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I. Introduction

Interest rate changes are broadly recognized as a major source of uncertainty for corporations. According to survey evidence by Graham and Harvey (2001), interest rate risk is perceived by U.S. firm managers as the second most important risk factor, only behind market risk. Financial theory states that movements in interest rates affect both the firm’s expectations about future corporate cash flows and the discount rate employed to value these cash flows and, hence, the value of the firm. The impact of interest rate fluctuations on the market value of companies has received a great deal of attention in the literature, although much of the empirical research has focused on financial institutions because of the particularly interest rate sensitive nature of the banking business (Flannery and James, 1984; Staikouras, 2003 and 2006; Hahm, 2004). Nevertheless, interest rate variations may also exert a significant influence on nonfinancial corporations, principally through their effect on the financing costs and the value of financial assets and liabilities held by these firms (Bartram, 2002).

The most usual practice has been to estimate interest rate exposure as the sensitivity of the value of the firm, proxied by the firm’s stock return, to changes in interest rates within the framework of a linear regression model. This is simply an extension to equities of the duration-based approach traditionally used in bond portfolio management. More recent empirical studies have utilized, however, more sophisticated time series econometric methods in the time domain, such as cointegration analysis, Granger causality tests, vector autoregressive (VAR) models or generalized autoregressive conditional heteroskedasticity (GARCH) specifications. These approaches consider one or at most two time scales, i.e. the short run and the long run. Nonetheless, financial security markets are complex systems consisting of thousands of heterogeneous traders and investors making decisions over different time horizons (from minutes to years), which collectively determine aggregate market behavior. Therefore, it is likely that the extent of connection between interest rates and stock prices varies across time scales according to the investment horizon of investors. For example, assuming that big institutional investors have long-term horizons and, consequently, follow macroeconomic fundamentals, it seems reasonable to expect that the association between interest rates and equity prices is stronger at coarsest time scales than at the finest ones, which are more closely linked to speculative trading. In
such a context, where the strength and direction of relationships between economic and financial variables may differ according to the time scale of the analysis, wavelet techniques are of particular interest. Wavelet analysis is a comparatively new, at least in finance, and powerful tool for signal processing that takes into account both time and frequency domains. The main advantages of wavelets are their ability to decompose any signal into their time scale components, their flexibility to handle non-stationary data and their capacity to provide an alternative representation of the variability and association structure between variables on a scale-by-scale basis.¹

The primary aim of this paper is to examine the link between changes in interest rates and the Spanish stock market at the industry level across different time scales by using wavelet methods. The application of wavelet decomposition enables us to study the dynamic linkages between interest rates and stock prices in time, as well as frequency domains, opening the path to a deeper understanding of the true relationship between these variables. Specifically, three alternative and complementary wavelet tools are utilized, namely wavelet variance, correlation and cross-correlation. Knowledge of the relationship between interest rates and stock prices at different time scales is of undoubted interest for investors, portfolio managers, corporate managers and policy makers, as it provides critical information for risk management, asset allocation, portfolio management or policy making decisions.

This study contributes to the extant literature in three ways. First, to the best of our knowledge, this paper is the first one to investigate the linkage between interest rates and stock prices in the Spanish case by using a wavelet-based approach. In this regard, the Spanish equity market offers an ideal setting to assess interest rate exposure due to the great relative importance of heavily indebted, regulated and financial firms, all of them being especially vulnerable to interest rate risk, in this market. In any case, the empirical studies on the interest rate-stock market nexus in other markets based on wavelet analysis are also very scarce. Second, no previous research has examined the connection between interest rates and stock prices at the industry level by employing wavelet methods. However, the analysis on an industry basis is appropriate because market aggregation may hide significant differences among industries in terms of interest rate sensitivity. Third, the present work is also unique in the sense that a novel

¹ As noted by Schleicher (2002), the wavelet decomposition of a signal can be compared to the activity of a camera-lens. Zooming out the lens brings a broad landscape, while zooming in the lens allows one to find details that are not observable in the landscape portrait.
and efficient discrete wavelet technique is applied, which was first introduced by Murtagh et al. (2004) and is known as the Haar à trous wavelet transform. This wavelet method has an easy implementation, relaxes the strict assumption of a dyadic length (i.e. the sample size is not restricted to be a power of two), is redundant and, consequently, shift invariant. In addition, unlike other conventional wavelet functions such as the maximal overlap discrete wavelet transform, the Haar à trous wavelet (hereafter referred to as HTW) is not subject to boundary effect problems. This implies that there is no need to remove the boundary coefficients in order to avoid bias in the results, which ensures the conservation of the whole information contained in the original data. Accordingly, the HTW transform makes it possible to capture more accurately the main features and reduce the noise of signals, thus leading to a better characterization of the real interactions between interest rates and the stock market.

The results of the empirical analysis show that the Spanish stock market exhibits a remarkable degree of interest rate exposure, although sizeable differences can be observed across industries and depending on the time horizon under consideration. Unsurprisingly, regulated industries such as Utilities, heavily indebted industries such as Real Estate, Utilities or Food and Beverages, and the Banking industry emerge as the most interest rate sensitive. On the contrary, there is a broad range of industries such as Chemicals and Paper, Financials, Construction, Health Car and Industrials hardly influenced by interest rate risk. Further, the link between movements in interest rates and industry equity returns is stronger at coarser scales (low frequencies), suggesting that the role of interest rates as a major determinant of stock prices may be held only in the long run for some specific industries. As expected, the interest rate exposure is predominantly negative, indicating that Spanish firms are, on average, adversely impacted by interest rate rises. In addition, a bidirectional relationship between changes in interest rates and industry stock returns is found at higher scales.

