Matteo Bonforte

Title: *Nonlinear and Nonlocal Diffusions. Smoothing effects, Green functions and functional inequalities*

Abstract:

We will consider the Cauchy problem for Nonlinear Diffusion equations of porous medium type $u_t = -\mathcal{L}u^m$, with $m > 1$ and investigate whether or not integrable data produce bounded solutions. The diffusion operator belongs to a quite general class of nonlocal operators, and we will see how different assumption on the operator imply (or not) smoothing properties. We will briefly compare the approach based on Moser iteration and the approach through Green functions. On one hand, we show that if the linear case ($m = 1$) enjoys smoothing properties, also the nonlinear will do. On the other hand, we see that in some cases the nonlinear diffusion enjoys the smoothing properties also when the linear counterpart does not, thanks to the convex nonlinearity.

Following Nash' ideas, we see how smoothing properties are often equivalent to the validity of Gagliardo-Nirenberg-Sobolev (and Nash) inequalities: we explore these implications also in the nonlinear and nonlocal context and the connection with dual inequalities (Hardy-Littlewood-Sobolev) and Green function estimates.

This is a joint work with J. Endal (UAM, Madrid).

If time allows, we will complete the panorama by showing related results on Euclidean bounded domains (joint works with Figalli, Ros-Oton, Sire, Vazquez) and/or on Riemannian Manifolds (joint works with Berchio, Ganguly, Grillo, Muratori), together with a small detour on the Fast diffusion case $m < 1$ (joint work with Ibarrondo and Ispizua).