The Rate example

OpenBUGS

The Rate Example Description

In this example we try to **estimate a rate θ**, that is, the proportion of values satisfying a given condition.

We assume that the prior of θ is uniform in the range $[0, 1]$, which is equivalent to assume that it is a beta $Beta(1, 1)$ random variable, i.e. with parameters 1 and 1,

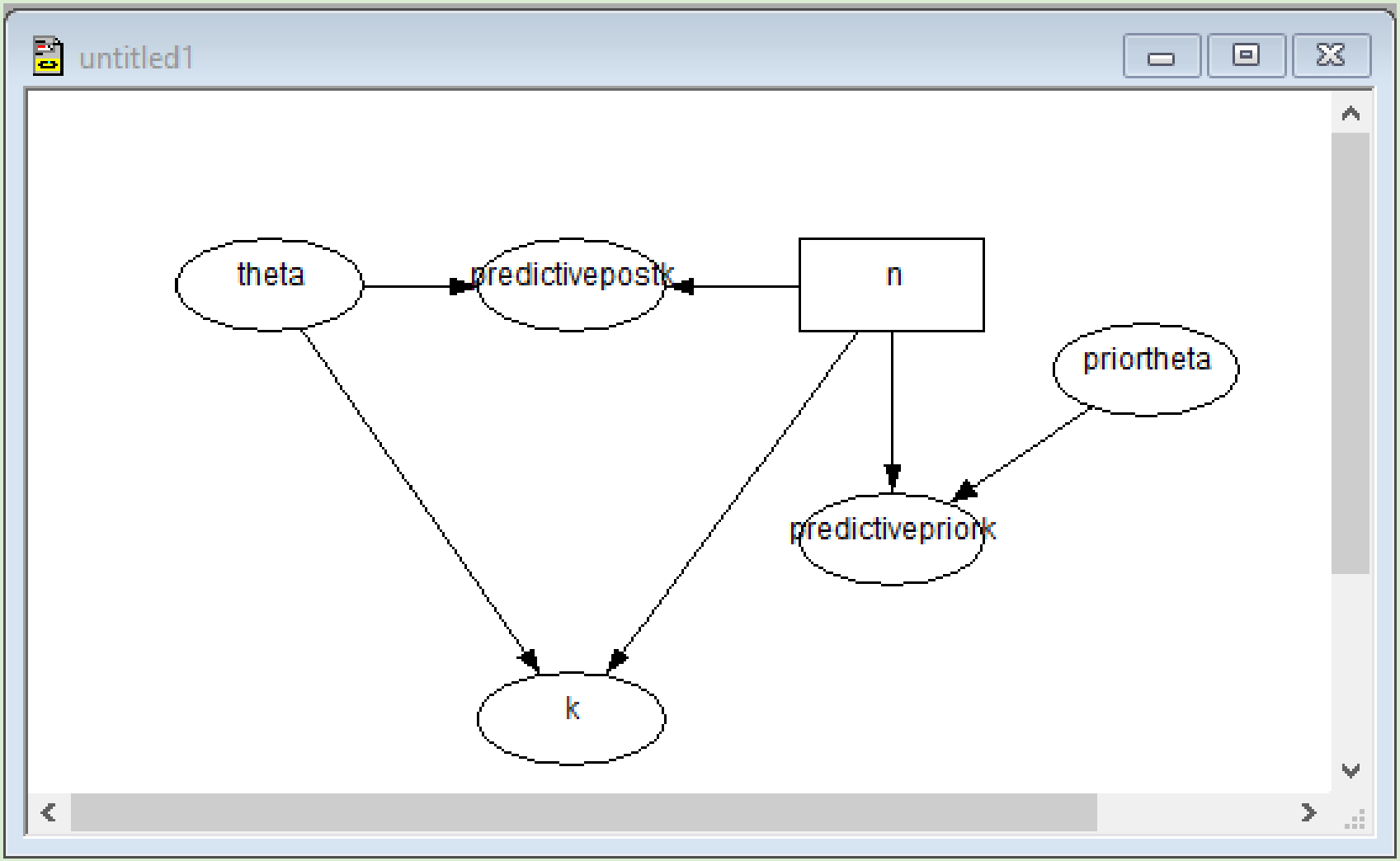
We force OpenBUGS to generate the prior and the posterior predictives for a new experiment, when the sample size is n. This means that the theoretical random number of successes k is given by a binomial random variable.

Finally, we assume that **we observe 9 successes of a total of $n = 10$ trials**, which is our observation data.

