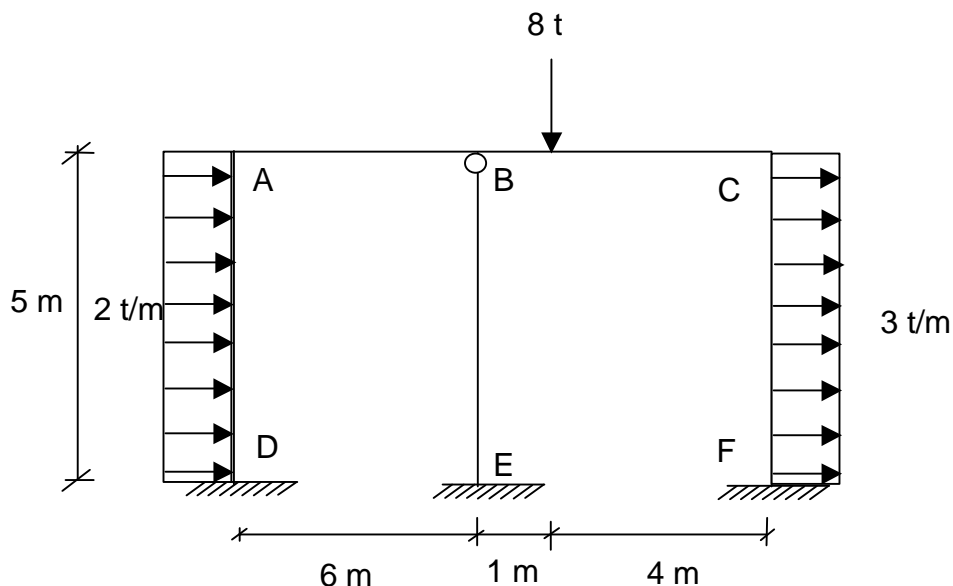


Hallar por el método de Cross los diagramas de momento flector y de esfuerzo cortante, así como las reacciones de la estructura de la figura, indicando la situación de los puntos más característicos de ambos diagramas utilizando el método de superposición.

Determinar el ángulo girado por C en la barra BC.

Los momentos de inercia de las barras horizontales es I_1 , mientras que el de las barras verticales es I_2 , sabiendo que la relación entre ellos es $I_1=2I_2$.



1º Coeficientes elásticos

NUDO A

$$K_{AD} = \frac{4 \cdot E \cdot I}{l} = \frac{4 \cdot E \cdot I_2}{5} = \frac{2 \cdot E \cdot I_1}{5}$$

$$\beta_{AD} = \frac{1}{2}$$

$$K_{AB} = \frac{4 \cdot E \cdot I}{l} = \frac{4 \cdot E \cdot I_1}{6} = \frac{2 \cdot E \cdot I_1}{3}$$

$$\beta_{AB} = \frac{1}{2}$$

$$r_{AD} = \frac{\frac{2}{5}}{\frac{2}{5} + \frac{2}{3}} = 0.37$$
$$r_{AB} = \frac{\frac{2}{3}}{\frac{2}{5} + \frac{2}{3}} = 0.63$$

NUDO B

$$K_{BA} = \frac{4 \cdot E \cdot I}{l} = \frac{4 \cdot E \cdot I_1}{6} = \frac{2}{3} \cdot E \cdot I_1$$

$$\beta_{BA} = \frac{1}{2}$$

$$K_{BC} = \frac{4 \cdot E \cdot I}{l} = \frac{4 \cdot E \cdot I_1}{5}$$

$$\beta_{BC} = \frac{1}{2}$$

$$K_{BE} = 0$$

$$\beta_{BE} = 1$$

$$r_{BA} = \frac{\frac{2}{3}}{\frac{2}{3} + \frac{4}{5} + 0} = 0.45$$

$$r_{BC} = \frac{\frac{4}{5}}{\frac{2}{3} + \frac{4}{5} + 0} = 0.55$$

$$r_{BE} = 0$$

NUDO C

$$K_{CB} = \frac{4 \cdot E \cdot I}{l} = \frac{4 \cdot E \cdot I_1}{5}$$

$$\beta_{CB} = \frac{1}{2}$$

$$K_{CF} = \frac{4 \cdot E \cdot I}{l} = \frac{4 \cdot E \cdot I_2}{5} = \frac{2 \cdot E \cdot I_1}{5}$$

