The Pearson correlation example

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The Pearson Correlation Definition

The **Pearson linear correlation coefficient**, denoted as ρ or r, is defined as:

$$\rho = \frac{S_{xy}}{\sqrt{S_{xx} \quad S_{yy}}} = \frac{\sum_{i=1}^{n}(x_i - \mu_X)(y_i - \mu_Y)}{\sqrt{\sum_{i=1}^{n}(x_i - \mu_X)^2 \sum_{i=1}^{n}(y_i - \mu_Y)^2}} \tag{7}$$

which **always takes values in the interval** $[-1, 1]$, that is, $-1 \leq \rho \leq 1$, and **the equalities hold only when the values of the two variables lie on a straight line**, getting apart from the equality when they separate from the linear trend.

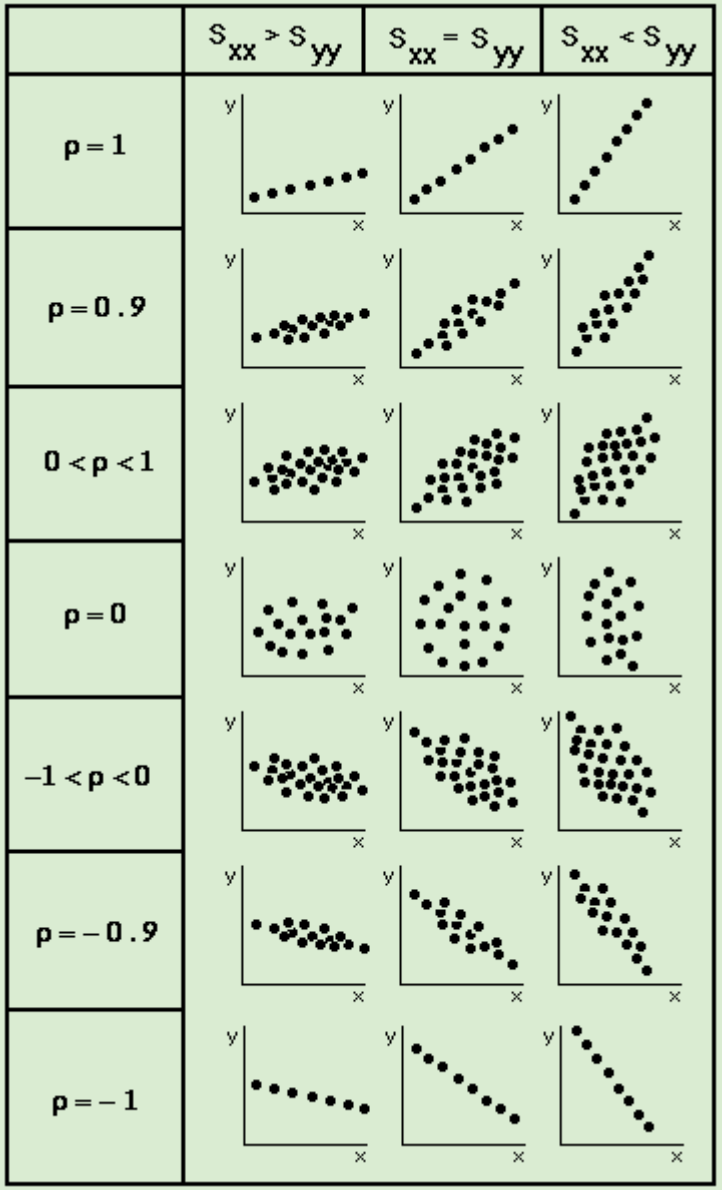
It can be easily demonstrated that the following relation holds:

$$Var[Y - aX - b] = Var[Y](1 - \rho^2). \tag{8}$$

Due to this property, **the correlation coefficient is used to measure the proportion of the variance of Y gained when predicting it with a linear law of the form $Y = aX + b$**.

