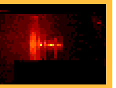
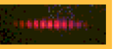




MIDDLE TERM EXAM A3. ENGLISH GROUP. DECEMBER 23th 2011

AGRICULTURAL ☐

FORESTRY ☐

FULL NAME ______________________________

Instructions. Available time for the exam: 2 h. Begin by answering the theory question using the colored sheet (neither books nor class notes are allowed for this part).Once you have handed in your answer to this question, you can use books and class notes to solve the problems.
Well-founded arguments are required in your answers. Gratuitous statements will not be accepted.
Put your full name on every sheet you use.
This sheet must be returned when the exam is over.

THEORY (3 p) Archimedes' Principle. Brief explanation.

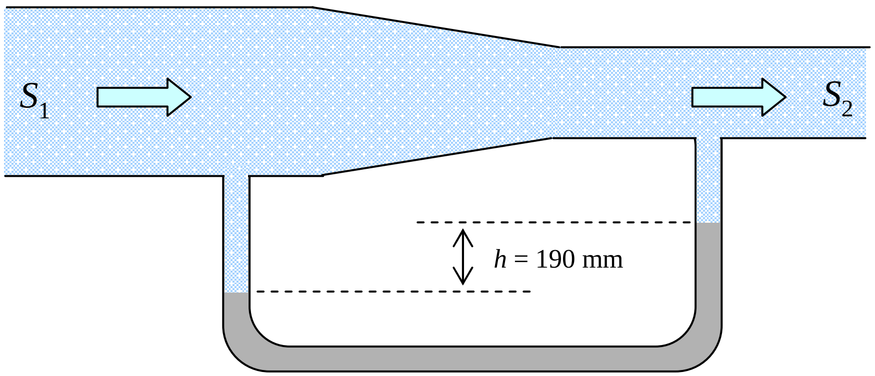
PROBLEM 1 (3 p)

A fluid of density $\rho = 1$ g/cm^3 passes through a pipe of cross-sectional area $S_1 = 16$ cm^2 that has a constriction of cross-sectional area $S_2 = 5$ cm^2. The two parts of the pipe are connected with a U-tube manometer partially filled with a liquid of density $\rho_f = 13.6$ g/cm^3. The pressure difference is measured by the difference in the levels of the liquid in the U-tube, $h = 190$ mm.

(a) Explain whether the velocity of the fluid at the wide section S_1 of the pipe is higher or lower than at the narrower section S_2.

(b) Find the flow rate of the fluid and the velocities at both sections of the pipe.

(c) Find the pressure difference between both sections of the pipe.



PROBLEM 2 (4 p)

Three moles of nitrogen at a pressure of $2.5 \cdot 10^5$ Pa initially occupy a volume of $3 \cdot 10^{-2}$ m^3. The gas is carried through a cycle consisting of three steps:

1→2. The gas is expanded isothermally until it reaches a volume of $5 \cdot 10^{-2}$ m^3.

2→3. The gas is heated at constant volume until it is back to its initial pressure of $2.5 \cdot 10^5$ Pa.

3→1. The gas is cooled at constant pressure until it is back to its initial state.

a) Plot a schematic diagram of the cycle in *p*-*V* coordinates.

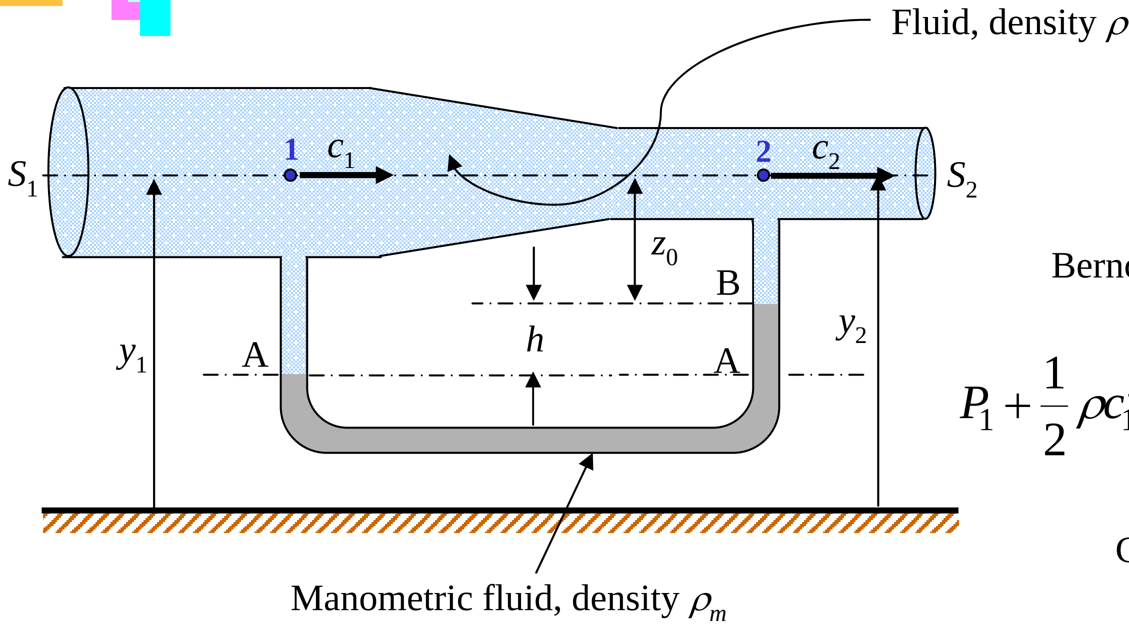
b) Calculate the *p*-*V*-*T* coordinates not given in the heading of each point 1, 2, 3.

c) Find the work of each step.

d) Find the total heat added to the gas during the cycle.