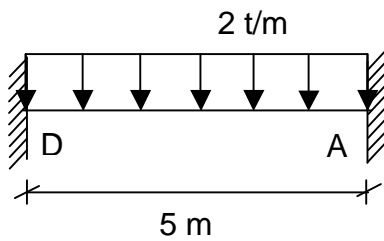
$$\beta_{CF} = \frac{1}{2}$$

$$r_{CB} = \frac{\frac{4}{5}}{\frac{4}{5} + \frac{2}{5}} = 0.67$$

$$r_{CF} = \frac{\frac{2}{5}}{\frac{4}{5} + \frac{2}{5}} = 0.33$$

2º. Calculamos los momentos y pares de empotramiento

Tramo DA

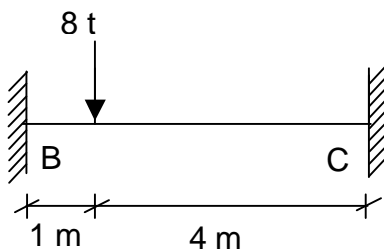


$$M_A = M_D = -\frac{q \cdot l^2}{12} = -\frac{2 \cdot 5^2}{12} = -4.17 \text{ t} \cdot \text{m}$$

$$m_D = +4.17 \text{ t} \cdot \text{m}$$

$$m_A = -4.17 \text{ t} \cdot \text{m}$$

Tramo BC



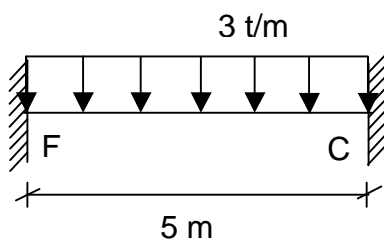
$$M_B = -\frac{P \cdot a \cdot b^2}{l^2} = -\frac{8 \cdot 1 \cdot 4^2}{5^2} = -5.12 \text{ t} \cdot \text{m}$$

$$M_C = -\frac{P \cdot a^2 \cdot b}{l^2} = -\frac{8 \cdot 1^2 \cdot 4}{5^2} = -1.28 \text{ t} \cdot \text{m}$$