The remainder of the paper is structured as follows. Section 2 provides an overview of the relevant empirical literature on the linkage between interest rates and stock returns. Section 3 introduces the wavelet-based approach. Section 4 describes the data used in this study. Empirical findings are presented in Section 5, while Section 6 displays the conclusions drawn from this paper.

II. Literature review
The influence of changes in interest rates on the value of companies has given rise to a prolific research activity during the past few decades. The bulk of this literature has concentrated on the banking industry due to the peculiar nature of the financial intermediation business. In particular, the maturity mismatch between banks’ financial assets and liabilities resulting from the maturity transformation function of banking institutions, i.e. the financing of long-term loans with short-term deposits, has been usually identified as the main factor responsible for the high interest rate sensitivity of banks (Flannery and James, 1984; Hahm, 2004; Czaja et al., 2009 and 2010).

Nevertheless, interest rate fluctuations may also have a significant influence on the value of nonfinancial corporations through several channels. First, interest rate rises increase the interest expense of highly leveraged companies, thus reducing cash flows available for future dividends with the consequent negative impact on share prices. Second, interest rate fluctuations have an impact on the market value of financial assets and liabilities held by nonfinancial firms. Third, movements in interest rates affect the opportunity cost of equity investments. Higher interest rates make bonds more attractive given their risk-return characteristics, which motivates investors to adjust their portfolios by buying bonds and selling stocks, thus depressing stock prices. Fourth, changes in interest rates may impact upon the level of real activity in the economy in the short to medium term, and this affects equity prices by altering the expectations of future cash flows.

A broad consensus emerges from this body of literature regarding several relevant issues. Firstly, the empirical research in this area generally reports a significant negative effect of changes in market interest rates on stock returns of both financial and nonfinancial companies (Lynge and Zumwalt, 1980; Prasad and Rajan, 1995; Dinenis and Staikouras, 1998; Reilly et al., 2007). However, more recent work (Ryan and Worthington, 2004; Czaja et al., 2009; Korkeamäki, 2011) suggests that the interest rate exposure has declined over time primarily due to the increased availability of improved tools for managing interest rate risk. The extraordinary growth in interest rate derivative markets and the expansion of corporate bond markets may have played a critical role in this context. Secondly, the sensitivity of stock returns to movements in long-term interest rates is substantially greater than the sensitivity to changes in short-term rates (Oertmann et al., 2000; Bartram, 2002; Czaja et al., 2009; Ferrer et al., 2010). Thirdly, nonfinancial firms in regulated and/or highly indebted industries such as Utilities,
Electricity, Real Estate and Technology and Telecommunications are commonly recognized as the most interest rate sensitive (Sweeney and Warga, 1986, Bartram, 2002; Reilly et al., 2007; Ferrer et al., 2010). Two primary reasons help to explain this result. First, the profits and, consequently, stock prices of heavily indebted corporations are strongly dependent on interest rate developments, as the cost of their debt is directly related to the level of interest rates. Second, regulated companies such as utilities adjust the prices of their products and services with some lag behind cost increases due to the constraints imposed by regulators. This contributes to strengthening the negative impact of interest rate rises on the stock prices of these firms.

So far, the empirical literature on the relationship between interest rates and stock prices has been developed essentially in the time domain by using a broad range of time series econometric models, including simple linear regression (Leibowitz, 1986; Hahm, 2004; Reilly et al., 2007; Korkeamäki, 2011), VAR techniques (Campbell and Ammer, 1993; Bjornland and Leitemo, 2009; Laopodis, 2010; Shah et al., 2012), cointegration analysis (Chan et al., 1997; Das, 2005; Hatemi-J and Roca, 2008), Granger causality tests (Rahman and Mustafa, 1997; Alagamar and Bhar, 2003; Panda, 2008; Farsio and Fazel, 2010), nonlinear models (McMillan, 2001; Bartram, 2002; Ferrer et al., 2010), and GARCH type models (Elyasiani and Mansur, 1998; Faff et al., 2005; Kasman et al., 2011).

There are, however, a few recent papers examining the interest rate-stock market link through the wavelet multiscaling approach (Kim and In, 2007; Çifter and Ozun, 2008; Hamrita and Trifi, 2011; Tiwari, 2012). These studies are based on national stock markets of various countries and conclude that the connection between interest rates and share prices is scale dependent, increasing in importance at coarser time scales. All these works, with the only exception of Tiwari (2012), utilize a variant of the discrete wavelet transform, which is known as the maximal overlap discrete wavelet transform (hereafter referred to as MODWT). This transform has become the most frequently used wavelet in prior research on the dynamic relationships between economic and financial variables due to various attractive properties, but, as argued by Jammazi (2012 b), it also has some drawbacks. First, the MODWT is affected by boundary conditions, which leads to biased estimates of the wavelet variance and, hence, to spurious and misleading results (Percival and Walden, 2000). To get an unbiased estimator of the wavelet variance, all coefficients influenced by boundary conditions are removed. Given that the
number of boundary elements increases with scale, there are many more boundary coefficients at higher scales. Thus, the wavelet variance estimator obtained is unable to adequately represent the full information contained in the original signal, as the remaining coefficients at coarser scales only include a small amount of data. This loss of edge information inherent to the MODWT causes, therefore, serious problems to characterize the real interactions between variables, especially at coarsest scales.