It **is used to measure the convenience of adjusting or not, a straight line a the cloud of data points**. It is an alternative to the covariance, but **has the advantage of being dimensionless**.

Graphical Illustration Of The Pearson Correlation



Multivariate normal

We consider a two-dimensional normal variable, with mean

$$\mu = (\mu_1, \mu_2),$$

and covariance matrix

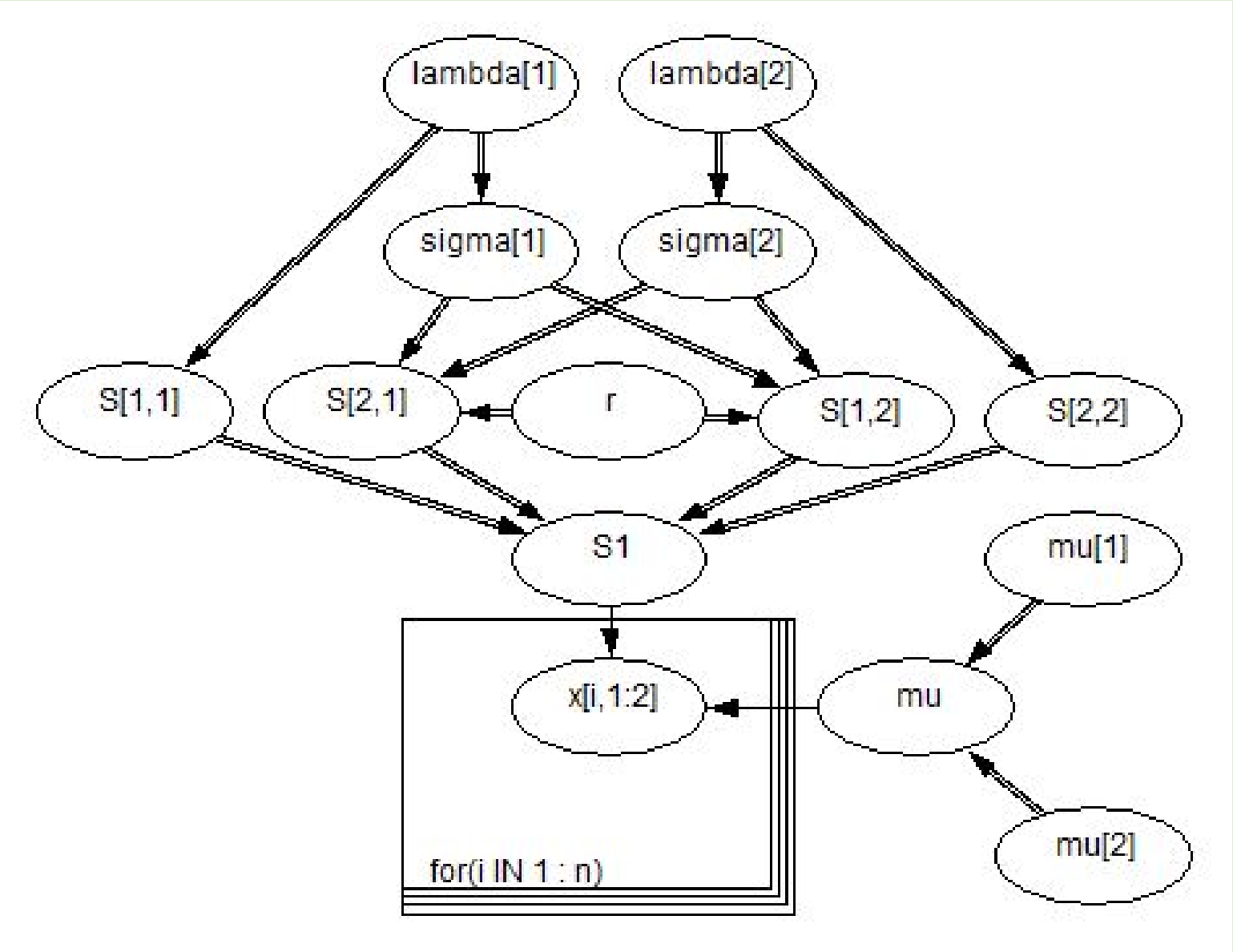
$$Covar = \begin{pmatrix} \sigma_1^2 & r\sigma_1\sigma_2 \\ r\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$$

where μ_1 and μ_2 are the marginal means of the two variables, σ_1 and σ_2 are their standard deviations, and r is the correlation coefficient.