$$m_B = +5.12 \text{ t} \cdot \text{m}$$

$$m_C = -1.28 \text{ t} \cdot \text{m}$$

Tramo FC

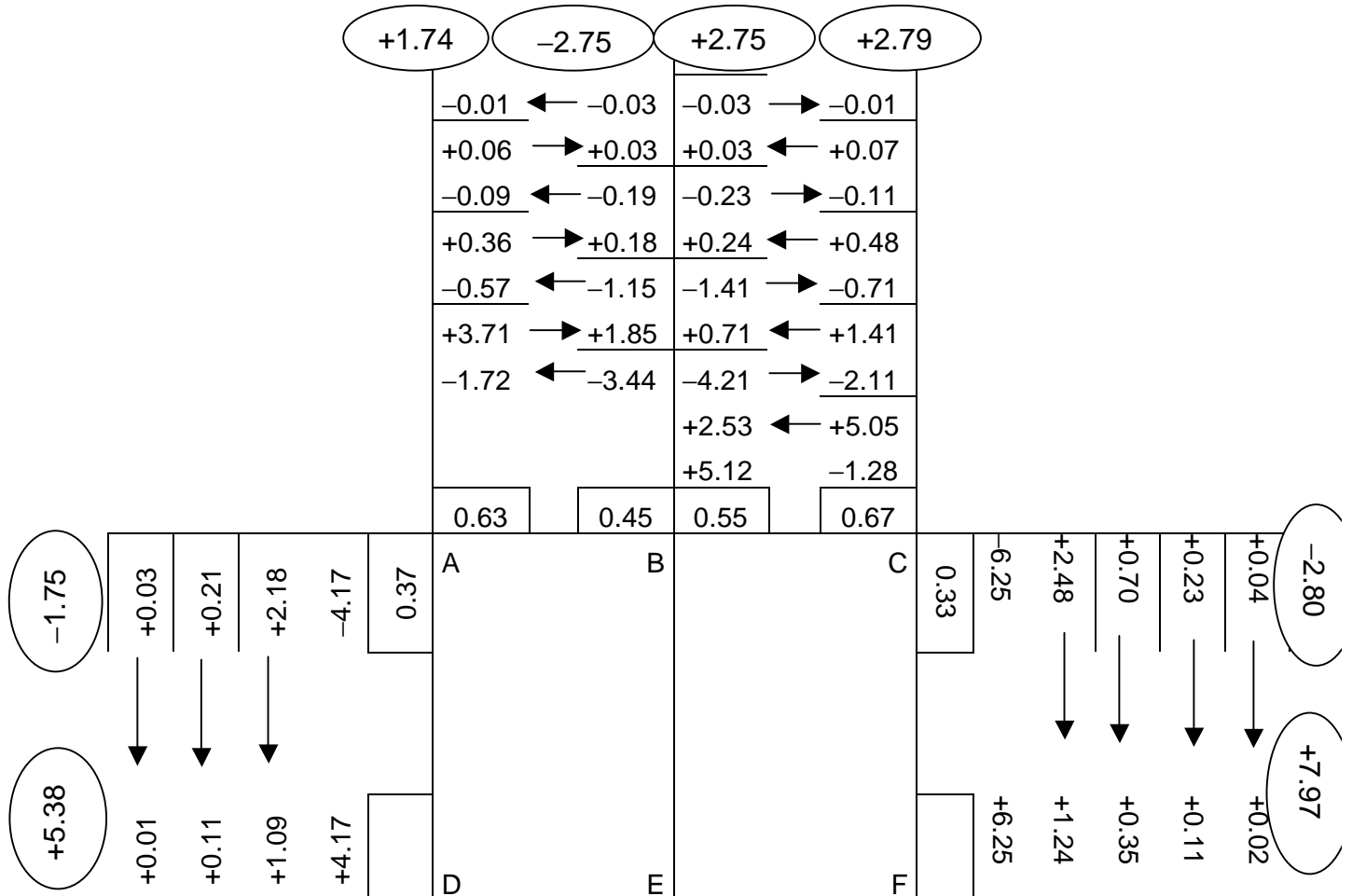


$$M_F = M_C = -\frac{q \cdot l^2}{12} = -\frac{3 \cdot 5^2}{12} = -6.25 \text{ t} \cdot \text{m}$$

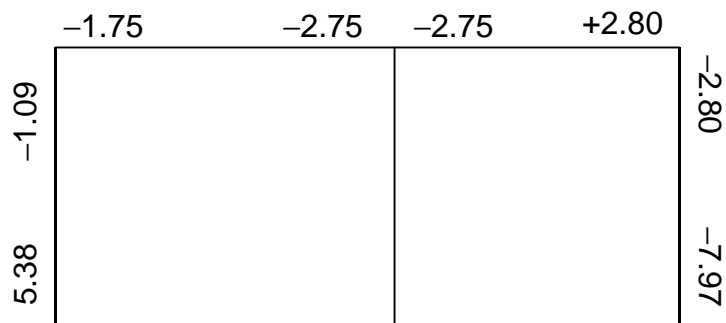
$$m_F = +6.25 \text{ t} \cdot \text{m}$$

$$m_C = -6.25 \text{ t} \cdot \text{m}$$

3º Cross: Transmisiones^(*)