Second, the MODWT displays a high sensitivity to the choice of the filter width. As is well known, there is a trade-off between the length of the filter and the boundary-affected coefficients. A longer filter width produces a smoother representation of the signal and reduces the possible appearance of artifacts in the results, but the number of coefficients influenced by boundary conditions is greater. The MODWT algorithm is very often associated to the Daubechies least asymmetric (LA) wavelet filter of length L=8, denoted by LA(8), as it allows the most accurate alignment in time between wavelet coefficients at various scales and the original time series. However, the LA(8) filter is not suitable for encircling the nonlinearity and chaotic behavior of many economic and financial time series (e.g. oil price, inflation, stock returns). Given the shortcomings of the LA(8) filter and in order to minimize the boundary coefficients, a smaller filter width should be employed. Nevertheless, the use of a restricted (smaller) filter width is not desirable since it may decrease the optimal degree of smoothness in the reconstructed series and yield more serially correlated coefficients at different scales.

This study differs from the above mentioned works in that we utilize the HTW transform, a new and robust wavelet technique rarely applied in the economic-finance area. As a matter of fact, the works of Jammazi and Aloui (2010) and Jammazi (2012 a, b) are the only ones that have employed so far the HTW within a financial framework, particularly with the aim of investigating the linkage between oil price changes and stock markets. The HTW algorithm overcomes the main weaknesses of the MODWT thanks to its properties of redundancy and lack of boundary problems, thereby enabling a better characterization of the real interactions between interest rates and the stock market.

Most of the literature on the interdependence between interest rates and stock prices has focused to date on a few countries with highly developed financial markets, especially the U.S. and more recently Germany, the U.K., Japan or Australia. Despite this, there
are some recent works addressing this issue in the Spanish case (Soto et al., 2005; Jareño, 2008, Ferrer et al., 2010; Jareño and Navarro, 2010). All these studies, based primarily on multifactor linear regression models, provide evidence of a significant effect of interest rate movements on firms’ stock returns, confirming the interest rate sensitive nature of the Spanish market.

III. Data Set

This investigation deals with the Spanish market over the period from January 1993 to December 2012. The sample period starts in January 1993 in order to avoid possible distortions in the interest rate-stock market link resulting from the turbulences in financial markets within the context of the European exchange rate mechanism (ERM) crisis during the second half of 1992. Following Campbell (1987), Kim and In (2007), Reilly et al. (2007) and Korkeamäki (2011), among others, monthly data series are employed (240 observations). Monthly data are preferred to weekly or daily data for a number of reasons. Firstly, monthly data contain less noise and can therefore better capture the interactions between interest rates and stock prices. Secondly, monthly data have smaller biases due to nonsynchronous trading of some individual stocks. Thirdly, the results in terms of smoothness and distinction among the different time horizons produced by wavelet analysis on monthly data are much harder to achieve with higher frequency data. Consequently, to find similar results to those obtained with monthly data, a very large number of decomposition levels are required when using weekly or daily data.

The equity market data being used consist of stock prices of all firms listed on the Spanish Stock Exchange for at least one full year of the sample period (a total of 249 companies), which have been extracted from the Madrid Stock Exchange database. Interest rates are measured by the 10-year Spanish government bond yield, collected from the Bank of Spain’s database. This choice has become increasingly popular in the literature on interest rate exposure (Elyasiani and Mansur, 1998; Oertmann et al., 2000; Faff et al., 2005; Ballester et al., 2011). Long-term interest rates incorporate market expectations about future prospects for the economy and determine to a large extent the cost of borrowing funds. Accordingly, long-term rates presumably will have a critical influence on investment decisions and profitability of firms and, hence, on their stock market performance. Both stock prices and interest rates represent end-of-the month observations.
In line with previous research on corporate interest rate exposure (Sweeney and Warga, 1986; Bartram, 2002; Reilly et al., 2007; Ferrer et al., 2010), this study is conducted at the industry level. Thus, value-weighted industry portfolios are constructed from individual stock prices for the fourteen main Spanish industries, namely Consumer Goods, Consumer Services, Technology and Telecommunications, Real Estate, Banking, Financials, Utilities, Construction, Chemicals and Paper, Basic Resources, Health Care, Food and Beverages, Industrials, and Energy. Various reasons are usually put forward to justify an industry-based approach. First, the formation of industry portfolios provides an efficient way of condensing a sizable amount of information regarding stock price behavior. Second, the use of portfolios helps to smooth the noise in the data produced by transitory shocks in individual stocks, thereby yielding more reliable results. It should also be noted that the analysis on a sector basis allows identifying differences among industries in terms of interest rate sensitivity, which may have relevant implications for risk management, asset allocation, monetary policy or asset pricing. Equity industry returns are defined as the first log difference of industry stock price indices. In turn, movements in interest rates are calculated as the first differences in the level of interest rates between two successive months.

