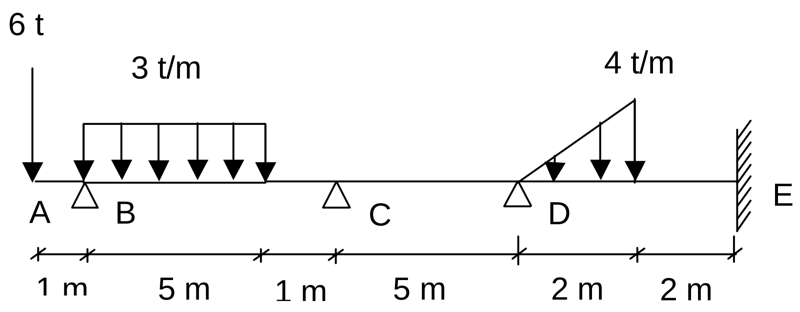
Hallar por el método de Cross los diagramas de momento flector y de esfuerzo cortante, así como las reacciones de la viga continua de la figura, empleando el método de superposición en las barras cargadas.

Suponemos que el momento de inercia de la viga es constante en toda su longitud.

En el vano CD, calcular la flecha y el ángulo de la deformada en los apoyos C y D, dejando los resultados en función de E·I.



1º Coeficientes elásticos

NUDO B

$$K_{BC} = \frac{4 \cdot E \cdot I}{l} = \frac{4 \cdot E \cdot I}{6} \qquad \beta_{BC} = 1/2$$

$$r_{BC} = 1$$

NUDO C

$$K_{CB} = \frac{4 \cdot E \cdot I}{l} = \frac{4 \cdot E \cdot I}{6} = \frac{2 \cdot E \cdot I}{3} \qquad \beta_{CB} = 1/2$$

$$K_{CD} = \frac{4 \cdot E \cdot I}{l} = \frac{4 \cdot E \cdot I}{5} \qquad \beta_{CD} = 1/2$$

$$r_{CB} = \frac{2/3}{2/3 + 4/5} = 0.45$$

$$r_{CD} = \frac{4/5}{2/3 + 4/5} = 0.55$$

NUDO D

$$K_{DC} = \frac{4 \cdot E \cdot I}{l} = \frac{4 \cdot E \cdot I}{5} \qquad \beta_{DC} = 1/2$$

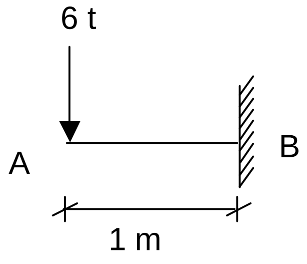
$$K_{DE} = \frac{4 \cdot E \cdot I}{l} = \frac{4 \cdot E \cdot I}{4} \qquad \beta_{DE} = 1/2$$

$$r_{DC} = \frac{4/5}{4/5 + 1} = 0.44$$

$$r_{DE} = \frac{1}{4/5 + 1} = 0.56$$

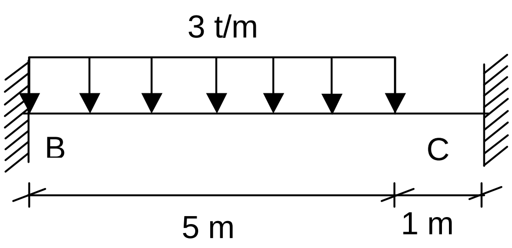
2º Momentos y pares de empotramiento

Tramo AB



$$M_B = -P \cdot l = -6 \cdot 1 = -6\ t \cdot m$$
$$m_B = -6\ t \cdot m$$

Tramo BC



a = 2.5 m
b = 3.5 m
c = 5 m
q = 3 t/m
l = 6 m

$$M_B = \frac{-q \cdot c^3}{12 \cdot l^2} \cdot \left(l - 3 \cdot b + \frac{12 \cdot a \cdot b^2}{c^2} \right)$$