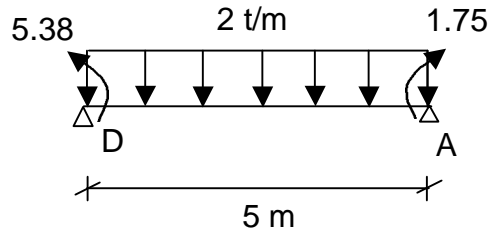
Momentos adoptados en los nudos



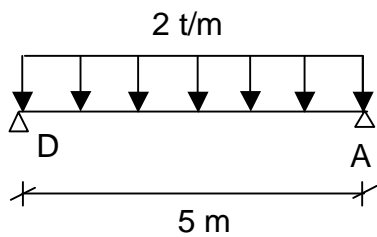
^(*) Redondeos según Norma UNE 7018. En los casos de redondeo de número que terminan en 5, se fuerza la cifra anterior si es par, y no se fuerza si es impar. Esto se hace con la intención de distribuir errores.

Obtención de los momentos máximos en las barras cargadas

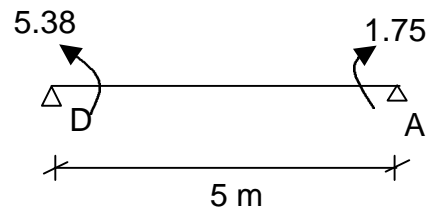
Tramo DA



[1]



[2]



[1]

$$M_{1DA} = \frac{q \cdot x}{2} \cdot (l - x) = \frac{2 \cdot x}{2} \cdot (5 - x) = 5 \cdot x - x^2$$

[2]

$$M_{2DA} = -\frac{M_a}{l} \cdot (l - x) - \frac{M_b}{l} \cdot x$$

$$M_{2DA} = -\frac{5.38}{5} \cdot (5 - x) - \frac{1.75}{5} \cdot x = -5.38 + 0.73 \cdot x$$

Superponiendo:

$$M_{DA} = -5 \cdot x - x^2 - 5.38 + 0.73 \cdot x = 5.73 \cdot x - x^2 - 5.38$$

$$M'_{DA} = 0; 5.73 - 2 \cdot x = 0 \rightarrow x = 2.78 \text{ m}$$

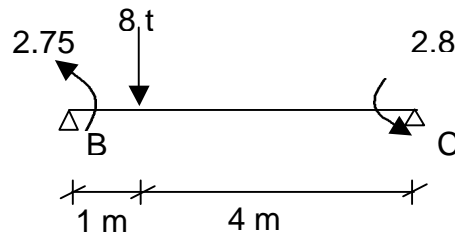
$$\text{En } x = 2.78, M_{x=2.78} = 5.73 \cdot 2.87 - 2.87^2 - 5.38 = 2.83 \text{ t} \cdot \text{m}$$

Puntos de momento nulo:

$$M_{DA} = 0$$

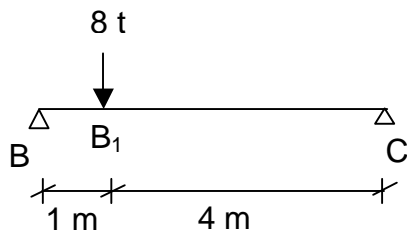
$$x^2 - 5.73 \cdot x + 5.38 = 0 \quad \left\{ \begin{array}{l} x_1 = 1.18 \text{ m} \\ x_2 = 4.55 \text{ m} \end{array} \right.$$

Tramo BC

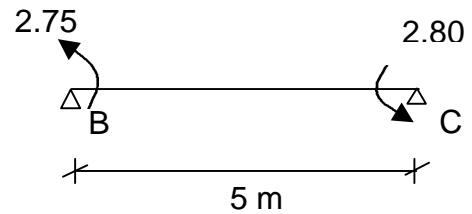


[1]

[2]



+



[1]

$$M_{BB1} = \frac{P \cdot b}{l} \cdot x = \frac{8 \cdot 4}{5} = 6.4 \cdot x$$

$$M_{B,C} = \frac{P \cdot a}{l} \cdot (l - x) = \frac{8 \cdot 1}{5} \cdot (5 - x) = 8 - 1.6 \cdot x$$