Consider the nitrogen as an ideal gas with $c_V = 5R/2$ J/(mol·K) and $\gamma = 1.4$.
Constant of the gasses $R = 8.314$ J/(K·mol).

PROBLEM 1



Bernoulli's equation

$$P_1 + \frac{1}{2}\rho c_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho c_2^2 + \rho g y_2$$

Continuity equation

$$S_1 \cdot c_1 = S_2 \cdot c_2 \qquad c_2 = \frac{S_1}{S_2} \cdot c_1$$

$$P_1 - P_2 = \frac{\rho}{2}\left(c_2^2 - c_1^2\right)$$

$$P_1 - P_2 = \frac{\rho c_1^2}{2}\left[\left(\frac{S_1}{S_2}\right)^2 - 1\right]$$

Getting an extra relationship between P_1 and P_2 (statics of fluids)

$$P_A = P_1 + \rho g\left(h + z_0\right) \qquad P_B = P_2 + \rho g z_0$$

$$P_A = P_B + \rho_m g h \qquad P_A - P_B = \rho_m g h$$

$$P_1 - P_2 = P_A - P_B - \rho g h = (\rho_m - \rho) g h$$

$$(\rho_m - \rho) g h = \frac{\rho c_1^2}{2}\left[\left(\frac{S_1}{S_2}\right)^2 - 1\right]$$

$$c_1 = \sqrt{\frac{2(\rho_m - \rho) g h}{\rho\left[\left(S_1 / S_2\right)^2 - 1\right]}}$$

PROBLEM 1 (Continued)

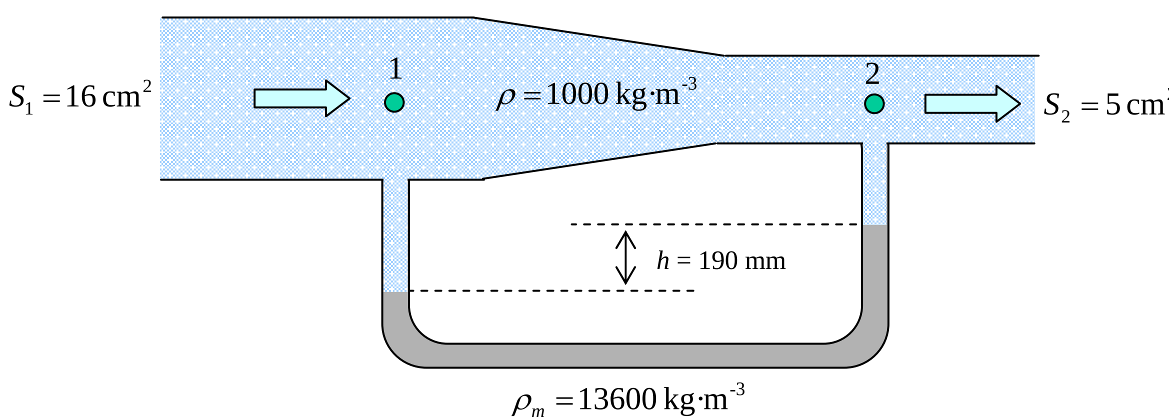
$$c_1 = \sqrt{\frac{2(\rho_m - \rho)gh}{\rho\left[(S_1/S_2)^2 - 1\right]}} = \sqrt{\frac{2(13600-1000)9.8\cdot 0.190}{1000\left[(16/5)^2 - 1\right]}} = 2.25\ \text{m/s}$$

$$c_2 = \frac{S_1}{S_2}\cdot c_1 = \frac{16}{5}2.25 = 7.20\ \text{m/s}$$

$$\dot{V} = S_1 \cdot c_1 = 16\cdot 10^{-4}\cdot 2.25 = 3.6\cdot 10^{-3}\ \text{m}^3/\text{s} = 3.6\ \text{liter/s}$$

$$\dot{m} = \rho\dot{V} = 1\frac{\text{kg}}{\text{liter}}3.6\frac{\text{liter}}{\text{s}} = 3.6\frac{\text{kg}}{\text{s}}$$

$$P_1 - P_2 = (\rho_m - \rho)gh = \frac{\rho\, c_1^2}{2}\left[\left(\frac{S_1}{S_2}\right)^2 - 1\right] = 24079\ \text{Pa}$$