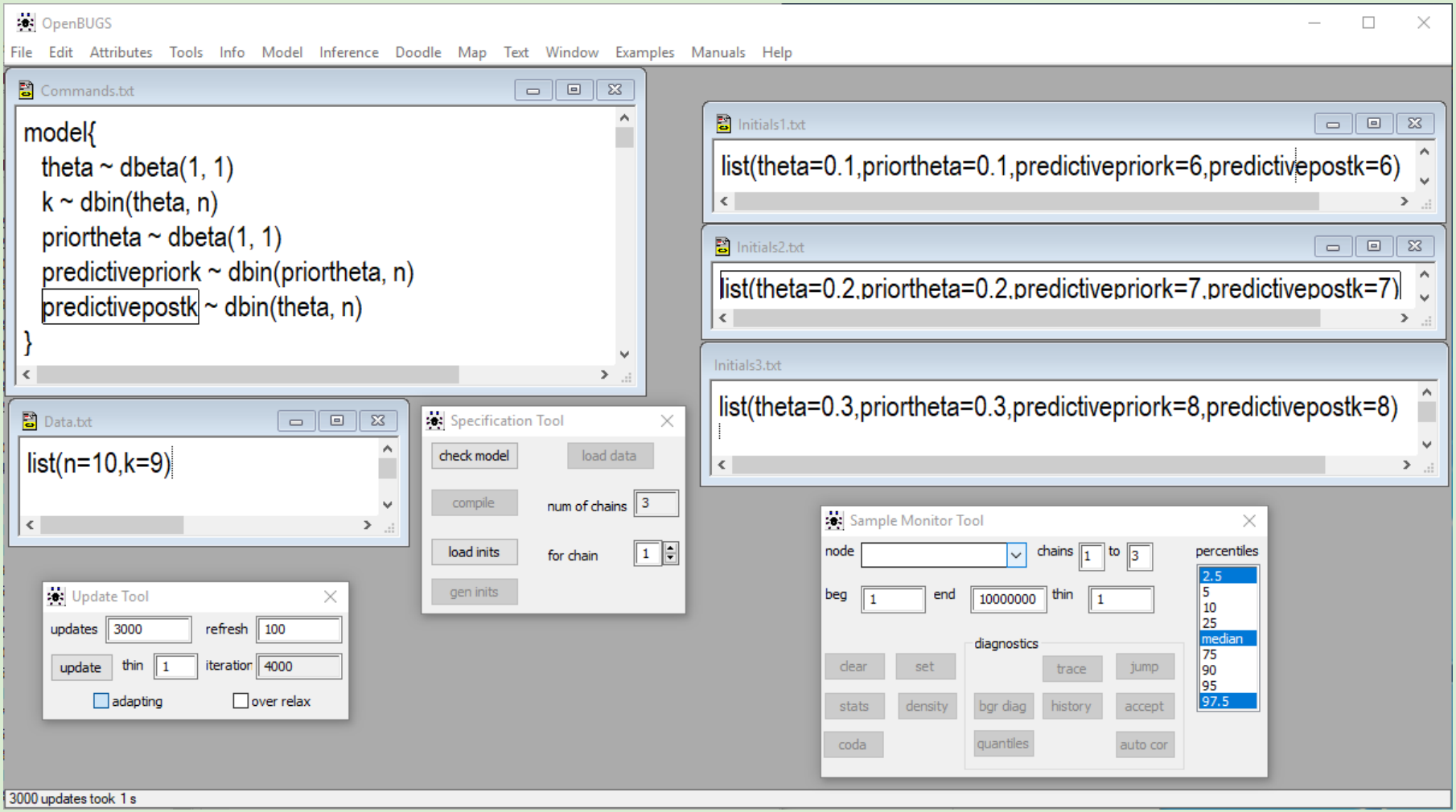
The Rate Example Commands

```
# Rate example

model {
    theta ~ dbeta(1,1) # uniform prior

# Data
    k ~ dbin(theta,n) # example data in , k = 9 and n = 10

# Predictive distributions
    predictivepriork~dbin(priortheta,n) # predictive prior k
    priortheta ~ dbeta(1,1) # uniform prior
    predictivepostk ~ dbin(theta,n) # predictive post k
}
```

The Doodle Model



The Rate Example Commands, Data and Initial Files



The Rate Script and Coda Files

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SCRIPT.txt

```
modelDisplay('log')
# Check model syntax
modelCheck('D:\Documentos\Enrique\CURSOS_INTERNET\OpenBUGS\Matl
ab/Commands.txt')
# Load data
modelData('D:\Documentos\Enrique\CURSOS_INTERNET\OpenBUGS\Matla
b/Data.txt')
# Compile with one chain
modelCompile(3)
# Load initial values for all chains
modelInits('D:\Documentos\Enrique\CURSOS_INTERNET\OpenBUGS\Matl
ab/Initials1.txt',1)
modelInits('D:\Documentos\Enrique\CURSOS_INTERNET\OpenBUGS\Matl
ab/Initials2.txt',2)
modelInits('D:\Documentos\Enrique\CURSOS_INTERNET\OpenBUGS\Matl
ab/Initials3.txt',3)
# Set nodes of interest
samplesSet('theta')
samplesSet('priortheta')
samplesSet('predictivepriork')
samplesSet('predictivepostk')
# Start with nburnin update burn-in
modelUpdate(1000)
```

Specification Tool

check model | load data | compile | num of chains 1 | load inits | for chain 1 | gen inits

predictivepostkCODAchain1.txt

1	7.0
2	8.0
3	10.0
4	8.0
5	10.0
6	9.0
7	10.0
8	7.0
9	9.0
10	10.0
11	10.0
12	8.0
13	7.0
14	8.0
15	10.0
16	9.0
17	10.0
18	5.0
19	10.0
20	7.0
21	7.0
22	9.0
23	9.0
24	8.0
25	10.0
26	8.0
27	9.0
28	8.0
29	8.0
30	6.0
31	6.0
32	10.0
33	10.0
34	6.0
35	10.0
36	10.0
37	9.0
38	9.0
39	9.0
40	10.0
41	9.0
42	10.0

