranted or potential rate of growth. The last one results from investing the part of capacity income ( $Y$ ) that it is not devoted to final consumption or lent to foreign importers (our exports). After dividing such expenditure by the capital installed and expressing the numerator and the denominator at the unit level we get:

$$[13] \quad g^* = \frac{1-(c-m)-x'}{k^*} = \frac{i'}{k^*}$$

The warranted rate is also called “steady state” because it requires output, aggregate demand, autonomous demand and the remaining economic variables to grow at the same rhythm:

$$[14] \quad g_y = g_d = g_z = g^*$$

We arrive at the most troublesome question. If long period equilibrium requires the autonomous trend ( $g_x$ ) to equal the warranted one ( $g^*$ ), how can we defend its autonomy? The supermultiplier model concludes that it is  $g^*$  which adapts to  $g_x$ . The warranted rate would be, so to speak, *endogenous*. It also specifies the mechanisms that make possible the adjustment of  $i'$  and  $x$  in a competitive market economy.

The instability of many multiplier-associated models derives from two sources. (1) The introduction of an autonomous trend higher than the maximum rate derived from technology and expenditure patterns. (2) The overreaction of investors to disequilibria in the rate of capacity utilization.

The stability of the system becomes more evident after splitting the economy into the “normal system” (the left-hand side term in the equations of [11]) and the “complementary system” (the right-hand side term within square brackets). The main system grows at the expected rate of demand ( $g_{dt}$ , embedded in the supermultiplier). If  $g_{dt}=g'_x$  capacity is operated at the normal rate. If  $g_{dt}<g_x$  firms are bound to overuse capacity (for a better visualization, think in night hours). This accelerates investment and raises demand expectations. The main system is stable provided the autonomous trend lies below the maximum rate that we shall analyze in a moment. The

complementary system can be associated to the operation of equipment at weekends until the extra capacity is built. The production of these machines involves multiplier effects on induced consumption; not acceleration effects on investment. As soon as the extra capacity ( $KJ_t$ ) is produced there is no need to operate capacity at weekends. No problems of instability can arise in this system governed by the multiplier.

The autonomy of  $g_z$  does not preclude the existence of technological and economic limits. Any economic model should respect them. Looking at the Keynesian multiplier ( $\mu=1/(1-c+m)$ ) we conclude that to obtain an economically-meaningful result (positive income) the aggregate propensity to consume domestic goods has to be lower than unity. In other words:  $(1-c+m)>0$ . Looking at the supermultiplier we conclude that a positive value requires that  $(1-c+m-k^* \cdot g_x)>0$ . The maximum autonomous trend compatible with the model becomes:

$$[15] \quad \hat{g}_x < \frac{1-c+m}{k^*}$$

As is apparent in the previous equation, the possibility to import the missing equipment raises the limit of the autonomous trend. But this is a temporary solution. Eventually the balance of payments constraint will check expansion ((McCombie, 1994) (Blecker, 2010)). International banks will deny credit to countries whose current deficit surpasses a given threshold of "creditworthiness". By and large, the external threshold will lie below the supply-side limit. Suppose that foreign trade was balanced in the base period ( $t=0$ ). This implies that the share of exports in income equals the propensity to consume:  $x=m$ . In this paper  $m$  is an exogenous parameter. It depends basically on technology, distribution and international prices in a given currency that we have considered constant. The export share turns out to be the inverse of the supermultiplier:

$$[16] \quad x = \frac{x_t}{Y_t} = \frac{x_0(1+g_x)^t}{\mu^* \cdot x_0(1+g_x)^t} = \frac{1}{\mu^*} = 1 - c + m - k^* \cdot g_x$$

After subtracting the import propensity we get the balance of trade in relative terms:

$$[17] \quad bt = x - m = 1 - c - k^* \cdot g_x$$

The result looks paradoxical. The acceleration in the growth of exports raises the trade deficit (as a percentage of income). We know the cause. Indeed, exports are increasing at a higher rate. But their share in income falls because during the traverse investment and income grew faster.