Table 1 provides some summary descriptive statistics of the variables. The mean monthly return over the study period is positive for most industries because of the overall increasing trend in stock prices. The mean of monthly changes in 10-year government bond yields is, however, negative, reflecting the downward trend in Spanish long-term rates over that period. Based on the standard deviation, all industry equity returns have, unsurprisingly, higher volatility than the series of changes in 10-year Spanish bond yields. The measures of skewness indicate that the majority of industry stock returns are negatively skewed, meaning that negative shocks are more common than positive ones in the Spanish market. Further, all equity industry return series exhibit a level of kurtosis substantially larger than three, thereby implying leptokurtic distributions with higher peaks and fatter tails around the mean in comparison to the standard normal distribution. The Jarque-Bera test statistics confirm this result, rejecting the null hypothesis of normal distribution in all cases at the 1% significance level. A similar distributional picture emerges for the 10-year government bond yield change series. In order to determine the order of integration of the variables, the standard Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) unit root tests
are conducted. The results from both tests indicate that the series of changes in 10-year bond yields (first differences) and market and industry equity returns (log differences) are all stationary (integrated of order zero). This finding is consistent with earlier works on the linkage between interest rate fluctuations and stock returns (Rahman and Mustafa, 1997; Broome and Morley, 2000; Çifter and Ozun, 2008; Czaja et al., 2009).

**INSERT TABLE 1 ABOUT HERE**

Figure 1 presents the evolution of the Spanish equity market index and the 10-year Spanish government bond yield over the period from January 1993 to December 2012. The Spanish stock market exhibits a general upward trend during most of the analysis period, only interrupted by the Internet bubble burst in March 2000 and the global financial crisis from late 2007. On the contrary, the yields on 10-year government securities display a downward trend up to the start of the sovereign debt crisis in the euro zone during the spring of 2010. As can be seen in this figure, the connection between stock prices and 10-year bond yields over the entire sample period is somewhat unclear. In particular, the Spanish market index and 10-year government bond yields have moved predominantly in opposite directions until approximately mid-1998. Since then, the link between both variables is more ambiguous, although a positive correlation seems to exist for most of the 2000s.

**INSERT FIG. 1 ABOUT HERE**

**IV. Econometric Methodology**

*Wavelet analysis: general considerations*

Wavelet analysis is a technique for signal decomposition that has its origins in classical Fourier analysis. However, the wavelet transform overcomes the major limitations of the Fourier transform as it combines information from both time and frequency domains, decomposing any signal into its time scale components. Further, it preserves time information and does not require stationarity of the signal. Although the wavelet theory has rapidly expanded in many areas of applied sciences (Physics, Engineering, signal and image processing, Astronomy, Climatology, etc.), its application in economics and finance is only a recent phenomenon (see, among others, Ramsey and Lampart, 1998 a, b; Gençay et al., 2002; Gallegati, 2012; Reboredo and Rivera-Castro, 2013 a, b). In addition, wavelet methods have been widely used to study interdependence between economic and financial time series, helping to uncover
interactions which are hard to detect by using any other econometric technique and which would otherwise remain hidden.

A wavelet is essentially a small wave which grows and decays in a limited time period. There are two main classes of wavelets: the continuous wavelet transform (CWT) and its discrete counterpart (DWT). As noted by Percival and Walden (2000), the majority of the wavelet analysis applications in the economic field concentrate exclusively on the DWT because it is a more natural way of handling discrete time series such as those commonly used in economics and finance. The intuitively most appealing procedure to compute the DWT is the pyramid algorithm derived by Mallat (1989), which consists of recursively applying two wavelet filter functions to a given time series: the father wavelet $\Phi$ and the mother wavelet $\Psi$. The father and the mother wavelets are formally defined by the functions:

$$\Phi_{j,k}(t) = 2^{-j/2}\Phi(2^j t - k) \quad (1)$$
$$\Psi_{j,k}(t) = 2^{-j/2}\Psi(2^j t - k) \quad (2)$$

where $j = 1, \ldots, J$ is the scaling parameter in a $J$-level decomposition and $k$ denotes a translation parameter. The father wavelet $\Phi$ integrates to one and is used to represent the smooth and low frequency parts of a signal, whereas the mother wavelet $\Psi$ integrates to zero and describes the details or high-frequency components of a signal. Father wavelets generate what are known as scaling or approximation coefficients and the mother wavelets generate the wavelet or detail coefficients. In other words, the father wavelet acts as a low-pass filter, whereas the mother wavelets act as high-pass filters. A number of wavelet families are available in the literature, for example Haar, Daubechies, Symmlets, Coiflets, Morlet, à trous, etc.

The basic aim of wavelet analysis is to project the time series of interest onto a sequence of father and mother wavelets from a specific family. Thus, the multiresolution representation of a signal $X_t$ is as follows:

$$X_t = \sum_k s_{j,k} \Phi_{j,k}(t) + \sum_k w_{j,k} \Psi_{j,k}(t) + \cdots + \sum_k w_{1,k} \Psi_{1,k}(t) \quad (3)$$

where $J$ is the number of multiresolution levels or scales and $k$ ranges from 1 to the number of coefficients in the specified level. The coefficients $s_{j,k}$ and $w_{j,k}$ represent the scaling and wavelet coefficients, respectively, and are given by:
The wavelet coefficients $w_{j,k}$ capture the high frequency content present in the signal, while the scaling coefficients $s_{j,k}$ represent the smooth behavior of the signal at the coarsest scale.