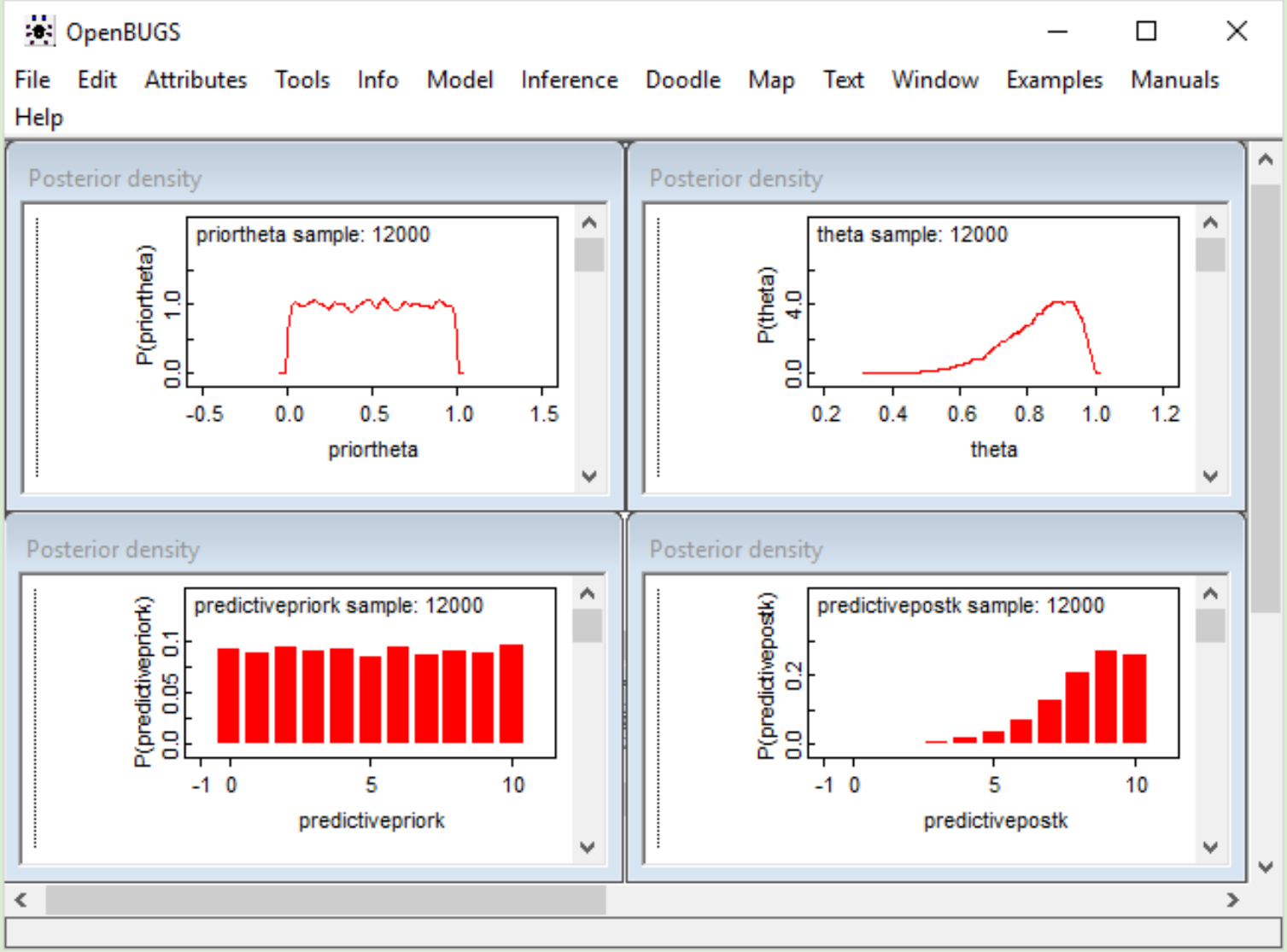
predictivepostkCODAchain2.txt

1	9.0
2	10.0
3	9.0
4	9.0
5	8.0
6	6.0
7	9.0
8	9.0
9	6.0
10	8.0
11	9.0
12	10.0
13	10.0
14	10.0
15	9.0
16	8.0
17	8.0
18	8.0
19	8.0
20	8.0
21	9.0
22	8.0
23	10.0
24	6.0
25	9.0
26	8.0
27	8.0
28	10.0
29	9.0
30	10.0
31	10.0
32	10.0
33	7.0
34	8.0
35	10.0
36	7.0
37	9.0
38	9.0
39	6.0
40	9.0
41	7.0
42	10.0