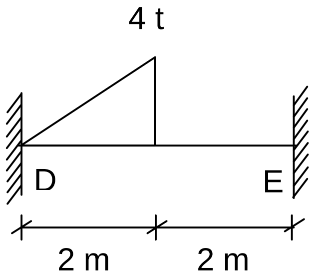
$$M_B = \frac{-3 \cdot 5^3}{12 \cdot 6^2} \cdot \left(6 - 3 \cdot 3.5 + \frac{12 \cdot 2.5 \cdot 3.5^2}{5^2} \right) = -8.85 \text{ t} \cdot \text{m}$$

$$M_C = \frac{-q \cdot c^3}{12 \cdot l^2} \cdot \left(l - 3 \cdot a + \frac{12 \cdot a^2 \cdot b}{c^2} \right)$$

$$M_C = \frac{-3 \cdot 5^3}{12 \cdot 6^2} \cdot \left(6 - 3 \cdot 2.5 + \frac{12 \cdot 2.5^2 \cdot 3.5}{5^2} \right) = -7.81 \text{ t} \cdot \text{m}$$

$$m_B = +8.85 \text{ t} \cdot \text{m}$$
$$m_C = -7.81 \text{ t} \cdot \text{m}$$

Tramo DE



a = 2 m
b = 2 m
l = 4 m
q = 4 t

$$M_D = -\frac{q \cdot a^2}{30 \cdot l^2} \cdot \left(10 \cdot l^2 - 15 \cdot a \cdot l + 6 \cdot a^2\right)$$

$$M_D = -\frac{4 \cdot 2^2}{30 \cdot 4^2} \cdot \left(10 \cdot 4^2 - 15 \cdot 2 \cdot 4 + 6 \cdot 2^2\right) = -2.13\ t \cdot m$$

$$M_E = -\frac{q \cdot a^3}{4 \cdot l} \cdot \left(1 - \frac{4 \cdot a}{5 \cdot l}\right)$$

$$M_E = -\frac{4 \cdot 2^3}{4 \cdot 4} \cdot \left(1 - \frac{4 \cdot 2}{5 \cdot 4}\right) = -1.20\ t \cdot m$$

$$m_D = +2.13\ t \cdot m$$

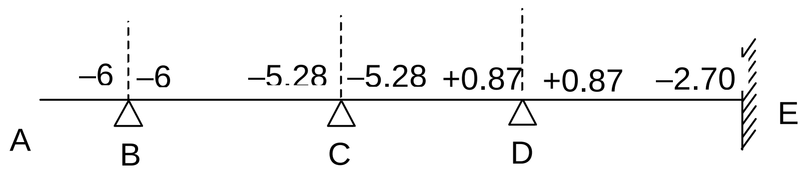
$$m_E = -1.20\ t \cdot m$$

3º Cross: Transmisiones

B (AB)	B (BC)		C (CB)	C (CD)		D (DC)	D (DE)		E
–6.0	**+6.0**		**–5.29**	**+5.27**		**+0.87**	**–0.87**		**–2.70**
	–0.03	→	–0.01	–0.01	←	–0.01	–0.02	→	–0.01
	+0.03	←	+0.05	+0.05	→	+0.03			
	–0.13	→	–0.07	–0.03	←	–0.07	–0.08	→	–0.04
	+0.13	←	+0.25	+0.31	→	+0.15			
	–0.73	→	–0.37	–0.19	←	–0.39	–0.50	→	–0.25
	+0.73	←	+1.46	+1.78	→	+0.89			
	–4.60	→	–2.30	–0.94	←	–1.88	–2.40	→	–1.20
	+1.75	←	+3.51	+4.30	→	+2.15			
–6	+8.85		–7.81				+2.13		–1.20
	1.0		0.45	0.55		0.44	0.56		