In turn, the original signal can be reconstructed as:

$$X_t = S_J(t) + W_J(t) + W_{J-1}(t) + \cdots + W_1(t)$$

where $S_J = \sum_{k} s_{j,k} \Phi_{j,k}(t)$ and $W_j = \sum_{k} w_{j,k} \psi_{j,k}(t)$, $j = 1, \ldots, J$. The functions $S_j$ and $W_j$ are known as approximation (smooth) and wavelet (detail) components, respectively. The approximation represents the trend component of the time series at the level $J$, while the wavelet or detail component captures deviations from that trend at each decomposition level $j$.

The DWT is often referred to as the decimated transform as the pyramid algorithm is derived by successive downsampling (reduction of the number of coefficients at longer scales), in which only alternate observations are picked up at each sub sampling stage. Nevertheless, the DWT has a number of serious shortcomings, such as the lack of translation invariance, the dyadic length requirement, and the existence of boundary effects. As a result, the application of an undecimated DWT (no down-sampling is applied) seems more appropriate.

*The Haar à trous wavelet (HTW) transform*

In this study, the Haar à trous (with holes) wavelet transform developed by Murtagh et al. (2004) is used as an alternative to conventional DWT. The HTW constitutes a non-decimated or redundant version of the usual Haar wavelet function and offers several advantages over standard wavelet algorithms that make it suitable for time series analysis, regression and forecast applications. First, the redundancy inherent in the à trous wavelet implies that all wavelet components have the same length as the original time series, so it is easy to relate information at each resolution scale for the same time point. This leads to smoother approximations of the signal by filling the “gap” caused by decimation (Zhang et al., 2001). The redundancy property also makes the à trous wavelet transform be shift invariant, i.e. the wavelet coefficients do not change with
arbitrary time shifts. Second, the Haar wavelet respects the asymmetric nature of the
time-varying signal, calculating the scaling and wavelet coefficients only from data
obtained previously in time. In this context, Murtagh et al. (2004) argue for using the
two wavelet techniques (à trous and Haar) together in order to gain the advantages of
both. Thus, the HTW transform provides a convincing and computationally very
straightforward solution to troublesome time series boundary effects that arise with
standard decimated wavelet transforms (Murtagh et al., 2004). In particular, the HTW
is not subject to downsampling and, therefore, considers the whole information
contained in the original series, preserving the intrinsic characteristics of the data when
decomposed into a scale-dependent set of components. In addition, the simplicity and
ease of implementation of this algorithm have also contributed to the choice thereof.

The HTW is a redundant version of the Haar wavelet transform that uses a simple non-
symmetric low-pass filter $h$ equal to $\left(\frac{1}{2}, \frac{1}{2}\right)$, and can be described as follows. The scaling
or smooth coefficients $s_{j+1,k}$ of a time series $X_t$ at any scale can be obtained by
convolving the smoothed version of the signal at the previous scale with the wavelet
filter $h = \left(\frac{1}{2}, \frac{1}{2}\right)$:

$$s_{j+1,k} = \frac{1}{2} \left( s_{j,k-1} + s_{j,k} \right)$$

(7)

In keeping with the literature, the finest scale $s_{0,t}$ is the original series $X_t$ ($s_{0,t} = X_t$).

Then, the detail or wavelet coefficients at higher scale can be computed as the
difference between the smoothed versions of the signal at two consecutive resolution
levels:

$$w_{j+1,k} = s_{j,k} - s_{j+1,k}$$

(8)

Clearly, the future data after $t$ are never involved in the calculation of the wavelet
coefficients at any time point $k$.

**Haar à trous wavelet (HTW) variance, correlation and cross-correlation**

Wavelet techniques can be adapted to accommodate the analysis of time-frequency
dependencies between pairs of series. As pointed by Gallegati (2012), the wavelet

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2 In particular, the boundary problem means that the decomposed values near both edges can be distorted. The boundary effects usually appear with symmetric wavelets, which take into account not only previous information but also future information and, therefore, those values should not be used for prediction.

3 For a more complete and detailed discussion of the Haar à trous wavelet transform, see Murtagh et al. (2004) and Jammazi (2012 b).
coefficients can be straightforwardly manipulated to achieve several recognizable statistical quantities such as wavelet variance, wavelet correlation, and wavelet cross-correlation. The wavelet variance decomposes the variance of a time series on a scale-by-scale basis and constitutes a useful tool for determining what time scales are the dominant contributors to the overall variability of a series (Percival and Walden, 2000). Let $X_t$ be a stationary stochastic process with variance $\sigma^2_{X}$. If $\sigma^2_{X}(\tau_j)$ denotes the wavelet variance at scale $\tau_j$, then the following relationship holds:

$$\sigma^2_{X} = \sum_{j=1}^{\infty} \sigma^2_{X}(\tau_j)$$

(9)

where $\sigma^2_{X}(\tau_j)$ represents the contribution of the changes at scale $\tau_j$ to the total variance of the process. This relationship states that the wavelet variance provides an exact decomposition of the variance of a time series into components associated to different time scales.