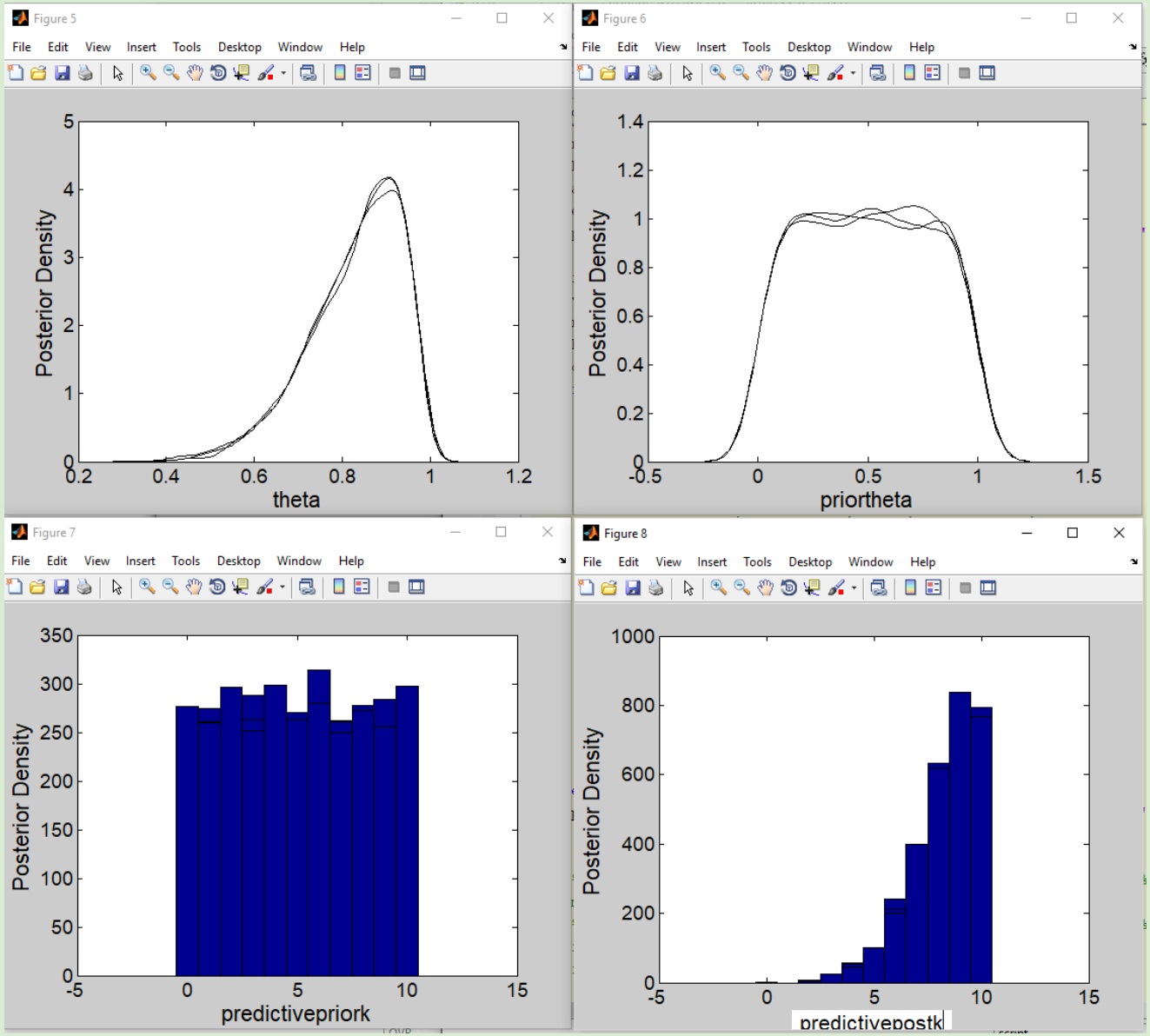
predictivepostkCODAchain3.txt

1	7.0
2	6.0
3	8.0
4	10.0
5	6.0
6	6.0
7	10.0
8	10.0
9	9.0
10	8.0
11	10.0
12	10.0
13	7.0
14	9.0
15	8.0
16	9.0
17	9.0
18	8.0
19	7.0
20	10.0
21	7.0
22	8.0
23	9.0
24	9.0
25	10.0
26	8.0
27	9.0
28	9.0
29	9.0
30	6.0
31	8.0
32	9.0
33	10.0
34	9.0
35	9.0
36	10.0
37	8.0
38	5.0
39	10.0
40	5.0
41	10.0
42	6.0

The Rate Example Prior, Posterior and Predictive Distributions



The Rate Example Prior, Posterior and Predictive Distributions



The RATE with unknown n example

The Rate With Unknown n Example Description

In this example we try to **estimate a rate θ**, that is, the proportion of values satisfying a given condition, when the sample size n is common but unknown.

We assume that the prior of θ is uniform in the range $[0, 1]$, which is equivalent to assume that it is a beta $Beta(1, 1)$ random variable, i.e. with parameters 1 and 1,

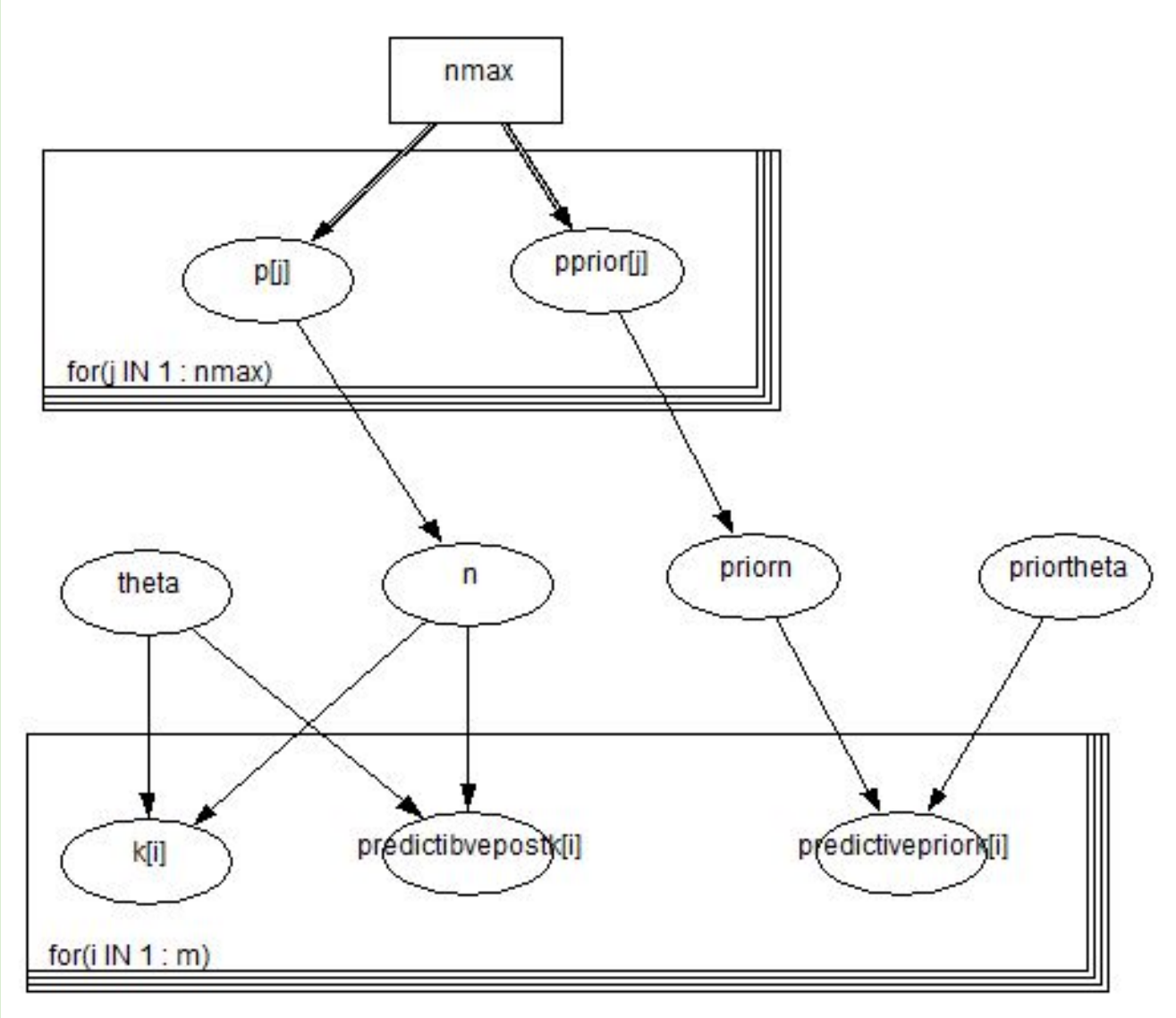
We force OpenBUGS to generate the prior and the posterior predictives for a new experiment, when the sample size is n. This means that the theoretical random number of successes k is given by a binomial random variable.

