Orthogonal Subspaces and Complements

Algebra

Iteration 1

a_1	v_1	v_2	v_3	v_4
1	1	0	0	0
1	0	1	0	0
0	0	0	1	0
1	0	0	0	1
t	1	1	0	1

Iteration 2

a_2	w_1	v_2	v_3	v_4
0	1	-1	0	-1
1	0	1	0	0
0	0	0	1	0
1	0	0	0	1
t	0	1	0	1

Iteration 3

a_3	w_1	w_2	v_3	v_4
0	1	-1	0	0
0	0	1	0	-1
-1	0	0	1	0
1	0	0	0	1
t	0	0	-1	1

Iteration 4

a_4	w_1	w_2	w_3	v_4
-1	1	-1	0	0
2	0	1	0	-1
1	0	0	-1	1
0	0	0	0	1
t	-1	3	-1	-1

	w_1	w_2	w_3	w_4
	1	-1	0	0
	1	-2	1	1
	-1	3	-2	-1
	-1	3	-1	-1

$$A = \begin{pmatrix} \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & 1 \\ -1 & 2 & 1 & 0 \end{pmatrix}$$

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JUSTIFICATION OF THE ORTHOGONALIZATION ALGORITHM

The algorithm is designed to obtain the orthogonal and complementary subspaces of a subspace $A = L\{a_1, a_2, \ldots, a_k\}$ in another subspace $B = L\{b_1, b_2, \ldots, b_r\}$. The latter is the one that must be arranged in the initial matrix B^1 matrix with which the algorithm starts.

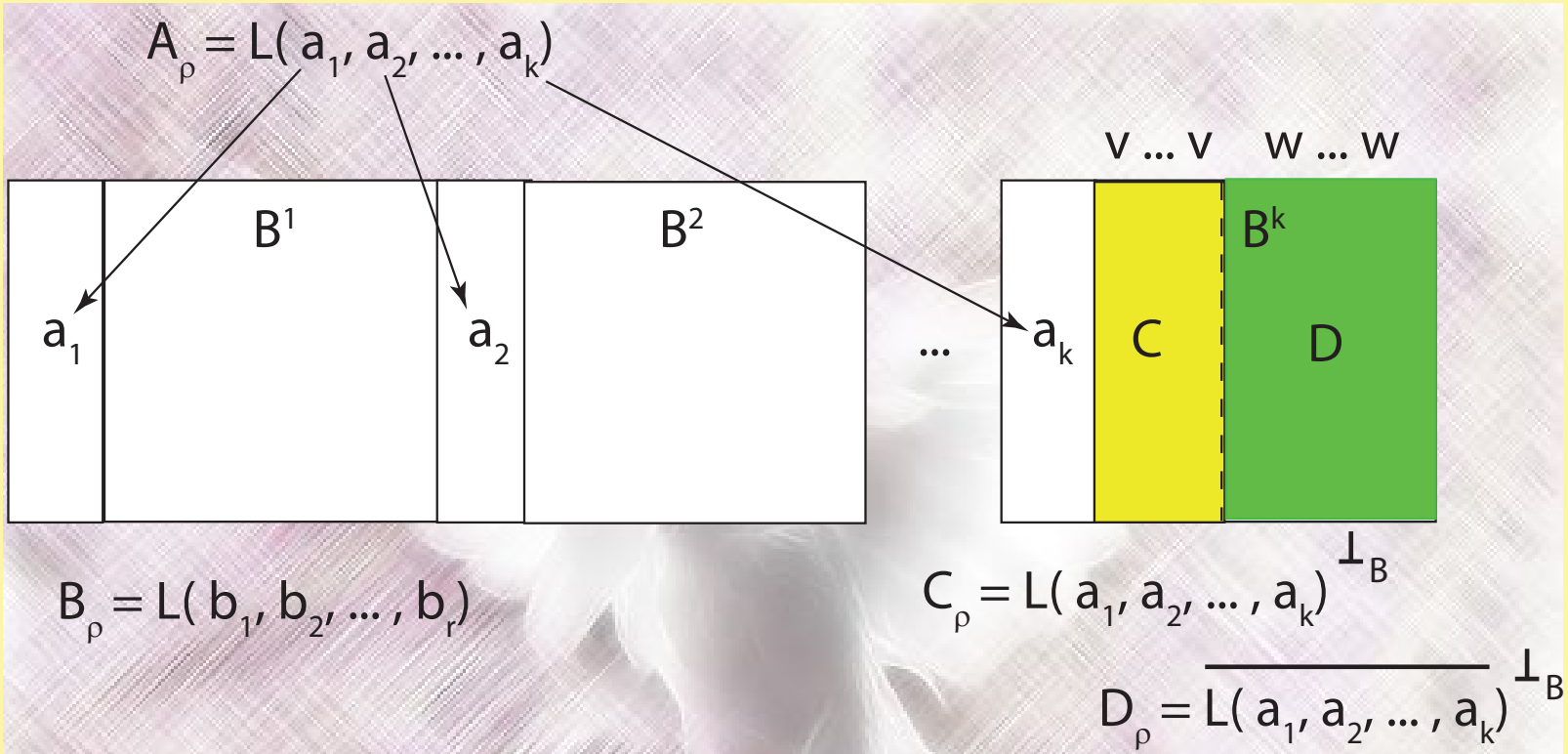

Diagram showing the stages of the algorithm and how the generators of the orthogonal and complementary subspaces are obtained.