J. A. Carrillo

Title

Nonlocal Aggregation-Diffusion Equations: entropies, gradient flows, phase transitions and applications

Abstract

This talk will be devoted to an overview of recent results understanding the bifurcation analysis of nonlinear Fokker-Planck equations arising in a myriad of applications such as consensus formation, optimization, granular media, swarming behavior, opinion dynamics and financial mathematics to name a few. We will present several results related to localized Cucker-Smale orientation dynamics, McKean-Vlasov equations, and nonlinear diffusion Keller-Segel type models in several settings. We will show the existence of continuous or discontinuous phase transitions on the torus under suitable assumptions on the Fourier modes of the interaction potential. The analysis is based on linear stability in the right functional space associated to the regularity of the problem at hand. While in the case of linear diffusion, one can work in the L^2 framework, nonlinear diffusion needs the stronger Linfty topology to proceed with the analysis based on Crandall-Rabinowitz bifurcation analysis applied to the variation of the entropy functional. Explicit examples show that the global bifurcation branches can be very complicated. Stability of the solutions will be discussed based on numerical simulations with fully explicit energy decaying finite volume schemes specifically tailored to the gradient flow structure of these problems. The theoretical analysis of the asymptotic stability of the different branches of solutions is a challenging open problem.

This overview talk is based on several works in collaboration with R. Bailo, A. Barbaro, J. A. Canizo, X. Chen, P. Degond, R. Gvalani, J. Hu, G. Pavliotis, A. Schlichting, Q. Wang, Z. Wang, and L. Zhang.

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