Finally, we assume that **we observe $(16, 18, 22, 15, 27)$ successes of a total of n unknown trials**, which is our observation data.

The Rate With Unknown n Example Commands

```
# Inferring Return Rate and sample size
model{
# Categorical probabilities
    for (j in 1:nmax){
        p[j] <- 1/nmax
        pprior[j] <- 1/nmax
    }
# Observed Returns
    for (i in 1:m){
        k[i] ~ dbin(theta,n)
        predictivepriork[i] ~ dbin(priortheta,priorn)
        predictivepostk[i] ~ dbin(theta,n)
    }
# Priors on Rate Theta and Number n
    theta ~ dbeta(1,1) # uniform prior
    n ~ dcat(p[])
    priortheta ~ dbeta(1,1) # uniform prior
    priorn ~ dcat(pprior[]) # categorical prior
}
```

The Rate With Unknown n Example Doodle Model



The Rate With Unknown n Example Commands, Data and Initial Files

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Commands.txt

```
# Inferring Return Rate and sample size
model{
# Observed Returns
  for (i in 1:m){
    k[i] ~ dbin(theta,n)
  }
# Priors on Rate Theta and Number n
  theta ~ dbeta(1,1) # uniform prior
  n ~ dcat(p[])
  for (i in 1:nmax){
    p[i] <- 1/nmax
    pprior[i] <- 1/nmax
  }
  for (i in 1:m){
   predictivepriork[i] ~ dbin(priortheta,priorn)
   predictivepostk[i] ~ dbin(theta,n)
  }
  priortheta ~ dbeta(1,1) # uniform prior
  priorn ~ dcat(pprior[]) # categorical prior
}
```

Specification Tool

check model | load data

compile | num of chains 3

load inits | for chain 1

gen inits

Data.txt

```
list(m=5,nmax=500,k=c(16.000000000000,    18.000000000000,
22.000000000000,    25.000000000000,    27.000000000000))
```

Initials1.txt

```
list(theta=0.1,n=100,priortheta=0.1,priorn=100,
predictivepriork=c(20.000000000000,    20.000000000000,
20.000000000000,    20.000000000000,    20.000000000000),
predictivepostk=c(20.000000000000,    20.000000000000,
20.000000000000,    20.000000000000,    20.000000000000))
```

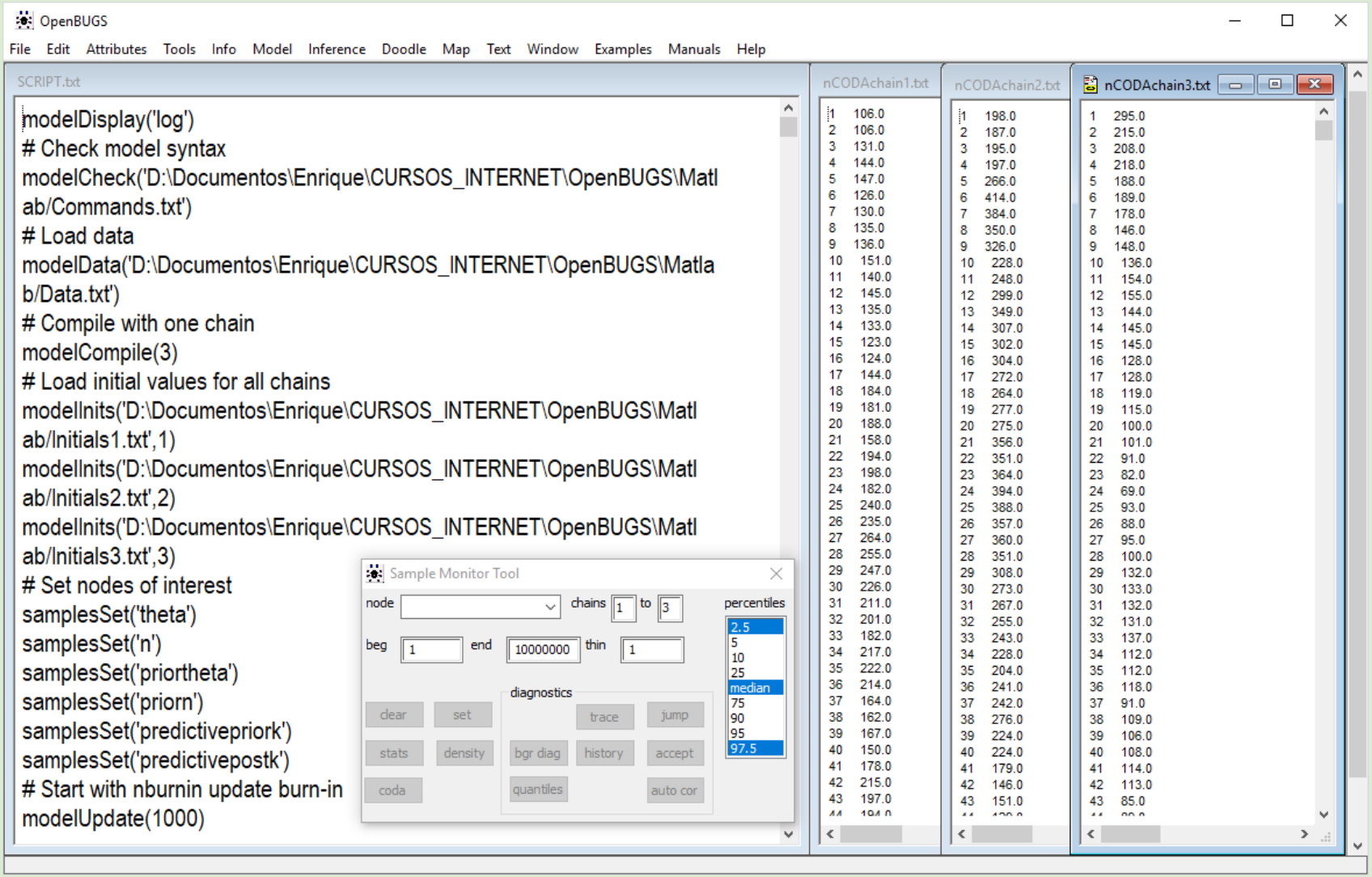
Initials2.txt

```
list(theta=0.2,n=200,priortheta=0.2,priorn=200,
predictivepriork=c(21.000000000000,    21.000000000000,
21.000000000000,    21.000000000000,    21.000000000000),
predictivepostk=c(21.000000000000,    21.000000000000,
21.000000000000,    21.000000000000,    21.000000000000))
```