For a stationary process, the wavelet variance at scale $\tau_j$ is independent of time and can be defined as (see Gallegati, 2008):

$$\sigma^2_{X}(\tau_j) = \frac{1}{2\tau_j} \text{var}(w_{j,t})$$

(10)

where $\tau_j = 2^{l-1}$ and $w_{j,t}$ are the wavelet coefficients of $X_t$ at scale $\tau_j$.

Thus, according to Jammazi (2012 b), the HTW variance estimator can be expressed as the normalized sum of the squared wavelet coefficients:

$$\hat{\sigma}^2_{X,HTW}(\tau_j) = \frac{1}{2/N_j} \sum_{t=0}^{N_j-1} \hat{w}_{j,t}^2 = \frac{1}{N} \sum_{t=0}^{N_j-1} \hat{w}_{j,t}^2$$

(11)

where $\hat{\sigma}^2_{X,HTW}(\tau_j)$ denotes the estimator of the wavelet variance at scale $\tau_j$, $N_j = N/2^j$ represents the number of wavelet coefficients at the resolution level $j$ and $N$ is the sample size.

As mentioned earlier, the HTW transform is not affected by boundary conditions. Therefore, unlike the standard wavelet transforms such as the MODWT, the HTW
variance estimator allows analyzing a signal by using all the wavelet coefficients (Jammazi, 2012 b).

As with the traditional wavelet transforms, the HTW framework enables to derive the wavelet covariance, correlation and cross-correlation in order to quantify the degree of association between two stochastic processes on a scale-by-scale basis. The wavelet covariance decomposes the sample covariance into different time scales (Whitcher et al., 2000). Similar to its classical counterparts, the wavelet covariance between two processes $X_t$ and $Y_t$ at scale $\tau_j$, $\gamma_{X,Y}(\tau_j)$, is given by:

$$
\gamma_{X,Y}(\tau_j) = \frac{1}{2\tau_j} \text{cov}(\tilde{W}_{j,t}^X, \tilde{W}_{j,t}^Y)
$$

As noted by Jammazi (2012 b), the HTW covariance estimator at scale $\tau_j$ can be also expressed in terms of the wavelet coefficients at that scale:

$$
\hat{\gamma}_{X,Y}^{HTW}(\tau_j) = \frac{1}{N} \sum_{t=0}^{N-1} \tilde{W}_{j,t}^X \tilde{W}_{j,t}^Y
$$

Since the covariance does not take into account the variation of the univariate time series, it is natural to introduce the concept of wavelet correlation, which represents a powerful tool for the detection of potential linkages between two variables for a given decomposition level $j$. The wavelet correlation coefficient, $\rho_{X,Y}(\tau_j)$, offers a standardized measure of the connection between the two processes $X_t$ and $Y_t$ on a scale-by-scale basis. As in the computation of the usual correlation coefficient, the wavelet correlation estimator is simply the ratio of the wavelet covariance for $\{X_t, Y_t\}$ and the square root of their wavelet variances at each scale:

$$
\hat{\rho}_{X,Y}(\tau_j) = \frac{\gamma_{X,Y}(\tau_j)}{\sigma_X(\tau_j)\sigma_Y(\tau_j)}
$$

Just like the classical correlation coefficient between two random variables,

$$
|\rho_{XY}(\tau_j)| \leq 1.
$$

The HTW correlation coefficient at scale $\tau_j$ can be derived as:

$$
\hat{\rho}^{HTW}_{X,Y}(\tau_j) = \frac{1}{N} \sum_{t=0}^{N-1} \tilde{W}_{j,t}^X \tilde{W}_{j,t}^Y
$$

Finally, the wavelet cross-correlation provides a convenient method of disentangling the cross-correlations on a scale-by-scale basis, thus enabling to determine the contribution of each scale to the overall cross-correlation dynamics. The cross-correlation function
measures the degree of association between two variables as a function of time lag ($\lambda$). The cross-correlation coefficient at scale $\tau_j$ and lag $\lambda$, $\rho_{X,Y,\lambda}(\tau_j)$, can be obtained by making use of the wavelet cross-covariance $\gamma_{X,Y,\lambda}(\tau_j)$ and the square root of the wavelet variances $\sigma_X(\tau_j)$ and $\sigma_Y(\tau_j)$ as follows:

$$\rho_{X,Y,\lambda}(\tau_j) = \frac{\gamma_{X,Y,\lambda}(\tau_j)}{\sigma_X(\tau_j)\sigma_Y(\tau_j)}$$ (16)

Just as the standard cross-correlation coefficients, the wavelet cross-correlation coefficients take values between 0 and 1.

In turn, the HTW cross-correlation coefficient at scale $\tau_j$ may be calculated as follows:

$$\hat{\rho}_{X,Y,\lambda}^{HTW}(\tau_j) = \frac{1}{N} \sum_{t=0}^{N-1} \hat{\omega}_{t+\lambda} X \hat{\omega}_{t} Y$$ (17)

V. Empirical findings

This section examines the link between changes in 10-year Spanish government bond yields and industry equity returns through wavelet analysis. Specifically, the HTW transform is applied and the data are decomposed up to scale 6. Since monthly data are used, the scale 1 represents the highest frequency and is associated with a time horizon of 2 to 4 months. In turn, scales 2 to 6 correspond to 4-8, 8-16, 16-32, 32-64 and 64-128 monthly periods, respectively.