A B C D E

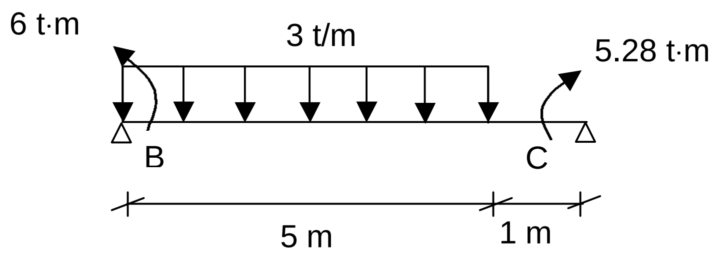
Momentos flectores en los nudos



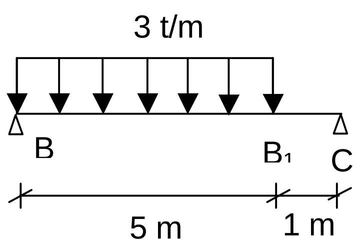
4º Diagrama de momentos

Obtención de los momentos máximos en los vanos

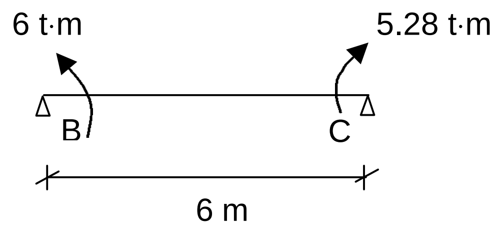
<u>Tramo BC</u>



[1] [2]



+



[1]

$$M_{BB_1} = \frac{q \cdot b \cdot c}{l} \cdot x - \frac{q}{2} \cdot \left[x - \left(a - \frac{c}{2} \right) \right]^2$$

$$M_{BB_1} = \frac{3 \cdot 3.5 \cdot 5}{6} \cdot x - \frac{3}{2} \cdot \left[x - \left(2.5 - \frac{5}{2} \right) \right]^2$$

$$M_{BB_1} = -1.5 \cdot x^2 + 8.75 \cdot x$$

[2]

$$M_{BC} = -\frac{M_b}{l} \cdot (l - x) - \frac{M_c}{l} \cdot x$$

$$M_{BC} = -\frac{6}{6} \cdot (6 - x) - \frac{5.28}{6} \cdot x$$

$$M_{BC} = 0.12 \cdot x - 6$$

Superponiendo:

$$M_{BB_1} + M_{BC} = 8.75 \cdot x - 1.5 \cdot x^2 + 0.12 \cdot x - 6 = -6 + 8.87 \cdot x - 1.5 \cdot x^2$$

Momento máximo:

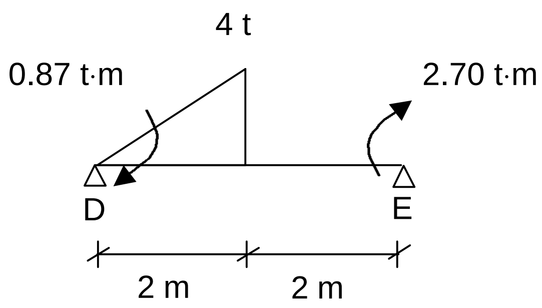
$$M' = 0$$

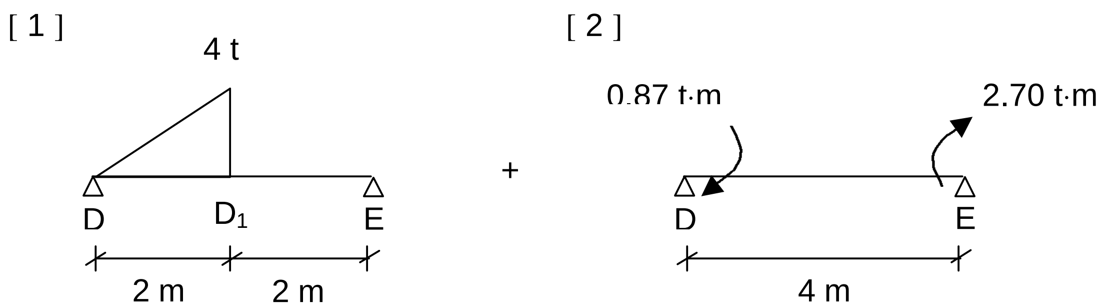
$$8.87 - 3 \cdot x = 0$$

$$x = 2.96 \text{ m}$$

$$M_{x=2.96} = -6 + 8.87 \cdot 2.96 - 1.5 \cdot 2.96^2 = 7.11 \, t \cdot m$$

Tramo DE





[1]

$$M_{DD_1} = q \cdot \frac{x}{6} \cdot \left[\left(3 - \frac{2 \cdot a}{l} \right) \cdot a - \frac{x^2}{2} \right]$$

$$M_{DD_1} = 4 \cdot \frac{x}{6} \cdot \left[\left(3 - \frac{2 \cdot 2}{4} \right) \cdot 2 - \frac{x^2}{2} \right] = \frac{8}{3} \cdot x - \frac{x^3}{3}$$