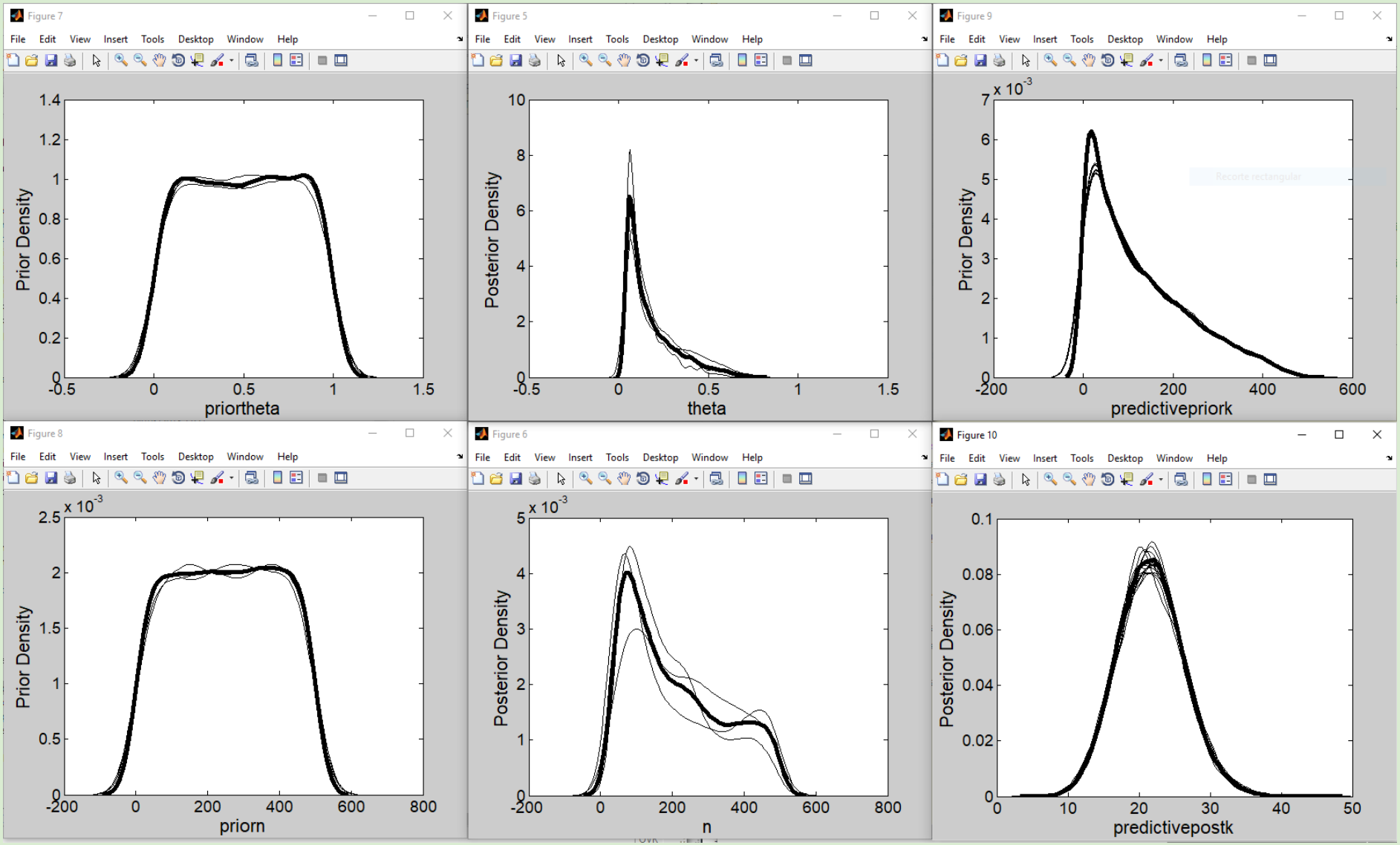
Initials3.txt

```
list(theta=0.3,n=300,priortheta=0.3,priorn=300,
predictivepriork=c(23.000000000000,    23.000000000000,
23.000000000000,    23.000000000000,    23.000000000000),
predictivepostk=c(23.000000000000,    23.000000000000,
23.000000000000,    23.000000000000,    23.000000000000))
```

