Compatibility of a linear system of equations

Algebra

COMPATIBILITY OF A COMPLETE LINEAR SYSTEM

$$
\begin{array}{cccccc}
a_{11}x_1 & +a_{12}x_2 & + & \cdots & +a_{1n}x_n & = & b_1 \\
a_{21}x_1 & +a_{22}x_2 & + & \cdots & +a_{2n}x_n & = & b_2 \\
\cdots & \cdots & & \cdots & \cdots & \cdots & \cdots \\
a_{m1}x_1 & +a_{m2}x_2 & + & \cdots & +a_{mn}x_n & = & b_m
\end{array}
$$

$$
x_1 \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix} + x_2 \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix} + \cdots + x_n \begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}
$$

The compatibility problem reduces to a membership problem

For the system to be compatible, the column vector of the independent terms must belong to the linear subspace generated by the column vectors of the coefficients.

EXAMPLE OF COMPATIBILITY OF A COMPLETE LINEAR SYSTEM OF EQUATIONS

$$\begin{aligned} x_1 \; -x_2 \; -x_3 &= a \\ x_1 \qquad\quad +x_3 &= 3a \\ x_2 \; +2x_3 &= b \end{aligned}$$

	Iteration 1		
a_1	$\mathbf{v}_1$	v_2	v_3
1	**1**	0	0
1	**0**	1	0
0	**0**	0	1
t	1	1	0

	Iteration 2		
a_2	w_1	$\mathbf{v}_2$	v_3
-1	1	**-1**	0
0	0	**1**	0
1	0	**0**	1
t	-1	1	1

	Iteration 3		
a_3	w_1	w_2	v_3
-1	0	-1	1
1	1	1	-1
2	0	0	1
t	1	2	0

	Final table		
	w_1	w_2	v_3
	0	-1	1
	1	1	-1
	0	0	1

$$(a, 3a, b) \cdot (1, -1, 1) = a - 3a + b = -2a + b = 0 \Leftrightarrow b = 2a$$