[2]

$$M_{DE} = \frac{M_d}{l} \cdot (l - x) - \frac{M_e}{l} \cdot x$$

$$M_{DE} = \frac{0.87}{4} \cdot (4 - x) - \frac{2.70}{4} \cdot x = 0.87 - \frac{3.57}{4} \cdot x$$

Superponiendo:

$$M_{DD_1} + M_{DE} = \frac{8}{3} \cdot x - \frac{x^3}{3} \quad + 0.87 - \frac{3.57}{4} \cdot x = 0.87 + 1.77 \cdot x - 0.33 \cdot x^2$$

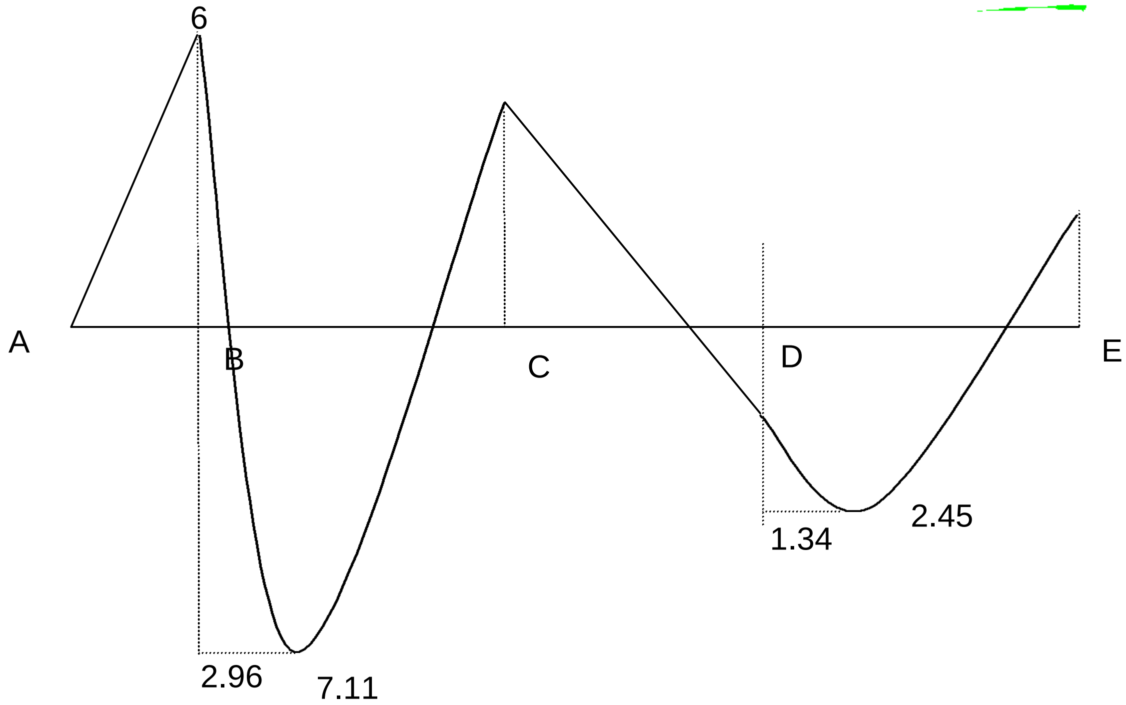
Momento máximo:

$$M' = 0$$

$$1.77 - 0.99 \cdot x^2 = 0$$

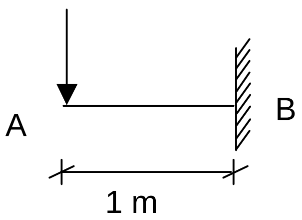
$$x = \pm 1.34 \text{ m}$$

$$M_{x=1.34} = 0.87 + 1.77 \cdot 1.34 - 0.33 \cdot 1.34^2 = 2.54 \text{ t} \cdot \text{m}$$