[2]

$$M_{BC} = -\frac{M_b}{l} \cdot (l - x) + \frac{M_c}{l} \cdot x$$

$$M_{BC} = -\frac{2.75}{5} \cdot (5 - x) + \frac{2.80}{5} \cdot x = -2.75 + 1.11 \cdot x$$

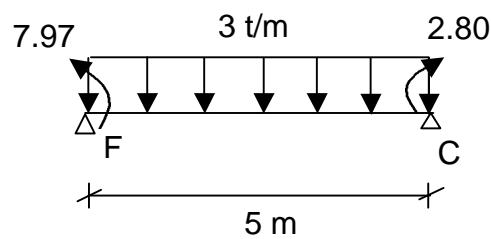
Superponiendo:

$$M_{BB_1} = 6.4 \cdot x - 2.75 + 1.11 \cdot x = -2.75 + 7.51 \cdot x$$

$$\text{Momento nulo: } M_{BB_1} = 0 \rightarrow x = \frac{2.75}{7.51} = 0.37 \text{ m}$$

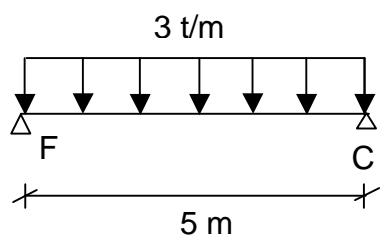
$$\text{Momento máximo: } M_{x=1} = 7.51 - 2.75 = 4.76 \text{ t} \cdot \text{m}$$

Tramo FC

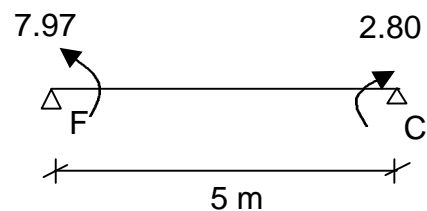


[1]

[2]



+



[1]

$$M_{1FC} = \frac{q \cdot x}{2} \cdot (l - x) = \frac{3 \cdot x}{2} \cdot (5 - x) = 7.5 - 1.5 \cdot x^2$$

[2]

$$M_{2FC} = -\frac{M_a}{l} \cdot (l - x) - \frac{M_b}{l} \cdot x$$

$$M_{2FC} = -\frac{7.97}{5} \cdot (5 - x) - \frac{2.80}{5} \cdot x = -7.97 + 1.03 \cdot x$$

Superponiendo:

$$M_{FC} = -7. + 1.03 \cdot x + 7.5 \cdot x - 1.5 \cdot x^2 = -1.5 \cdot x^2 + 8.53 \cdot x - 7.97$$

$$M'_{FC} = 0; 8.53 - 3 \cdot x = 0 \rightarrow x = 2.84 \text{ m}$$

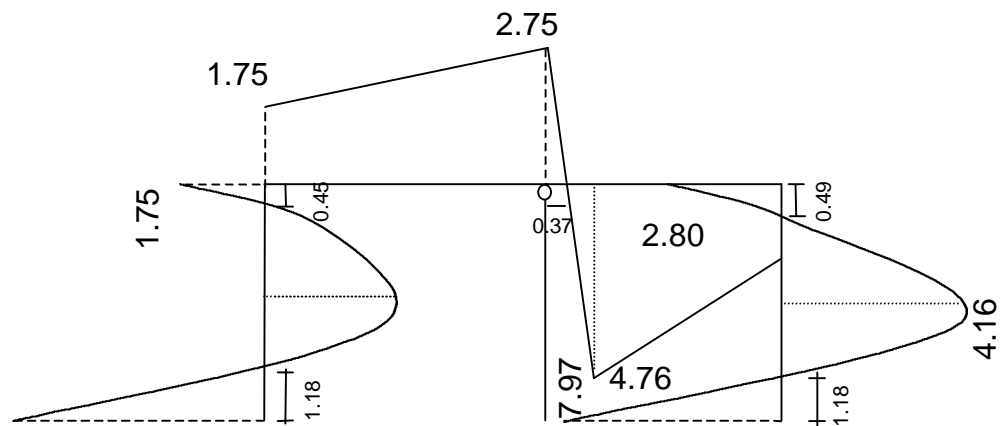
$$\text{En } x = 2.84, M_{x=2.84} = 8.53 \cdot 2.84 - 1.5 \cdot 2.84^2 - 7.97 = 4.16 \text{ t} \cdot \text{m}$$