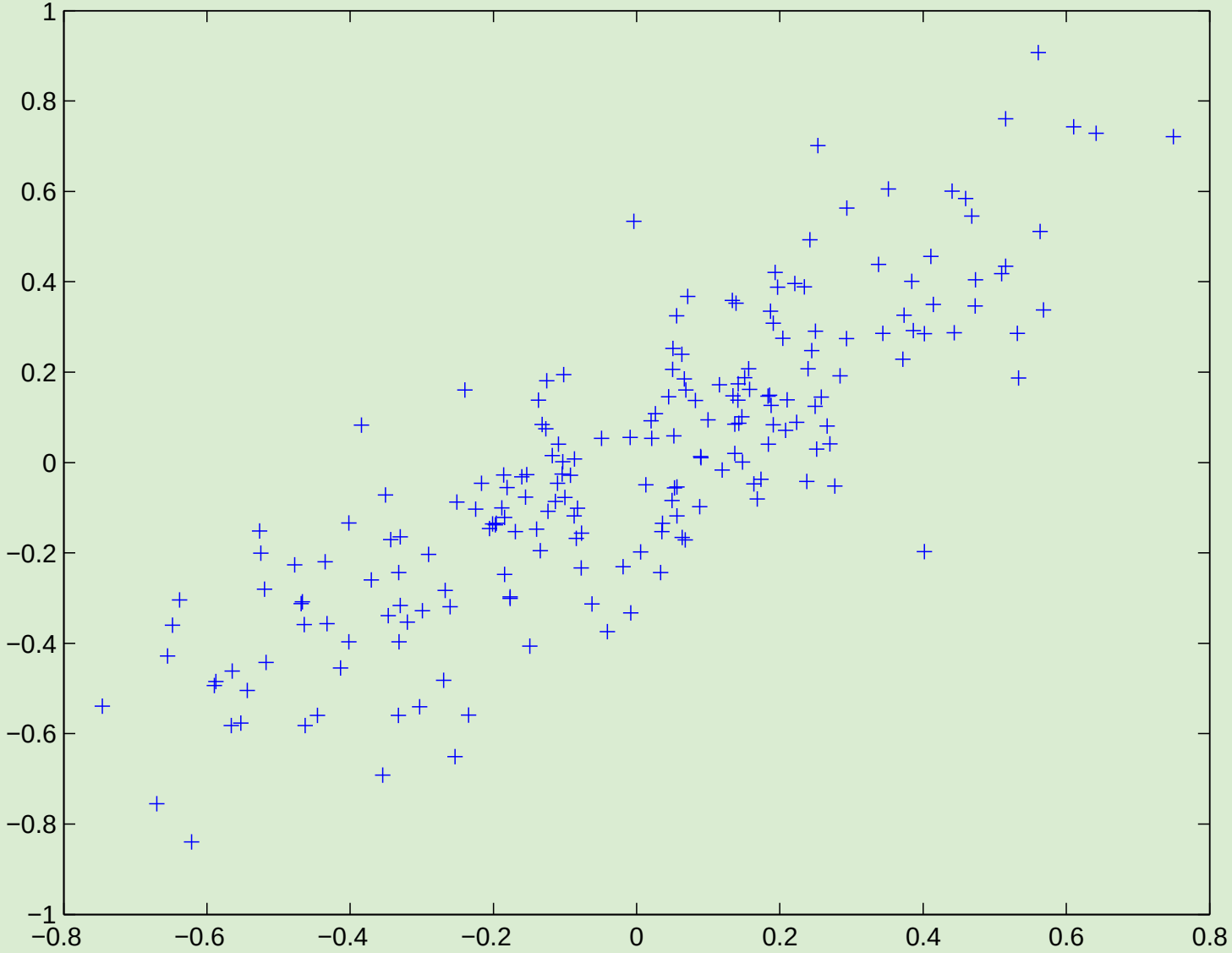
The Pearson Correlation Commands

```
# Pearson Correlation
model{
# Data
    for (i in 1:n){x[i,1:2] ~ dmnorm(mu[],S1[,])}
# Priors
    mu[1] ~ dnorm(0,.001)
    mu[2] ~ dnorm(0,.001)
    lambda[1] ~ dgamma(.001,.001)
    lambda[2] ~ dgamma(.001,.001)
    r ~ dunif(-1,1)
# Reparameterization
    sigma[1] <- 1/sqrt(lambda[1])
    sigma[2] <- 1/sqrt(lambda[2])
    S[1,1] <- 1/lambda[1]
    S[1,2] <- r*sigma[1]*sigma[2]
    S[2,1] <- r*sigma[1]*sigma[2]
    S[2,2] <- 1/lambda[2]
    S1[1:2,1:2] <- inverse(S[1:2,1:2])
}
```

