

A mathematical framework for Hyperelasticity based on (one-point) nonlocal gradients over bounded domains

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Abstract

This talk is composed of final results from [4] ([3]) and some recent follow-up ones. In particular, after some previous studies regarding bond-based peridynamics and fractional gradient functional, some relevant tools such as **nonlocal versions of the Fundamental Theorem of Calculus and Poincaré-Sobolev inequalities** are obtained for nonlocal gradients defined for $u : \Omega \cup B(0, \delta) \rightarrow \mathbb{R}$ as

$$D_{\delta}^s u(x) = c_{n,s} \int_{B(x,\delta)} \frac{u(x) - u(y)}{|x - y|} \frac{x - y}{|x - y|} \frac{w_{\delta}(x - y)}{|x - y|^{n-1+s}} dy, \quad x \in \Omega.$$

This means we would be dealing with a more realistic framework for applications, in particular Solid Mechanics than the (pure) fractional one [1], since the operators in the latter are defined over the whole space. Both cases were considered as an alternative to nonlinear bond-based peridynamic energies (double integrals) since in such case very few hyperelastic functions could be recovered after the localization process [2]. This nonlocal vector analysis is used in order to tackle a **nonlocal model of Hyperelasticity** under nonlocal Dirichlet conditions over a collar of width 2δ . Additionally, some recent results and current research lines in collaboration other authors may include further nonlocal vector calculus, Helmholtz decompositions or quasi-convexity as a characterization of weak lower semi-continuity for these operators.

References

- [1] José C. Bellido, Javier Cueto and Carlos Mora-Corral. *Fractional Piola identity and polyconvexity in fractional spaces. Annals. Poincaré.*, 2020.
- [2] José C. Bellido, Javier Cueto and Carlos Mora-Corral. *Bond-based Peridynamics does not converge to Hyperelasticity as the horizon goes to zero. Journal of Elasticity.*, 2020
- [3] José C. Bellido, Javier Cueto and Carlos Mora-Corral. *Nonlocal gradients in bounded domains motivated by Continuum Mechanics: fundamental theorem of Calculus and embeddings Preprint, Submitted.*
- [4] Javier Cueto, *Mathematical analysis of fractional and nonlocal models from nonlinear Solid Mechanics*, PhD thesis, Universidad de Castilla-La Mancha 2022