Puntos de momento nulo:

$$M_{FC} = 0$$

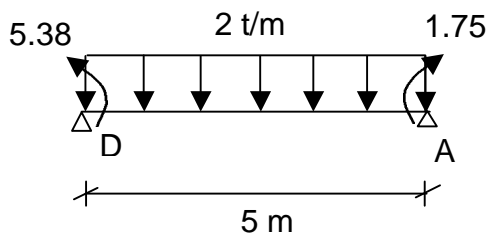
$$1.5 \cdot x^2 - 8.53 \cdot x + 7.97 = 0 \quad \left\{ \begin{array}{l} x_1 = 1.18 \text{ m} \\ x_2 = 4.51 \text{ m} \end{array} \right.$$

4º. Diagrama de momentos flectores



5º Reacciones y diagrama de esfuerzo cortante

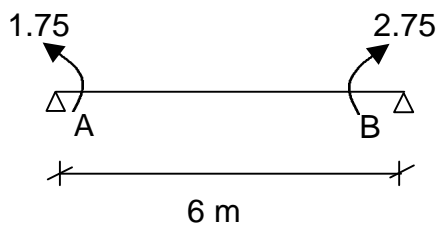
Tramo DA



$$\begin{aligned}\sum M_A &= 0 \\ R_D \cdot 5 + 1.75 - 5.38 - 2 \cdot 5 \cdot 2.5 &= 0 \\ R_D &= 5.73 \text{ t}\end{aligned}$$

$$\begin{aligned}\sum M_D &= 0 \\ R_A \cdot 5 + 5.38 - 1.75 - 2 \cdot 5 \cdot 2.5 &= 0 \\ R_A &= +4.27 \text{ t}\end{aligned}$$

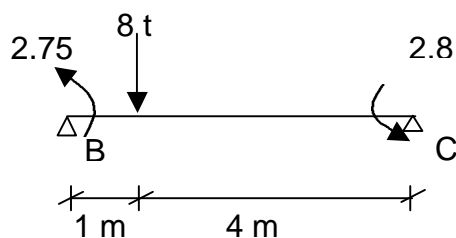
Tramo AB



$$\begin{aligned}\sum M_A &= 0 \\ R_B \cdot 6 + 1.75 - 2.75 &= 0 \\ R_B &= 0.17 \text{ t}\end{aligned}$$

$$\begin{aligned}\sum M_B &= 0 \\ R_A \cdot 6 + 2.75 - 1.75 &= 0 \\ R_A &= -0.17 \text{ t}\end{aligned}$$

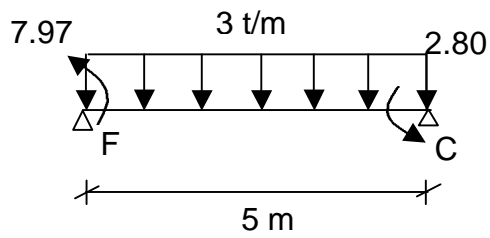
Tramo BC



$$\begin{aligned}\sum M_C &= 0 \\ R_B \cdot 5 - 8 \cdot 4 - 2.80 - 2.75 &= 0 \\ R_B &= 7.51 \text{ t}\end{aligned}$$

$$\begin{aligned}\sum M_B &= 0 \\ R_C \cdot 5 - 8 \cdot 1 + 2.75 + 2.80 &= 0 \\ R_C &= 0.49 \text{ t}\end{aligned}$$