### 3. Controversies and bridges.

#### 3.1 *Surprising rejection of the accelerator by Keynes and Kalecki*

At first sight, the accelerator appears to be the perfect companion of the multiplier. Both of them respond to the principle of effective demand and help understand the dynamics of modern capitalism, where quantity adjustments are embedded in mass production systems ((Nell, 1998), chapters 10-11). Why did Keynes and Kalecki reject the acceleration principle? This is the first enigma we want to clarify.

In a letter to Harrod, dated 12 April 1937, Keynes argued that the accelerator ('the relation' as it was called at that time) was too mechanical, leaving no room for entrepreneurial expectations:

"So far, we have excluded the possibility of changes in expectations. In fact, however, the rate of investment does not depend on current consumption, but on expectations (though the latter are, of course influenced, perhaps unduly, by current consumption). Thus, unless expectations are of a constant character, one would anticipate short-period changes in the *relation*" ((Keynes, 1973) v. 14, p. 172)).

In 1939 Kalecki considered that the accelerator was too rigid; incompatible with adjustments in the degree of capacity utilization that it is the usual lever of capitalist firms:

"The argument is apparently based on the unrealistic assumption that the degree of use of equipment is constant while it is clear from trade cycle statistics that it is precisely the fluctuation in the use of equipment which accounts chiefly for changes in output, and the proportionate increase or decrease of equipment is of minor importance" (Kalecki, 1971, p. 65).

Such criticisms may be justified in relation to the traditional ‘retrospective accelerator’ (Samuelson, 1939). In the ‘prospective and flexible accelerator’ presented here, entrepreneurial expectations play a key role. Keynes’s ‘animal spirits’ have been materialized: they refer to the expected rate of growth of the markets. On the other hand, our accelerator is flexible enough to allow adjustments via inventories and capacity utilization. As a matter of fact, these are the short term mechanisms of adjustment after a shock.

### **3.2 Harrod’s instability puzzles**

Harrod used a multiplier-accelerator model to explore the long-term dynamics of a capitalist system. Unfortunately, his starting point was not the best one. His aggregate demand consists only of induced consumption and expansionary investment:  $Y = C + I = c \cdot Y + k^* \cdot g^* \cdot Y$ . From here he derives the warranted rate:  $g^* = (1-c)/k^* = s/k^*$ . This rate depends only on “natural” forces (technology and psychological propensities) that are supposed to evolve smoothly and usually in the same direction. None of these variables are suitable for adjusting to a new autonomous trend.

In this paper we have showed that in a general model, with proper autonomous demand, the structure of demand is the hidden variable to achieve such an adjustment. After a permanent rise of the autonomous trend, the share of investment in income will rise at the expenses of the share of autonomous demand.

What about the extreme instability of Harrod’s model? In our opinion it is not caused by the interaction between the multiplier and accelerator. It rather results from the bizarre reaction function foisted on the entrepreneurs. Whenever capacity is overutilized, Harroddian entrepreneurs raise growth expectations. In our analysis, overutilization simply calls for a supplement to investment over the level decided by the pure acceleration mechanism. If the extra capacity is not enough to return to the normal rate, firms will be invited to revise upwards their demand expectations ( $g_{dt}$ ). After Allain (2013), we may accept that  $g_x$  influences  $g_d$ . But we emphasize that the autonomy of  $g_x$  should be always preserved.

### **3.3 Endogenous distribution: Arguments by old post-Keynesians and modern Marxians**

The first group of post-Keynesian economists ascertain that expenditure patterns to consume and save depend on income distribution ((Kaldor, 1955-56); (Kaldor, 1961); (Robinson, 1956), (Robinson, 1962)). Even if the propensity to consume (or save) out of wages and profits is constant, the aggregate propensity will decrease (or increase) if there is a transfer of income from wages to profits. In their opinion, this is the lever of a demand-led growth previously hidden that the "Cambridge equation" helped to uncover.