Fig. 2 illustrates the HTW variance of industry stock returns and 10-year bond yield movements over time scales up to scale 6 for the whole sample. Two things are noteworthy from this figure. First, there is an approximate linear relationship with negative slope between the wavelet variance and the scale, suggesting that the HTW variances of interest rate changes and industry returns decline as the wavelet scale increases. This greater stability in the long run seems to indicate that investors with short-term horizons face higher risks than investors with longer investment horizons. Second, the HTW variances of all industry equity returns are higher than those of 10-year government bond yield fluctuations over all time scales. This finding is consistent with that of Kim and In (2007) for the G7 countries, confirming that the stock market is more volatile than the public debt market regardless of the investment horizon. It is also worth noting that all industry returns show a similar behavior of wavelet variances.

INSERT FIG. 2 ABOUT HERE
The results from the application of wavelet correlation analysis on bivariate systems composed of movements in 10-year government bond yields and each of industry equity returns all over the six time scales are presented in Fig. 3. The dashed lines depict the 95% confidence interval of the corresponding estimates of HTW correlation, which are represented by the solid black line. As can be seen, the estimated wavelet correlation between 10-year bond yield changes and stock returns is highly dependent on the time scale, in the sense that the HTW correlation tends to increase (in absolute value) as the time scale increases. Thus, the wavelet correlation is not significantly different from zero at the shortest scales, i.e. scales 1 and 2, for a large number of industries. In contrast, the HTW correlation is statistically significant at the coarsest scales, i.e. scales 5 and 6, for most industries.

Furthermore, the wavelet correlation analysis reveals a considerable heterogeneity among industries in terms of the extent of interest exposure. In particular, the Utilities, Real Estate, Banking, Food and Beverages, and Technology and Telecommunications industries emerge as the most vulnerable to interest rate risk. This finding is in accordance with the pattern of interest rate sensitivity reported by, among others, Sweeney and Warga (1986), Bartram (2002), Reilly et al. (2007) and Ferrer et al. (2010), based on different methodologies (primarily OLS) and covering various countries over different time periods. On the contrary, a good number of industries such as Chemicals and Paper, Financials, Construction, Industrials, and Health Care are identified as sectors hardly influenced by interest rate changes.

The sign of the HTW wavelet correlation coefficients is mostly negative, implying that Spanish firms are, on average, hampered by interest rate rises. This result is in line with prior empirical research focused on different countries (Kim and In, 2007; Reilly et al., 2007; Ferrer et al., 2010; Korkeamäki, 2011) and is also consistent with the conventional wisdom that there exists an inverse relationship between interest rates and equity prices. Interestingly, a pattern of positive significant interest rate exposure is, however, observed at the coarsest scales for some industries such as Basic Resources, Chemicals and Paper, and Health Care. A possible explanation for this finding is related to the pro-cyclical nature of these sectors. Specifically, the low and relatively stable levels of interest rates in force for most of the period of study can be interpreted as a sign of a weak economy. Given that the long-term stock market performance of pro-cyclical industries is largely determined by economic prospects, it is not surprising that
a positive significant correlation between changes in 10-year bond yields and stock returns of these industries is found over long time horizons (more than 32 months). The wavelet correlation analysis shows, therefore, that 10-year interest rates can be regarded as a major determinant of Spanish stock prices in the medium and long-term, but not in short time horizons. This result corroborates those previously obtained by Kim and In (2007), Çifter and Ozun (2008) and Hamrita and Trifi (2011) for other countries also using wavelet methods.

Since wavelet correlation does not provide information regarding possible lagged interest rate effects on equity returns or vice versa, wavelet cross-correlation is computed in order to shed light on the lead-lag relationship between these variables across different frequencies. Fig. 4 reports the HTW cross-correlation between 10-year bond yield fluctuations at time \( t \) and industry stock returns at time \( t-\lambda \) and \( t+\lambda \) up to 24-month lags, with the corresponding approximate confidence intervals, against time leads and lags for all scales.\(^4\) The estimated HTW cross-correlation coefficients show that, just like the contemporaneous HTW correlation, the magnitude of the association increases with the wavelet scale. At the shortest scales, i.e. scales 1 and 2, the lead-lag relationship is insignificant for virtually all leads and lags regardless of the industry. However, at the coarsest scales, mainly at scales 5 and 6, the cross-correlation dynamics becomes more complex. In particular, a significant bidirectional relationship is found at scales 5 and 6 between movements in 10-year bond yields and industry equity returns, where stock returns lead positively and interest rate fluctuations lead negatively, and the leading as well as the lagging periods increase as the time scale increases. The existence of feedback effects between 10-year government bond yields and stock prices over long time horizons is not an abnormal finding since bonds and equities are close substitutes for investors with long horizons and, consequently, it seems natural that they move together and mutually influence each other.

Overall, the empirical evidence from the wavelet multiresolution analysis indicates that the link between changes in 10-year bond yields and industry equity returns is not fixed over various time scales, confirming that the degree of connection between interest rates

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\(^4\) Due to space restrictions and given that the HTW cross-correlation analysis shows a similar pattern for all industries, Fig. 4 only displays the cross-correlation coefficients corresponding to Utilities, Banking, and Consumer Goods industries. The full set of results from the wavelet cross-correlation analysis is available from the authors upon request.
and the stock market increases with the time frame (from daily, to monthly and to quarterly data), a result well documented in the empirical literature on this issue.