PROBLEM 2

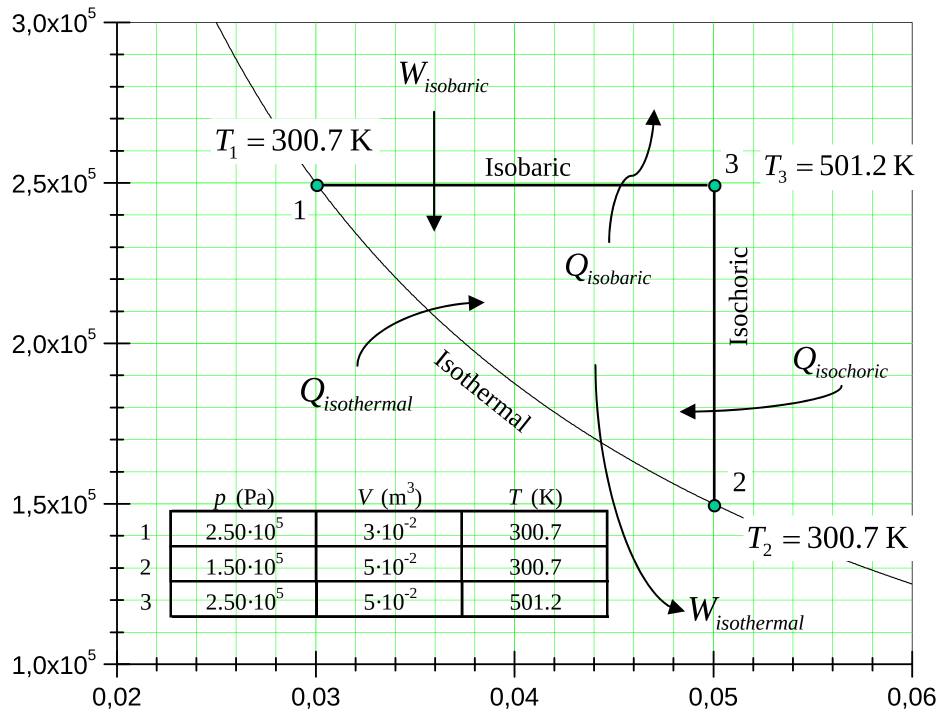
(a), (b) Calculating temperature at point 1 ($p_1 = 2.5\cdot10^5$ bar, $V_1 = 3\cdot10^{-2}$ m^3)

$$T_1 = \frac{p_1\,V_1}{n\cdot R} = \frac{2.5\cdot10^5\cdot3\cdot10^{-2}}{3\cdot8.314} = 300.7\ \text{K}$$

Isothermal process $1 \rightarrow 2$ implies $T_1 = T_2 = 300.7$ K

Point 2 $\quad T_2 = 300.7$ K $\quad V_2 = 5\cdot10^{-2}$ m^3 $\Rightarrow$ $p_2 = \dfrac{n\cdot R\,T_2}{V_2} = \dfrac{3\cdot8.314\cdot300.7}{5\cdot10^{-2}} = 1.50\cdot10^5$ Pa

P (Pa)



	p (Pa)	V (m^3)	T (K)
1	$2.50\cdot10^5$	$3\cdot10^{-2}$	300.7
2	$1.50\cdot10^5$	$5\cdot10^{-2}$	300.7
3	$2.50\cdot10^5$	$5\cdot10^{-2}$	501.2

V (m^3)

Point 3 $\quad V_3 = V_2 = 5\cdot10^{-2}$ m^3 $\quad p_3 = p_1 = 2.5\cdot10^5$ bar

$$T_3 = \frac{p_3\,V_3}{n\cdot R} = \frac{2.5\cdot10^5\cdot5\cdot10^{-2}}{3\cdot8.314} = 501.2\ \text{K}$$

c) Computing work $\quad W_{isothermal} = n\cdot R\,T_1 \ln\dfrac{V_2}{V_1}$

$W_{isothermal} = 3\cdot8.314\cdot300.7\ \ln\dfrac{5}{3} = 3831$ J $\quad$ Work done by the gas

$W_{isobaric} = P_1\left(V_1 - V_3\right) = 2.5\cdot10^5\,(3-5)\cdot10^{-2} = -5000$ J $\quad$ Work done on the gas

$W_{isochoric} = 0$ $\quad$ (volume does not change)

$\Delta U_{isothermal} = Q_{isothermal} - W_{isothermal} = 0$

d) Heat. During an isothermal process, the energy of the ideal gas remains constant $\quad Q_{isothermal} = W_{isothermal} = 3831$ J

$$Q_{isochoric} = n\cdot c_V\,(T_3 - T_2) = 3\cdot(5/2)\cdot8.314\,(501.2 - 300.7) = 12500\ \text{J}$$

$$Q_{isobaric} = n\cdot c_P\,(T_1 - T_3) = n\cdot\gamma\,c_V\,(T_1 - T_3) = 3\cdot1.4\cdot(5/2)\cdot8.314\,(300.7 - 501.2) = -17500\ \text{J}$$

$$Q = Q_{isothermal} + Q_{isochoric} + Q_{isobaric} = 3831 + 12500 - 17500 = -1168.8\ \text{J}$$