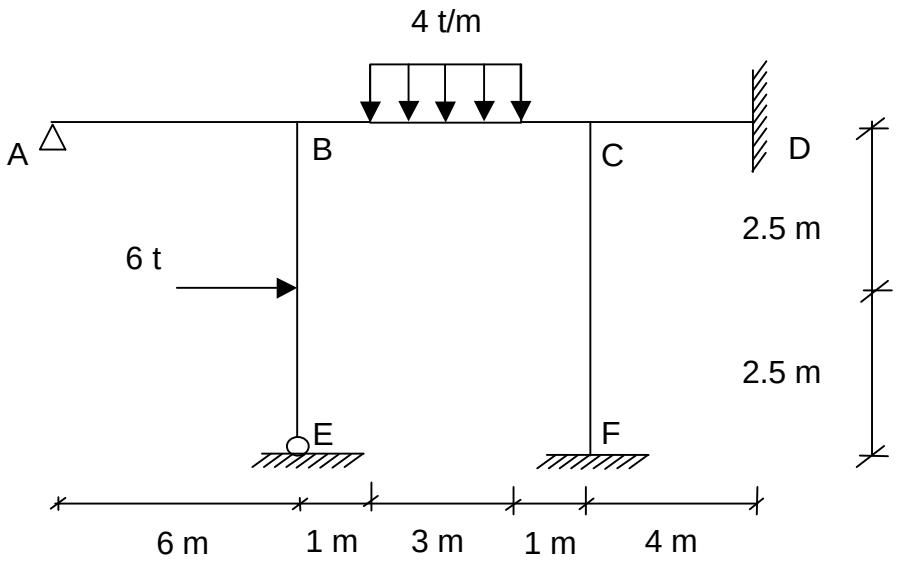
Hallar por el método de Cross los diagramas de momento flector y de esfuerzo cortante, así como las reacciones de la estructura de la figura, indicando la situación de los puntos más característicos de ambos diagramas utilizando el método de superposición.

Determinar el ángulo girado por C en la barra BC.

Los momentos de inercia de las barras horizontales es I_1, mientras que el de las barras verticales es I_2, sabiendo que la relación entre ellos es $I_1=2 \cdot I_2$.



1º Coeficientes elásticos

NUDO B

$$K_{BA} = \frac{3 \cdot E \cdot I}{l} = \frac{3 \cdot E \cdot I_1}{6} = 0.5 \cdot E \cdot I_1 \qquad \beta_{BA} = 0$$

$$K_{BC} = \frac{4 \cdot E \cdot I}{l} = \frac{4 \cdot E \cdot I_1}{5} = 0.8 \cdot E \cdot I_1 \qquad \beta_{BC} = 1/2$$

$$K_{BE} = \frac{3 \cdot E \cdot I}{l} = \frac{3 \cdot E \cdot I_2}{5} = \frac{3 \cdot E \cdot I_1}{10} = 0.3 \cdot E \cdot I_1 \qquad \beta_{BE} = 0$$

$$r_{BA} = \frac{0.5}{0.5+0.8+0.3} = 0.31$$
$$r_{BC} = \frac{0.8}{0.5+0.8+0.3} = 0.50$$
$$r_{BE} = \frac{0.3}{0.5+0.8+0.3} = 0.19$$

NUDO C

$$K_{CB} = \frac{4 \cdot E \cdot I}{l} = \frac{4 \cdot E \cdot I_1}{5} = 0.8 \cdot E \cdot I_1 \qquad \beta_{CB} = 1/2$$
$$K_{CD} = \frac{4 \cdot E \cdot I}{l} = \frac{4 \cdot E \cdot I_1}{4} = 1.0 \cdot E \cdot I_1 \qquad \beta_{CD} = 1/2$$
$$K_{CF} = \frac{4 \cdot E \cdot I}{l} = \frac{4 \cdot E \cdot I_2}{5} = \frac{4 \cdot E \cdot I_1}{10} = 0.4 \cdot E \cdot I_1 \qquad \beta_{CF} = 1/2$$
$$r_{CB} = \frac{0.8}{0.8+1.0+0.4} = 0.37$$
$$r_{CD} = \frac{1.0}{0.8+1.0+0.4} = 0.45$$
$$r_{CF} = \frac{0.4}{0.8+1.0+0.4} = 0.18$$

NUDOS A, E

Articulaciones

K =0 β =1 r =0

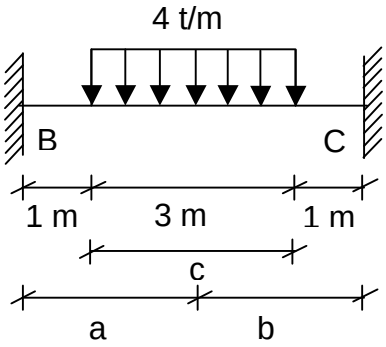
NUDOS D,F

Empotramientos perfectos

K =∞ β =0 r =0

2º Momentos y pares de empotramiento

Tramo BC



a =2.5 m
b =2.5 m
c =3 m
q =4 t/m

$$M_B = \frac{-q \cdot c^3}{12 \cdot l^2} \cdot \left(l - 3 \cdot b + \frac{12 \cdot a \cdot b^2}{c^2} \right)$$

$$M_B = \frac{-4 \cdot 3^3}{12 \cdot 5^2} \cdot \left(5 - 3 \cdot 2.5 + \frac{12 \cdot 2.5 \cdot 2.5^2}{3^2} \right) = -6.6\ t \cdot m$$