The Pearson Correlation Doodle Model



The Pearson Correlation Example Data



The Pearson Correlation Commands, Data and Initial Files

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Commands.txt

```
# Pearson Correlation
model{
# Data
for (i in 1:n){
x[i,1:2] ~ dmnorm(mu[],S1[,])
}
# Priors
mu[1] ~ dnorm(0,.001)
mu[2] ~ dnorm(0,.001)
lambda[1] ~ dgamma(.001,.001)
lambda[2] ~ dgamma(.001,.001)
r ~ dunif(-1,1)
# Reparameterization
sigma[1] <- 1/sqrt(lambda[1])
sigma[2] <- 1/sqrt(lambda[2])
S[1,1] <- 1/lambda[1]
S[1,2] <- r*sigma[1]*sigma[2]
S[2,1] <- r*sigma[1]*sigma[2]
S[2,2] <- 1/lambda[2]
S1[1:2,1:2] <- inverse(S[1:2,1:2])
```

Sample Monitor Tool

node | chains 1 to 3 | percentiles: 2.5, 5, 10, 25, median, 75, 90, 95, 97.5

beg 1 | end 10000000 | thin 1

clear | set | diagnostics | trace | jump | stats | density | bgr diag | history | accept | coda | quantiles | auto cor

Specification Tool

check model | load data | compile | num of chains 3 | load inits | for chain 1 | gen inits

Data.txt

```
list(n=200,
x=structure(
.Data=c( -0.2052362, -0.1458369,
  0.3735175,  0.3261632,
 -0.2398440,  0.1603157,
 -0.3508905, -0.0719385,
 -0.2673868, -0.2829939,
 -0.1810925, -0.0553196,
 -0.1766704, -0.3005456,
  0.0564088, -0.0542254,
 -0.0622531, -0.3133023,
  0.1854494,  0.1491656,
 -0.2693903, -0.4820510,
  0.2530836,  0.7012880,
 -0.4773157, -0.2263578,
  0.2769757, -0.0522873,
 -0.0767768, -0.1562384,
  0.0527510, -0.0560750,
 -0.6215200, -0.8392631,
 -0.4016318, -0.1337631,
  0.3716218,  0.2288494,
  0.6416768,  0.7282485,
 -0.0870124,  0.0079959,
```

Initials1.txt

```
list(mu=c(    0.000000000000,    0.000000000000),
lambda=c(    10.000000000000,   10.000000000000),
r=0.8)
```