The problem with such a lever is that it may move in the wrong direction. For the new (higher) autonomous trend to be sustainable, it requires the transfer of income from workers to capitalists or rentiers whose propensities to save are higher. But, is it plausible to expect a fall in wages during a boom period? In principle, we should expect the opposite.

Nowadays, some Marxian economists have explored new distributive variables able to lead a demand-led growth in a given technological context. Skott points to the profit margin that is expected to rise in boom periods when capacity is overutilized (Skott, 2010). Even if nominal wages do also increase, the rise in prices is supposed to favour capitalists. They would be able to increase the internal savings required for expansionary investment.

In our opinion, the outcome envisaged by Skott is possible and plausible in the short run, not in the long run. In our model we have allowed for a higher actual profit rate ( $r_t$ ) associated with the overuse of capacity. A rise in prices may reinforce this outcome. But when capacity returns to the normal level we can expect that the distributive variables will recover their initial values. It is difficult to imagine that in the new path of growth, with higher employment rates, workers accepting a permanent fall in wages<sup>6</sup>.

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<sup>6</sup> Dumenil and Levy (1999) point at a different distributive variable: the interest rate. To check inflation, central banks are supposed to raise the official rate boom periods. This implies a redistribution

### **3.4 Endogenous capacity utilization in neo-Kaleckian models**

Nowadays, the main defense for demand-led growth comes from Kaleckian economists ((Lavoie, 2010); (Hein, Lavoie, & Treeck, 2011), (Lavoie, 2013b)). In their opinion, the rate of capacity utilization is the variable that allows the sustainability of any autonomous path of growth both in the short and in the long run. It is also able to restate, in the long period, the validity of the Kaleckian paradox of cost and the Keynesian paradox of thrift.

Hein et al. (2011) make accumulation depend on the expected growth of demand and the actual deviation from the normal rate of capacity utilization (as we did in [6] and [9]). Surprisingly, despite the acceleration of investment whenever  $u''_t > 0$ , capacity never returns to its normal level. In our opinion they do not take the consequences of their own premises seriously. This explains why most Kaleckians relate investment to the actual level of capacity ( $u_t$ ) instead of to the deviations from the normal rate of capacity ( $u''_t$ ) that, in principle, should imply a convergence towards normality.

In our model the over-acceleration of investment when  $u''_t > 1$  brings about a rise in the investment share ( $i_t$ ) that raises the expected growth of demand and the actual growth of output until they coincide with the new autonomous trend.

A permanent overutilization could only be justified in the absence of autonomous demand. This is more a text-book exercise than a description of the real world, but let us explore it within our own model (a numerical illustration appears in Dejuan, 2005). Imagine an economy where there is only induced consumption and expansionary investment. Demand expectations grow at the warranted rate, so capacity is used at the normal rate. Suddenly the ‘animal spirits’ of entrepreneurs become buoyant and their growth expectations rise. Firms overuse capacity and then raise investment above the level required by the pure accelerator principle. They don’t manage to

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of income towards rentiers whose propensity to save is higher. In our opinion the solution is not appropriate because it depends on a policy variable that, in principle, should be treated as autonomous.

recover normal utilization but neither is there an explosion of the model as in Harrod. The rate of utilization will eventually converge to the value given by the ratio between the expected growth of demand that has risen to  $g'_d$  and the warranted rate that now coincides with Harrod's one ( $s/k^*$ )<sup>7</sup>:  $u^\# = g'_d/g^*$

In defense of Kaleckian economists we should recognize that most of them are engaged in applied economics. In this area there is no need to pre-suppose normal capacity utilization because their determinants are continuously changing. It is when they derive general conclusions (for instance, the long period validity of the paradox of cost) that they should pay attention to the gravity centres of economic variables suggested by Sraffian economists.