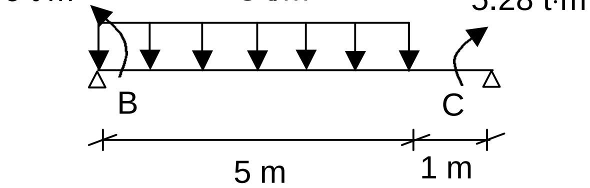
5º Cálculo de reacciones y diagrama de esfuerzo cortante

Tramo AB



$$R_B = +6\ t$$

Tramo BC



$$\sum M_B = 0$$

$$R_C \cdot 6 + 6 - 5.28 - 15 \cdot 2.5 = 0$$

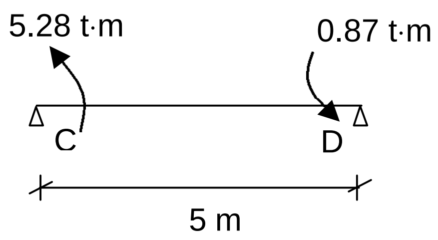
$$R_C = +6.13\ t$$

$$\sum M_C = 0$$

$$R_B \cdot 6 - 6 + 5.28 - 15 \cdot 3.5 = 0$$

$$R_B = +8.87\ t$$

Tramo CD



$$\sum M_C = 0$$

$$R_D \cdot 5 + 5.28 + 0.87 = 0$$

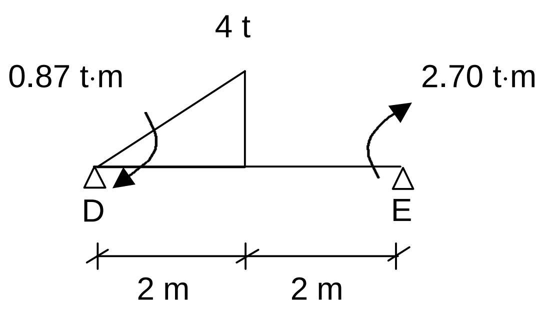
$$R_D = -1.23\ t$$

$$\sum M_D = 0$$

$$R_C \cdot 5 - 5.28 - 0.87 = 0$$

$$R_C = +1.23\ t$$

Tramo DE



$$\sum M_D = 0$$

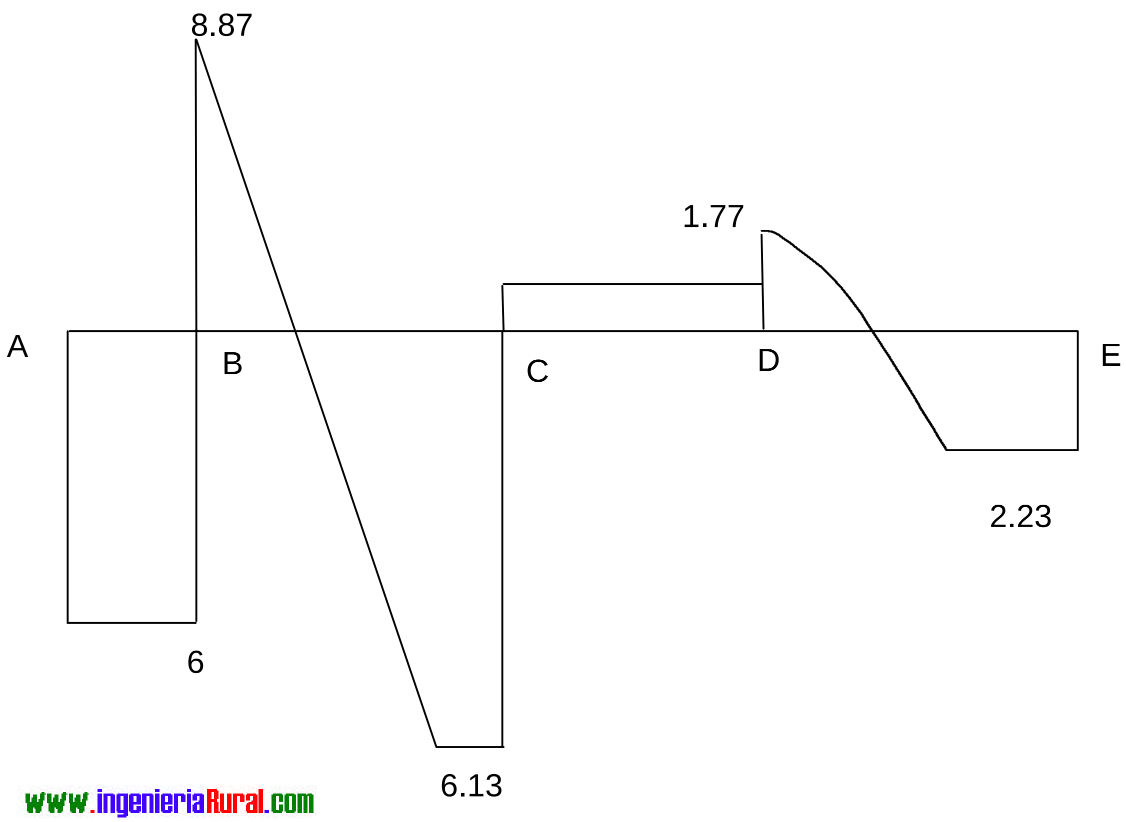
$$R_E \cdot 4 - 0.87 - 2.70 - \frac{1}{2} \cdot 4 \cdot 2 \cdot \frac{2}{3} \cdot 2 = 0$$