INSERT FIG. 4 ABOUT HERE

VI. Concluding Remarks

This paper investigates the relationship between movements in interest rates and the Spanish stock market from an industry perspective across time and frequencies, by using a wavelet-based framework. Wavelet analysis is based on decomposing time series at different time scales and provides a natural platform for characterizing the structure of the association between time series on a scale-by-scale basis. In particular, a novel and flexible non-decimated wavelet technique known as the Haar à trous wavelet transform is employed, developed by Murtagh et al. (2004) and which overcomes the main limitations of standard wavelets.

Our empirical results indicate that Spanish industries show, in general, a significant interest rate sensitivity, although the extent of interest rate exposure varies greatly across industries and depending on the time horizon under consideration. In the first place, it is found that changes in 10-year Spanish government bond yields seem to have very little effect on the stock returns of a wide array of industries such as Chemicals and Paper, Financials, Construction, Health Care, and Industrials. In contrast, Utilities, Banking, Real Estate and Food and Beverages are identified as the industries most significantly affected by 10-year bond yield fluctuations. This marked heterogeneity across sectors is in line with earlier work in this area (Sweeney and Warga, 1986, Bartram, 2002; Reilly et al., 2007; Ferrer et al., 2010), confirming that regulated, heavily indebted or with substantial financing needs and banking industries are also the most susceptible to interest rate risk in the Spanish case.

Likewise, the link between movements in interest rates and industry equity returns is weak at the shortest scales, but it becomes stronger at longer horizons corresponding to low frequencies. This finding agrees with the idea that investors with long-term horizons are more likely to follow macroeconomic fundamentals, such as interest rates, in their investment decisions than investors with shorter perspectives. Therefore, it can be stated that the role of interest rates as a key driver of stock market performance of Spanish industries only holds in the long run and for certain industries. As expected, the linkage between interest rate changes and stock returns is primarily negative, suggesting
that Spanish companies are, in general, favoured by falls in interest rates. In addition, the wavelet cross-correlation analysis provides evidence of feedback effects between interest rate fluctuations and industry equity returns at the longest scales.

The evidence presented in this study may be very helpful for the assessment of potential sector-based diversification opportunities by investors, for the design and implementation of adequate interest rate risk management strategies by firm managers and investors, for asset allocation decisions by portfolio managers and for the formulation of appropriate monetary policy measures by governments.
References


22


Table 1. Sample descriptive statistics of industry equity returns and 10-year Spanish government bond yield changes, January 1993-December 2012

<table>
<thead>
<tr>
<th>Returns and factors</th>
<th>Mean</th>
<th>Median</th>
<th>Min.</th>
<th>Max.</th>
<th>Std. Dev.</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>JB statistic</th>
<th>ADF statistic</th>
<th>PP statistic</th>
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<tbody>
<tr>
<td>Consumer Goods</td>
<td>0.0094</td>
<td>0.0101</td>
<td>-0.2173</td>
<td>0.2039</td>
<td>0.0659</td>
<td>-0.54**</td>
<td>4.16***</td>
<td>24.85***</td>
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<td>Consumer Serv.</td>
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<td>0.2395</td>
<td>0.0643</td>
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<td>6.05***</td>
<td>115.45***</td>
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<td>0.0047</td>
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<td>0.0804</td>
<td>0.03</td>
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<td>-15.87***</td>
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<td>0.0909</td>
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<td>6.63***</td>
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<td>-0.56**</td>
<td>5.26***</td>
<td>63.26***</td>
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<td>0.0000</td>
<td>-0.2544</td>
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<td>0.0549</td>
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<td>299.28***</td>
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<td>Basic Resources</td>
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<td>Food &amp; Beverages</td>
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<td>0.0029</td>
<td>-0.1558</td>
<td>0.2125</td>
<td>0.0405</td>
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<td>8.25***</td>
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<td>38.36***</td>
<td>-14.71***</td>
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Notes: The table presents some descriptive statistics of the monthly industry equity returns and 10-year government bond yield changes, including mean, median, standard deviation (Std. Dev.), minimum (Min.) and maximum (Max.) values and also skewness and kurtosis measures. JB denotes the statistic of the Jarque-Bera test for normality. The last two columns present the results of the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) unit root tests, respectively. *, **, *** denote statistical significance at the 10%, 5% and 1% levels, respectively.
Fig. 1. Evolution of the Spanish equity market index and the 10-year Spanish government bond yield from January 1993 to December 2012
Fig. 2. Wavelet variance of changes in 10-year Spanish government bond yields and industry equity returns
Fig. 3. Wavelet correlation between 10-year Spanish government bond yield changes and industry equity returns
Fig. 3. (Cont.). Wavelet correlation between 10-year Spanish government bond yield changes and industry equity returns.
Fig. 4. Wavelet cross-correlation between 10-year Spanish government bond yield changes and industry equity returns

a. Consumer Goods
Fig. 4. (Cont). Wavelet cross-correlation between 10-year Spanish government bond yield changes and industry equity returns

b. Banking

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Fig. 4. (Cont.). Wavelet cross-correlation between 10-year Spanish government bond yield changes and industry equity returns

c. Utilities

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