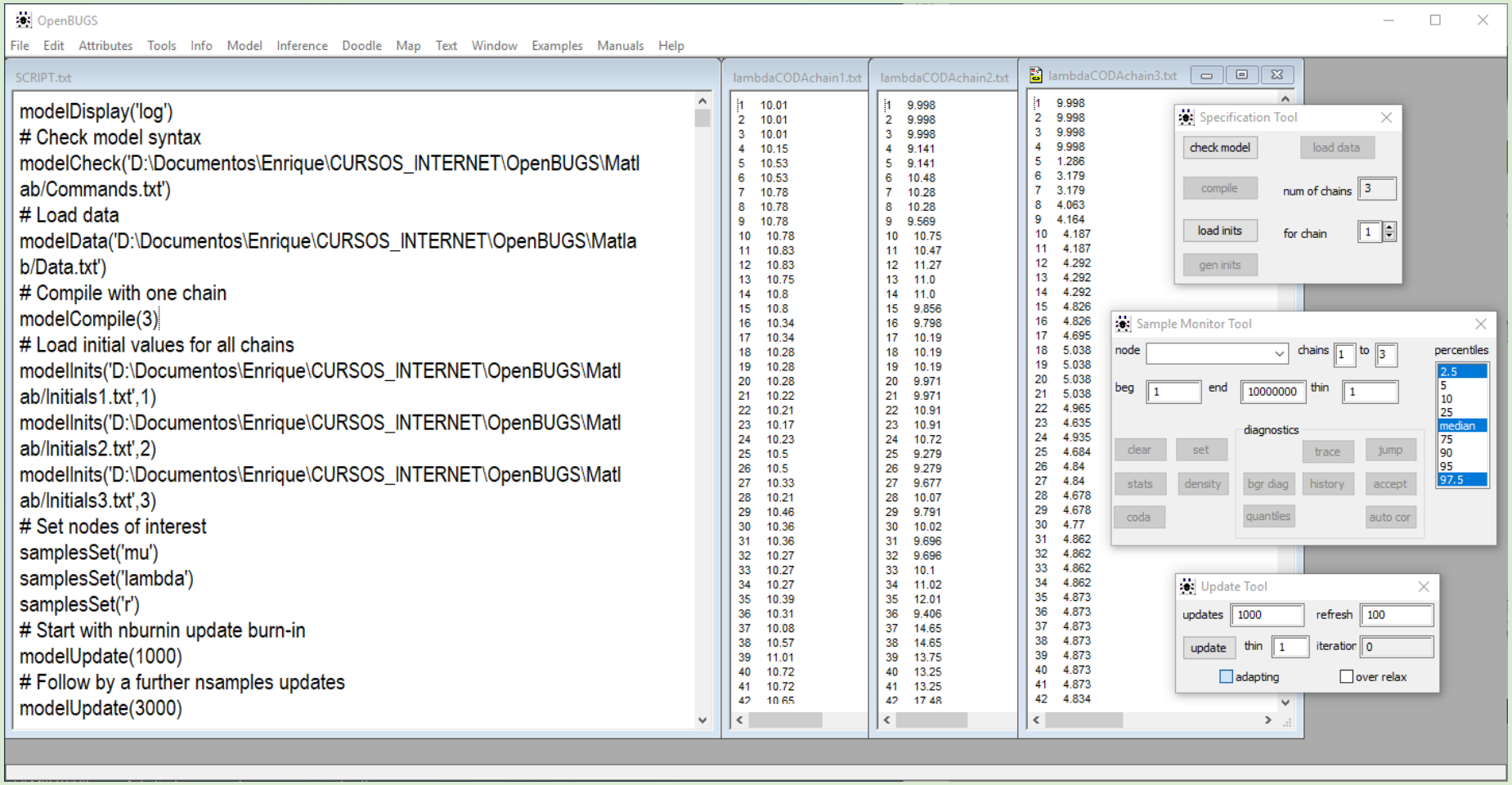
Initials2.txt

```
list(mu=c(    0.000000000000,    0.000000000000),
lambda=c(    10.000000000000,   10.000000000000),
r=0)
```

