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Title:

Optimal design via nonlocal basis pursuit

Abstract

This is a joint work with Jose C. Bellido from UCLM.

We consider a nonlocal version of the classical optimal design problem of distributing a limited amount of conductive material in a given design domain [1, 2]. Starting from a dual variational formulation of this problem in terms of two-point nonlocal fluxes we turn our attention to identifying the limiting optimal problem arising when the amount of available material is driven to zero.

Such a limiting process continues to attract attention of both theoreticians and practitioners of optimal design; for example in the case of linear local elasticity the resulting limiting problem corresponds to the celebrated case of Michell trusses [3, 4].

We establish the existence of optimal two-point fluxes for the limiting nonlocal problem in Lebesgue spaces with mixed exponents. This result is in stark contrast with the local case, where optimal fluxes are Radon measures.

Finally, we turn our attention to studying the relationship between the local and the nonlocal problems, both in the case of the vanishing amount of conducting material, as the nonlocal interaction horizon is driven to zero. By relaxing the antisymmetry requirement on the nonlocal two-point fluxes and generalizing some of the well-known nonlocal estimates [5] to the case of mixed Lebesgue exponents we are able to establish the desired connection.

References:

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