A bridge between the supermultiplier supporters and Kaleckians has been recently built by Allain (2013) and given its blessing by Lavoie (2013a). They recognize that the existence of non-capacity-creating autonomous demand provides a new way of adjustment. They prove mathematically the convergence towards the final equilibrium where the rate of growth of all variables (including savings and investment) coincides. "Of course, since it depends on the parameters, this solution to the Harrod knife-edge problem remains fragile. But it opens a door that has never been opened before" (Allain, 2013).

In this paper we have relied on Allain's hypothesis. It strengthens the stability of a model that eventually depends on the persistence of the flow in demand of an autonomous sector growing at  $g_x$ . Here we have contended that a flexible accelerator grants the convergence towards the new autonomous trend provided the extra investment coincides with the shortages of capacity ( $KJ_t$ ) and only a part of it ( $a \cdot KJ_t$ , being  $0 < a < 1$ ) is reproduced. We have also emphasized the limits of capacity overutilization. Equipment cannot be operated more than 24 hours a day!

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<sup>7</sup> In a model without proper autonomous demand, the actual rate of growth of output can be computed by  $g_y = g_d = s/k_t = s \cdot (u_t/k^*)$ . Clearing for the utilization rate we get:  $u_t = g_d \cdot (k^*/s) = g_d/g^*$ .

### **3.5 Sraffian controversies on the long period theory of output**

In the initial agenda of Sraffian economics was the project to build a long-period theory of output based on the principle of effective demand and compatible with the Classical long-period theory of value and distribution. This is the surplus approach whose major defense can be found in (Garegnani, 1978-79) and (John Eatwell & Milgate, 1983) (see the *Introduction* and *Conclusions* of the book).

Surprisingly, the bulk of Sraffian economists gave up the project, following new ideas by Garegnani (See: (Ciccone, 1986), (Ciccone, 1987); (Garegnani, 1992; Trezzini, 1995) (Palumbo & Trezzini, 2003), (Smith, 2013)). Since the autonomous trend lacks the persistency required to become a gravity center it should not be treated on the same footing as the technology and distribution that determine prices of production. The most they can say about dynamics of output is that production adapts to the ups and downs of aggregate demand via capacity utilization. As in the Kaleckian model, the rate of capacity seems unbounded and lacks a natural value playing the role of a gravity center.

Not all Sraffians have abandoned the original project of the surplus approach. In the Conference in Memorial of P. Garegnani, (John Eatwell, 2012) insisted on building a long-period theory of output that completes the Sraffian surplus approach. (Cesaratto, 2012) argues that, so far, the supermultiplier used by (Serrano, 1995), (Bortis, 1997), and (Dejuán, 2005), provides a plausible and most promising way out. (White, 2006) and (F. Freitas & Serrano, 2007) lean also in this direction.

The gap between the two Sraffian strands may be smaller than it appears at first sight. I admit that the autonomous trend lacks the persistency of the typical exogenous variables of the surplus approach (technology and distribution). I simply claim that, if the autonomous trend lasts long enough, it will deploy structural changes allowing the economy to grow at this rate with normal capacity utilization. Despite the volatility of the autonomous trend, the forces leading to the structural changes previously analyzed are always in operation. By the way, the same happens to prices of production that are affected by technical change even when this is so accelerated that

a new innovation enters on the stage before prices have adjusted to the previous ones.

Palumbo & Trezzini (2003) dislike the supermultiplier model because it goes back to steady state analysis leading to a stable rate of growth. These results are considered incompatible with Keynesian demand-led growth. As a matter of fact, the eventual convergence of the rates of growth of all the variables is not an unjustified assumption but the result of a model based on the principle of effective demand and a flexible accelerator. We only assume that firms adjust production to expected demand which is fed by the autonomous trend. Errors in foresight are admitted ... and corrected via an investment function which takes into account the shortages of capacity and inventories in order to remove them. If the current autonomous trend persisted long enough, all economic variables will eventually grow at the same rate. Since this is not the case, we cannot expect a stable and steady growth. Our claim is that the instability of capitalist economies is not a result of the multiplier-accelerator mechanism but of the performance of autonomous demand.