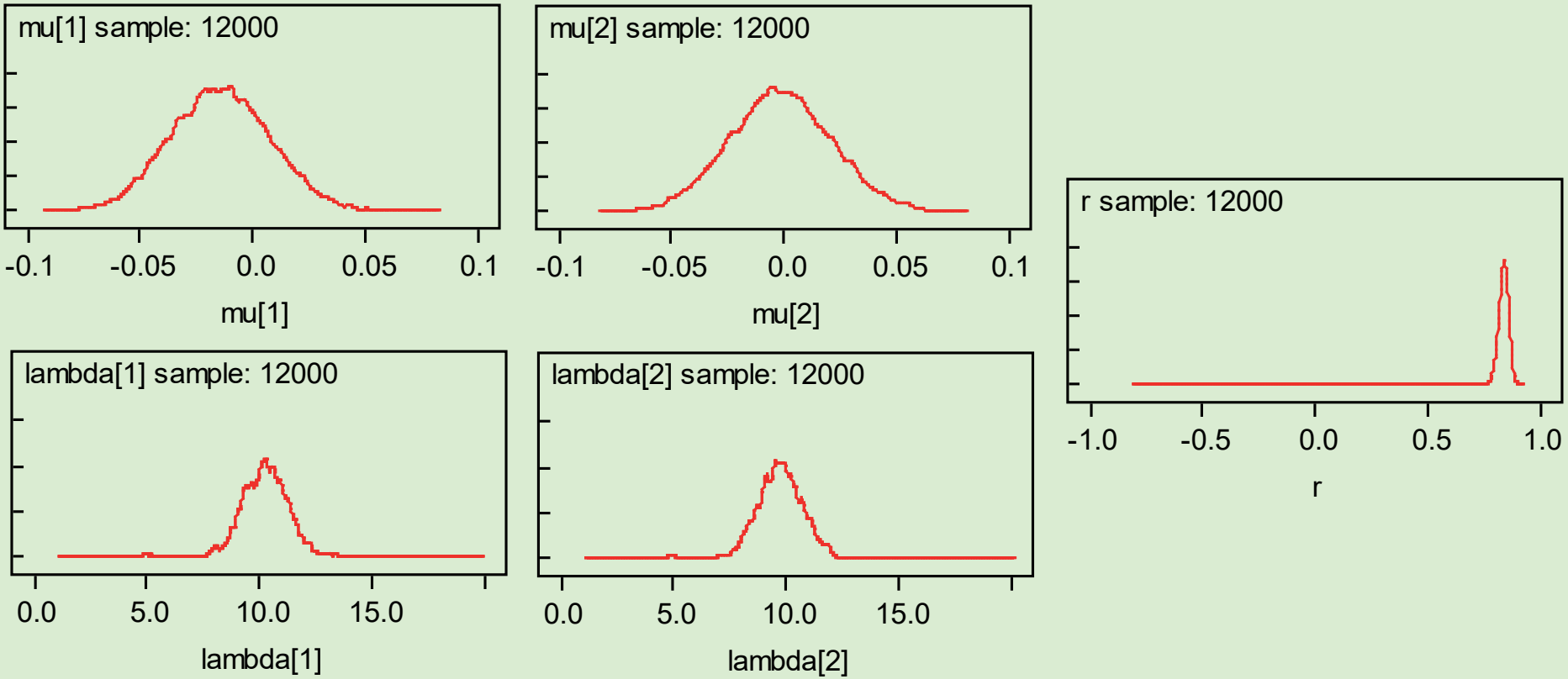
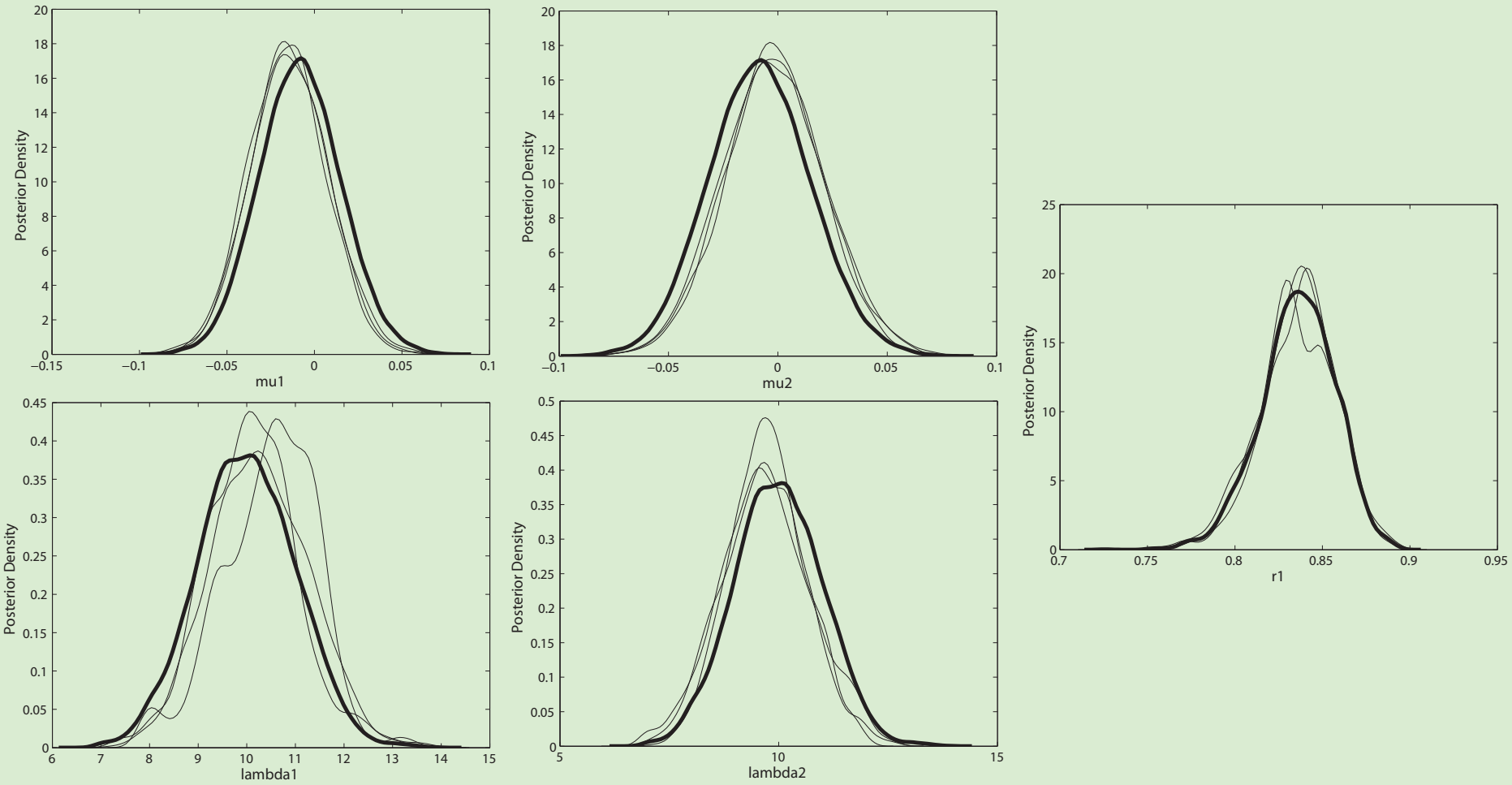
Initials3.txt

```
list(mu=c(    0.000000000000,    0.000000000000),
lambda=c(    10.000000000000,   10.000000000000),
r=-0.8)
```

The Pearson Correlation Script and Coda Files



The Pearson Correlation Posterior Densities of $\mu_1, \mu_2, \lambda_1, \lambda_2$ and r



The Pearson Correlation Posterior Densities of $\mu_1, \mu_2, \lambda_1, \lambda_2$ and r