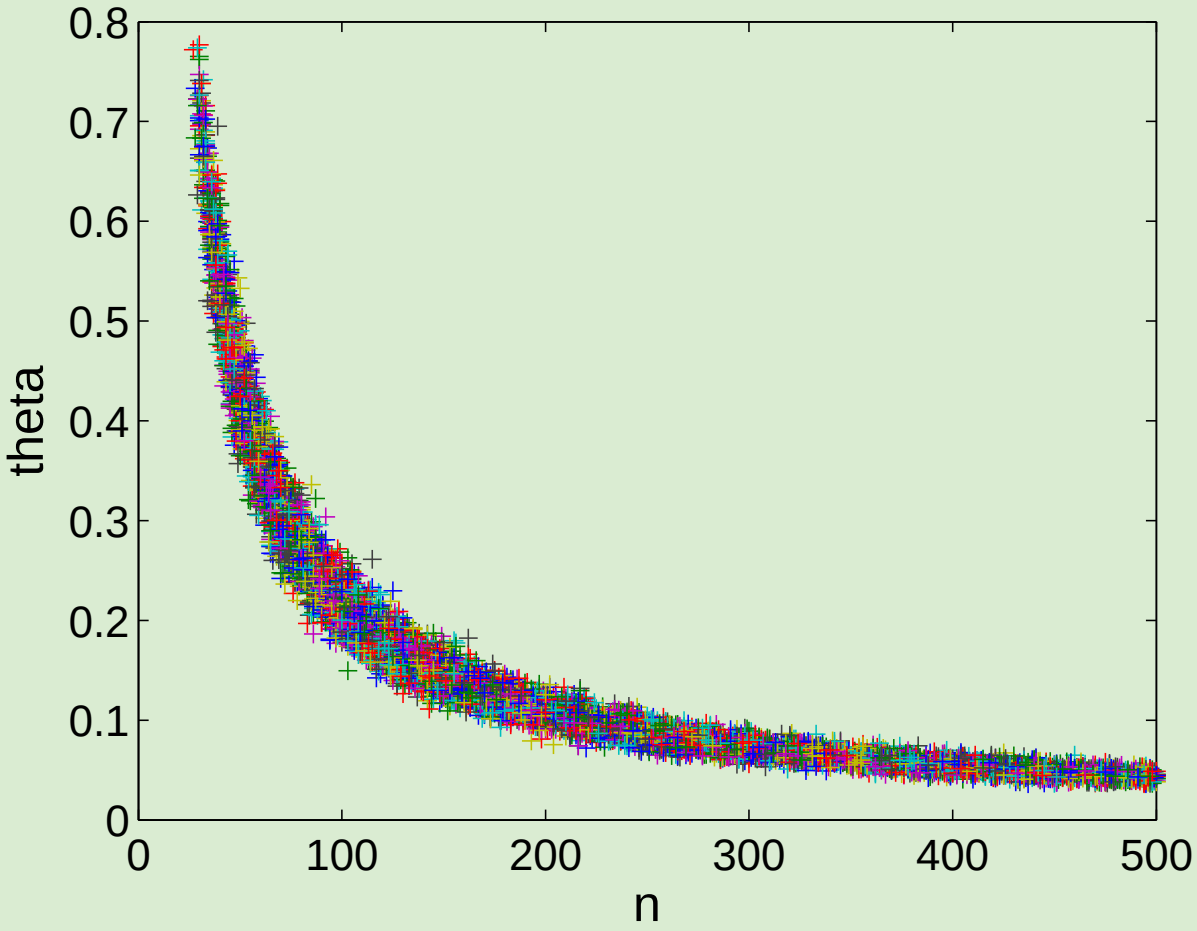
The Rate With Unknown n Example Script and Coda Files



