Intersection of two subspaces

Algebra

Taking into account the following equalities:

$$S_1 \cap S_2 = S_1 \cap \left({S_2}^{\perp}\right)^{\perp} = \left({S_1}^{\perp}\right)^{\perp} \cap S_2$$

new ways of seeing the intersection of two subspaces emerge

Three of them are:

1. $S_1 \cap S_2$: **The common elements to both subspaces**
2. $S_1 \cap \left({S_2}^{\perp}\right)^{\perp}$: **The orthogonal subspace to** ${S_2}^{\perp}$ **in** S_1.
3. $S_2 \cap \left({S_1}^{\perp}\right)^{\perp}$: **The orthogonal subspace to** ${S_1}^{\perp}$ **in** S_2.

Example 1 (Intersection of subspaces) We want to obtain the subspace intersection of the two subspaces:

$$S_1 = \mathcal{L}((1, 0, -1, 0, 1), (0, 1, 1, 1, 1)))$$
$$S_2 = \mathcal{L}((1, -1, -2, -1, 0), (0, 0, 1, 1, 0)));$$

It will be obtained in two different ways. First through

$$S_1 \cap S_2 = S_1 \cap \left(S_2^{\perp}\right)^{\perp}$$

and then through

$$S_1 \cap S_2 = \left(S_1^{\perp}\right)^{\perp} \cap S_2$$

$$S_1 = \mathcal{L}((1,\ \ 0,\ -1,\ \ 0,\ 1), (0, 1, 1, 1, 1)))$$

$$S_2 = \mathcal{L}((1, -1, -2, -1, 0), (0, 0, 1, 1, 0)))$$

$$S_1 \cap S_2 = S_1 \cap \left(S_2{}^{\perp}\right)^{\perp}$$

Iteration 1					
a_1	v_1	v_2	v_3	v_4	v_5
1	1	0	0	0	0
-1	0	1	0	0	0
-2	0	0	1	0	0
-1	0	0	0	1	0
0	0	0	0	0	1
t	1	1	-2	-1	0

Iteration 2					
a_2	w_1	v_2	v_3	v_4	v_5
0	1	1	2	1	0
0	0	1	0	0	0
1	0	0	1	0	0
1	0	0	0	1	0
0	0	0	0	0	1
t	0	0	1	1	0

Iteration 3					
	w_1	v_2	w_2	v_4	v_5
	1	1	2	-1	0
	0	1	0	0	0
	0	0	1	-1	0
	0	0	0	1	0
	0	0	0	0	1

Iteration 1		
a_1	v_1	v_2
1	1	0
1	0	1
0	-1	1
0	0	1
0	1	1
t	1	1

Iteration 2		
a_2	w_1	v_2
-1	1	-1
0	0	1
-1	-1	2
1	0	1
0	1	0
t	0	0

Iteration 3		
a_3	w_1	v_2
0	1	-1
0	0	1
0	-1	2
0	0	1
1	1	0
t	1	0

Final table		
	w_1	v_2
	1	-1
	0	1
	-1	2
	0	1
	1	0

$$S_1 \cap S_2 = \mathcal{L}(-1, 1, 2, 1, 0) \equiv \mathcal{L}\ (1, -1, -2, -1, 0)$$

$$S_1 = \mathcal{L}((1,\ \ 0,\ -1,\ \ 0,\ 1), (0, 1, 1, 1, 1)))$$

$$S_2 = \mathcal{L}((1, -1, -2, -1, 0), (0, 0, 1, 1, 0)))$$

$$S_1 \cap S_2 = \left(S_1{}^{\perp}\right)^{\perp} \cap S_2$$

Iteration 1

a_1	v_1	v_2	v_3	v_4	v_5
1	1	0	0	0	0
0	0	1	0	0	0
-1	0	0	1	0	0
0	0	0	0	1	0
1	0	0	0	0	1
t	1	0	-1	0	1

Iteration 2

a_2	w_1	v_2	v_3	v_4	v_5
0	1	0	1	0	- 1
1	0	1	0	0	0
1	0	0	1	0	0
1	0	0	0	1	0
1	0	0	0	0	1
t	0	1	1	1	1

Final table

	w_1	w_2	v_3	v_4	v_5
	1	0	1	0	-1
	0	1	-1	-1	-1
	0	0	1	0	0
	0	0	0	1	0
	0	0	0	0	1

Iteration 1

a_1	v_1	v_2
1	1	0
-1	-1	0
1	-2	1
0	-1	1
0	0	0
t	0	1

Iteration 2

a_2	v_1	w_1
0	1	0
-1	-1	0
0	-2	1
1	-1	1
0	0	0
t	0	1

Iteration 3

a_3	v_1	w_1
-1	1	0
-1	-1	0
0	-2	1
0	-1	1
1	0	0
t	0	0

Final table

	v_1	w_1
	1	0
	-1	0
	-2	1
	-1	1
	0	0

$$S_1 \cap S_2 = \mathcal{L}(1, -1, -2, -1, 0)$$