Rank of a matrix

Algebra

$$A = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 3 & 2 & 3 \\ 2 & 4 & 2 & 4 \end{pmatrix}$$

Rank(A) = 2

Iteration 1

a_1	v_1^*	v_2	v_3	v_4
1	1	0	0	0
1	0	1	0	0
0	0	0	1	0
1	0	0	0	1
t	1	1	0	1

Iteration 2

a_2	w_1	v_2^*	v_3	v_4
0	1	-1	0	-1
1	0	1	0	0
1	0	0	1	0
1	0	0	0	1
t	0	1	1	1

Iteration 3

a_3	w_1	w_2	v_3	v_4
1	1	-1	1	0
3	0	1	-1	-1
2	0	0	1	0
3	0	0	0	1
t	1	2	0	0

Iteration 4

a_4	w_1	w_2	v_3	v_4
2	1	-1	1	0
4	0	1	-1	-1
2	0	0	1	0
4	0	0	0	1
t	2	2	0	0

$$1\begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} + 2\begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 2 \\ 3 \end{pmatrix} \qquad 2\begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} + 2\begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 2 \\ 4 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 1 & 2 \\ 1 & 1 & 3 & 4 \\ 0 & 1 & 2 & 2 \\ 1 & 1 & 3 & 4 \end{pmatrix}$$

Rank(A) = 2

Iteration 1				
a_1	v_1^*	v_2	v_3	v_4
1	1	0	0	0
0	0	1	0	0
1	0	0	1	0
2	0	0	0	1
t	1	0	1	2

Iteration 2				
a_2	w_1	v_2^*	v_3	v_4
1	1	0	-1	-2
1	0	1	0	0
3	0	0	1	0
4	0	0	0	1
t	1	1	2	2

Iteration 3				
a_3	w_1	w_2	v_3	v_4
0	1	0	-1	-2
1	-1	1	-2	-2
2	0	0	1	0
2	0	0	0	1
t	-1	1	0	0

Iteration 4				
a_4	w_1	w_2	v_3	v_4
1	1	0	-1	-2
1	-1	1	-2	-2
3	0	0	1	0
4	0	0	0	1
t	0	1	0	0

$$-1\begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix} + 1\begin{pmatrix} 1 \\ 1 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 2 \end{pmatrix}$$

$$1\begin{pmatrix} 1 \\ 1 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 3 \\ 4 \end{pmatrix}$$

OBTAINING A BASIS

The described method to obtain the rank of a matrix is very useful also to obtain a basis, since it determines:

1. - **if a new vector depends linearly of the previous ones,**
2. - **on which generators it depends and**
3. - **with what coefficients.**

Therefore, given a set of generators of a subspace, that is, a set whose linear combinations generate all and only the subspace vectors, it is enough to sequentially enter the generators into the algorithm, and eliminate those that depend on the previous ones.

OBTAINING A SELECTIVE BASIS

If you want to prioritize some vectors over others, to keep them into the basis, it is enough to sort them by priorities, so that the last ones are eliminated.