$$R_E = +2.23\ t$$

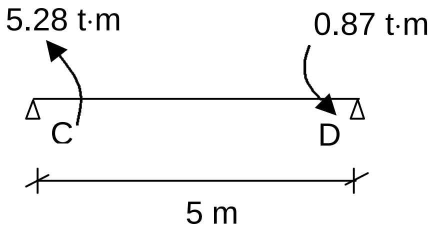
$$\sum M_E = 0$$

$$R_D \cdot 4 + 0.87 + 2.70 - \frac{1}{2} \cdot 4 \cdot 2 \cdot \left(\frac{1}{3} \cdot 2 + 2 \right) = 0$$

$$R_D = +1.77\ t$$



6º Flecha y ángulo de la deformada en los apoyos C y D



Ecuación de la elástica

$$y_{CD} = -\frac{M_a \cdot x}{6 \cdot E \cdot I} \cdot (l - x) \cdot \left[1 + \frac{l - x}{l} - \frac{M_b}{M_a} \cdot \left(1 + \frac{x}{l}\right)\right]$$

$$y_{CD} = -\frac{5.28 \cdot x}{6 \cdot E \cdot I} \cdot (5 - x) \cdot \left[1 + \frac{5 - x}{5} - \frac{0.87}{5.28} \cdot \left(1 + \frac{x}{5}\right)\right]$$

$$y_{CD} = \frac{1}{E \cdot I} \cdot \left[-0.20 \cdot x^3 + 2.63 \cdot x^2 - 8.10 \cdot x\right]$$

Para calcular la flecha

$$y' = \frac{1}{E \cdot I} \cdot \left[-0.60 \cdot x^2 + 5.26 \cdot x - 8.10\right] = 0$$

$$0.60 \cdot x^2 - 5.26 \cdot x + 8.10 = 0$$

$x_1 = 1.99$ m
$x_2 = 6.77$ m $\notin$ CD

$$y_{x=1.99} = \frac{1}{E \cdot I} \cdot \left[-0.20 \cdot 1.99^3 + 2.63 \cdot 1.99^2 - 8.10 \cdot 1.99\right]$$

$$y_{x=1.99} = \delta = \frac{-7.28}{E \cdot I}$$

Angulo girado

$$\varphi_C = -\frac{l}{6 \cdot E \cdot I} \cdot (2 \cdot M_c - M_d)$$
$$\varphi_C = -\frac{5}{6 \cdot E \cdot I} \cdot (2 \cdot 5.28 - 0.87) = \frac{-8.075}{E \cdot I} \text{ rad}$$

$$\varphi_D = \frac{l}{6 \cdot E \cdot I} \cdot (M_c - 2 \cdot M_d)$$
$$\varphi_D = \frac{5}{6 \cdot E \cdot I} \cdot (5.28 - 2 \cdot 0.87) = \frac{2.95}{E \cdot I} \text{ rad}$$