The Prior, Posterior and Predictive Densities



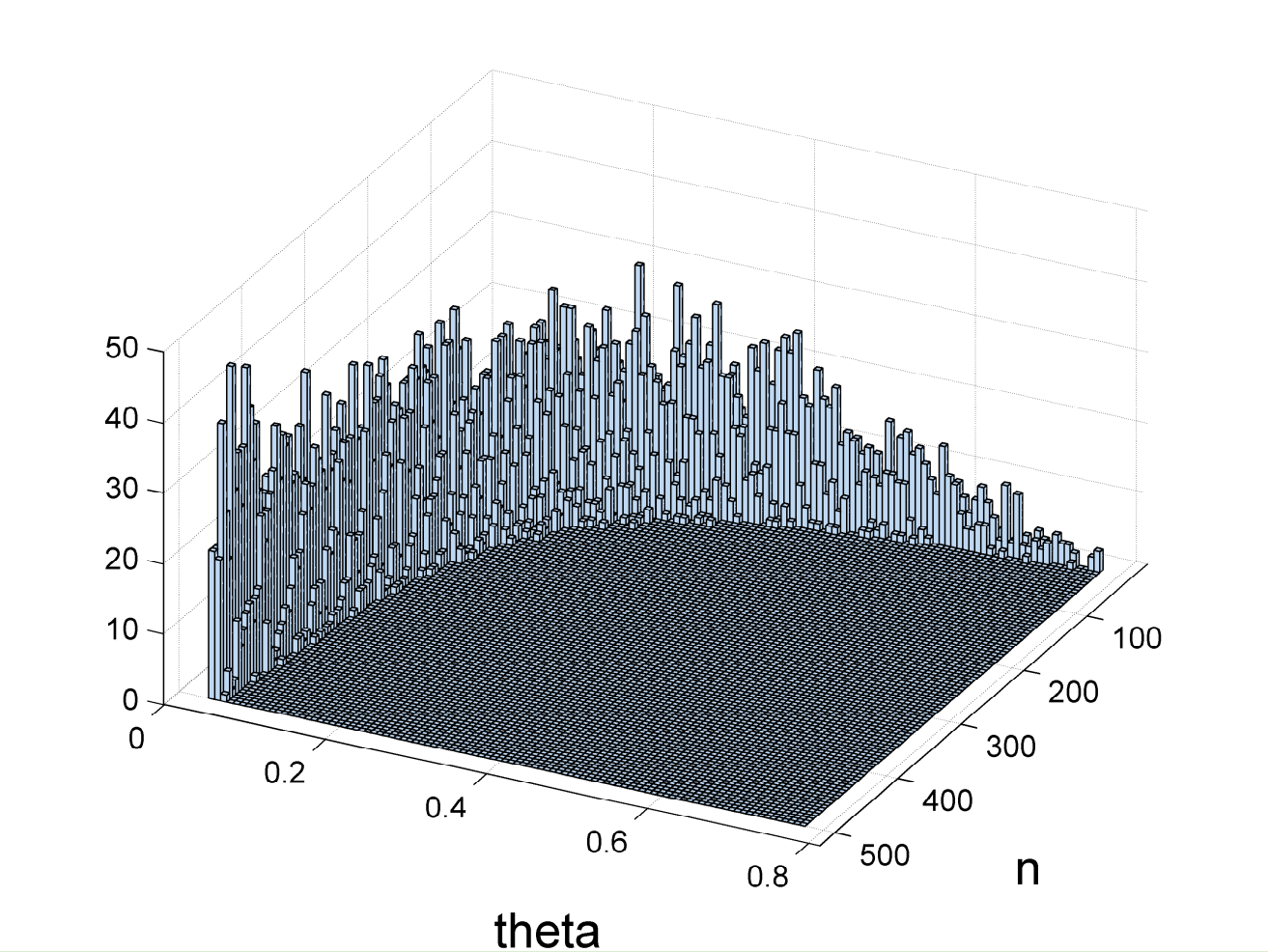
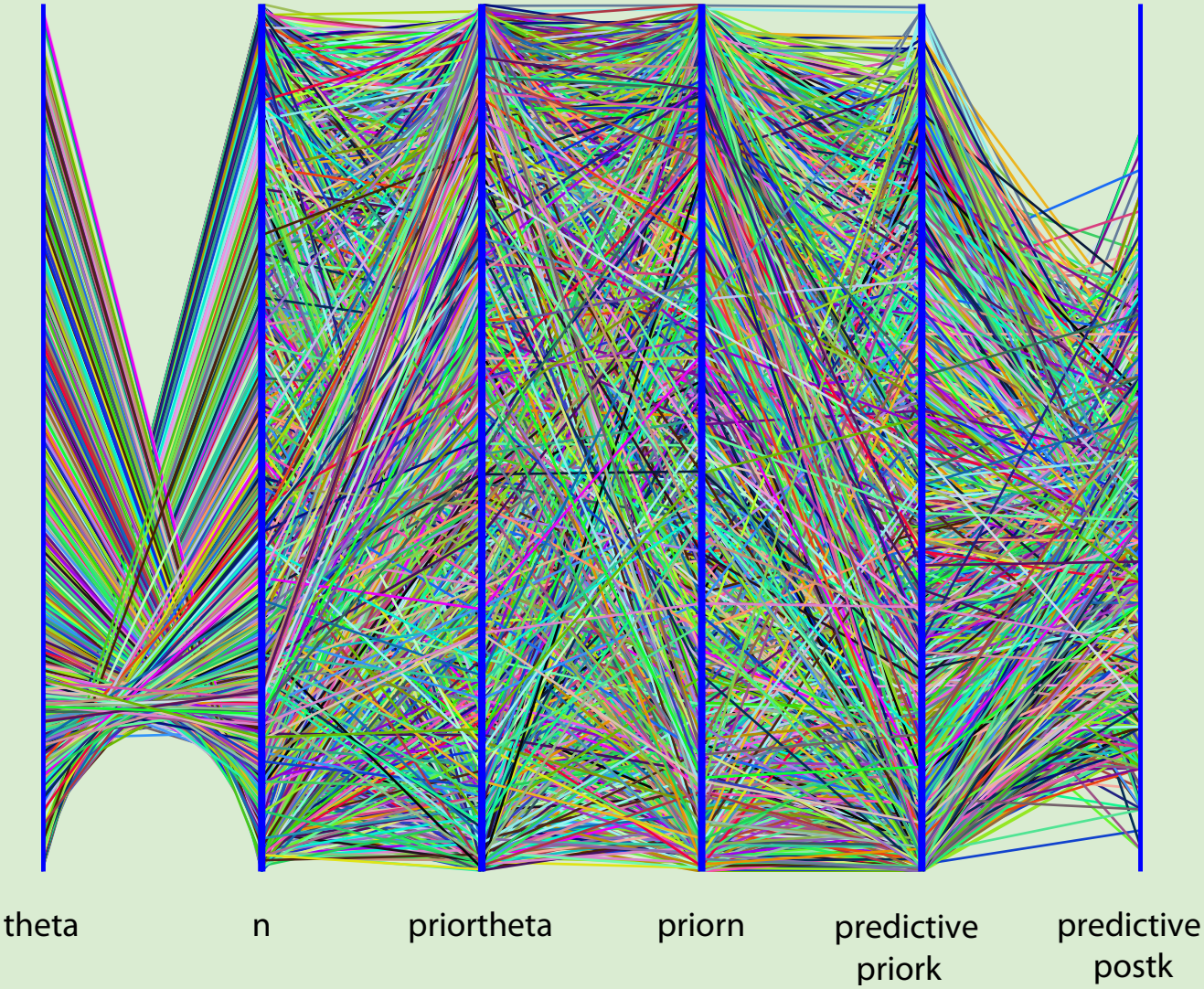
The Joint n and theta Posterior Plot



Even though the prior distributions of θ and n parameters were assumed to be independent, the data reveals a clear dependence of hyperbolic shape.

Note that the total number of successes ($\theta \times n$ ranges between 16 and 27.

The Joint n and theta Posterior Plot



Posterior Sample Dependence Plot