Empirical evidence has never been enough to solve the big economic questions but it may help. Two results would support the supermultiplier model: (1) The difference between fast growing countries and slow ones is not reflected so much in the rate of utilization but on the share of investment. (2) The standard deviations of the rate of utilization is rather low. The rate rises with every new boom but soon returns to its normal level. Consequently, the difference between the average rate in long waves of prosperity and depression is rather meagre.

## 5. Conclusions

The structure of the dynamic surplus approach can be represented by a triangle (we borrow the simile from Cesaratto, 2012). The vertices correspond to three basic premises: (1) The Keynesian-Kaleckian principle of effective demand whose dynamic corollary is an exogenous demand-led growth; (2) The exogeneity of distribution that influences *aggregate* expenditure patterns, although the propensity to consume and

save of different social groups may be taken as data; (3) The exogeneity of technology: competition compels firms to invest in the best available technology and to use it in the best conditions (at normal capacity utilization).

To cope with the instability problems of a demand-led growth, already detected by Harrod (1939), the first post-Keynesian economists set aside the exogeneity of distribution. Some Marxian economists have followed suit, introducing new distributive variables on to the economic stage. NeoKaleckians have given up the techno-economic requirement of a tendency towards normal capacity utilization. A solution that has been accepted by a sector of Sraffian economists.

In this paper, using the supermultiplier model we have shown that it is possible to preserve the independence of the three pillars. A demand-led growth is possible, even with constant distribution and normal capacity utilization in the long run, if the structure of demand / production changes in the appropriate way.

This is an automatic process brought about by our prospective and flexible accelerator. A rise in the autonomous trend forces firms to overutilize capacity. Then they raise investment above the level suggested by the pure acceleration mechanism. If the installment of the extra capacity is not enough to return to normal utilization, firms understand that they need to raise their demand-growth expectations. The long-term effect is a change in output composition. The share of capital goods in output and the share of expansionary investment in demand rise, at the expense of the autonomous part. A new fully adjusted path is achieved when capacity returns to its normal level and all the variables grow along the same path. In this sense, the potential or warranted rate of growth can be considered endogenous.

The main features of our multiplier-accelerator model on which this paper relies are the following:

- (1) It works both in equilibrium and disequilibrium situations. It explains the working of an economy along its fully adjusted path of growth but also the traverse to a different autonomous trend. It does not require, therefore, perfect foresight of effective demand.

- (2) Contrary to Harrod's "knife edge" metaphor, the multiplier-accelerator mechanism is a stable and stabilizing mechanism. The conditions for achieving such a result are:
- a. The autonomous trend is truly autonomous. It does not depend on the ups and downs of income and the rate of capacity. Its autonomy is enhanced by the fact that it is usually financed by credit.
  - b. The autonomous trend should lie below the limit set by technology and expenditure patterns. In an open economy, the possibility to import equipment relaxes this constraint in the short run. In the long run, however, the balance of payments constraint usually shows up before the supply side restrictions.
  - c. Investors take into account the shortages / excesses of capacity to get rid of them. They reproduce the whole or part of the inventories used up; not more. They do not overreact in inappropriate ways.
- (3) The convergence of the rates of growth (*steady growth*, in the traditional parlance) is not an assumption but the result of the internal forces of the model when there is only one source of autonomous growth which persists long enough. In practice there are many sources, each one with its own rhythm and length. So we can expect that the actual dynamics of capitalist economies will resemble self-contained chaotic movements.
- (4) The model does not imply that capitalism is a stable system. It simply suggests that economic instability usually stems from the volatility of the autonomous trend, not from the accelerator mechanism.

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