Tramo FC



$$\sum M_C = 0$$

$$R_F \cdot 5 - 5 \cdot 3 \cdot 2.5 - 7.97 - 2.80 = 0$$

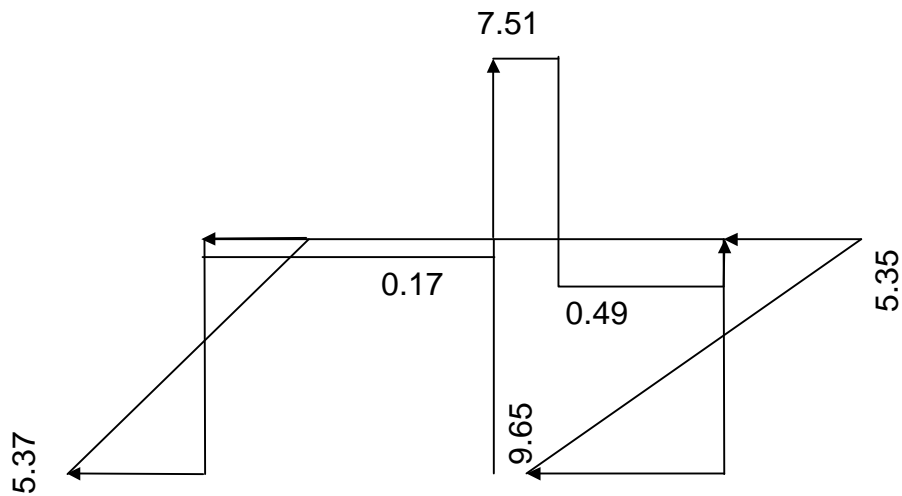
$$R_F = 9.65 \text{ t}$$

$$\sum M_F = 0$$

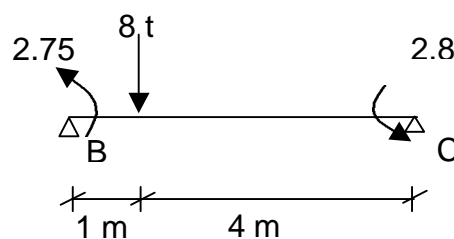
$$R_C \cdot 5 - 5 \cdot 3 \cdot 2.5 + 2.80 + 7.97 = 0$$

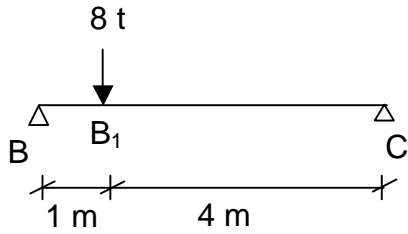
$$R_C = 5.35 \text{ t}$$

Diagrama de esfuerzos cortantes

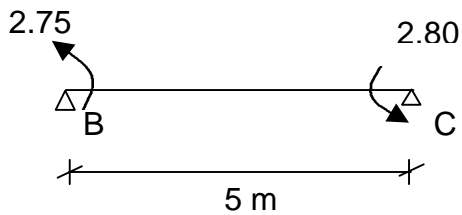


6º Angulo girado en C por la barra BC





$$\varphi_C = -\frac{P \cdot a \cdot b}{6 \cdot E \cdot I \cdot l} \cdot (l + a)$$



$$\varphi_C = \frac{l}{6 \cdot E \cdot I} \cdot (M_b + 2 \cdot M_c)$$

$$\varphi_C = -\frac{5}{6 \cdot E \cdot I} \cdot (2.75 - 2 \cdot 2.80) - \frac{8 \cdot 1 \cdot 4}{6 \cdot E \cdot I \cdot 5} \cdot (5 + 1) = -\frac{14.25}{6 \cdot E \cdot I} - \frac{32}{5 \cdot E \cdot I} = -\frac{8.78}{E \cdot I} \text{ rad}$$

(E, I en t y m)