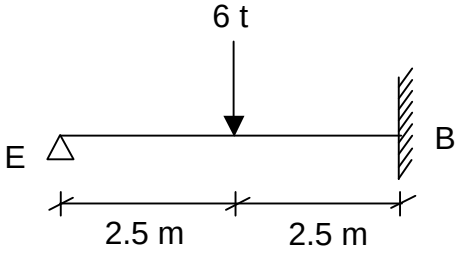
$$M_C = \frac{-q \cdot c^3}{12 \cdot l^2} \cdot \left(l - 3 \cdot a + \frac{12 \cdot a^2 \cdot b}{c^2} \right)$$

$$M_C = \frac{-4 \cdot 3^3}{12 \cdot 5^2} \cdot \left(5 - 3 \cdot 2.5 + \frac{12 \cdot 2.5^2 \cdot 2.5}{3^2} \right) = -6.6\ t \cdot m$$

$$m_B = -6.6\ t \cdot m$$

$$m_C = +6.6\ t \cdot m$$

Tramo EB



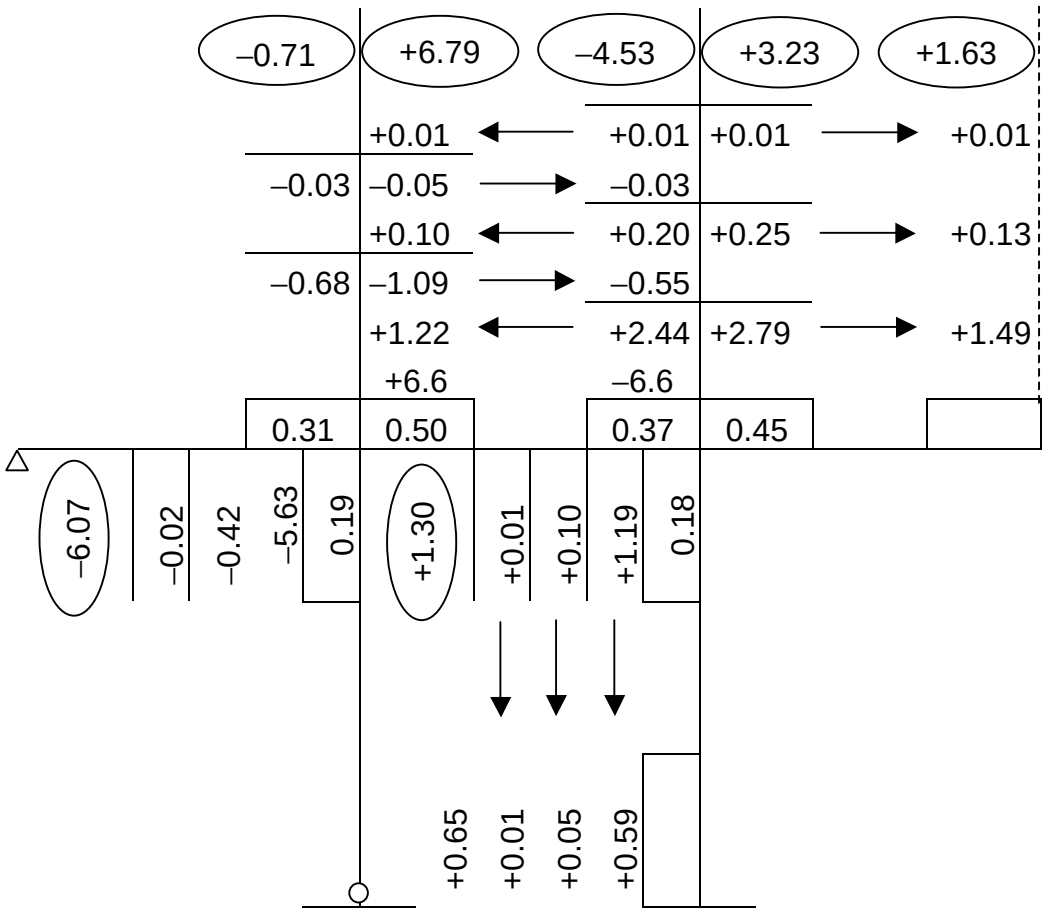
$M_E = 0$

$M_B = -\frac{3}{16} \cdot p \cdot l = -\frac{3}{16} \cdot 6 \cdot 5 = -5.625\ t \cdot m$

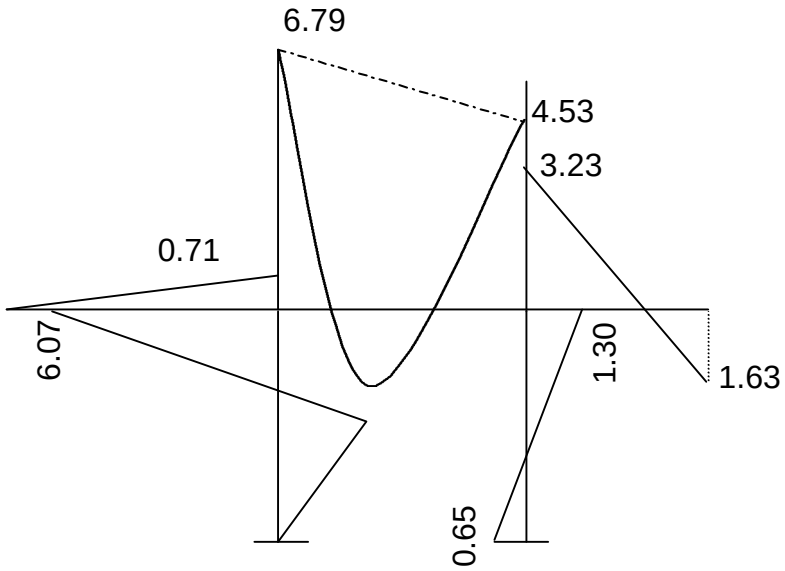
$m_E = 0$

$m_B = -5.625\ t \cdot m$

3º Cross: Transmisiones

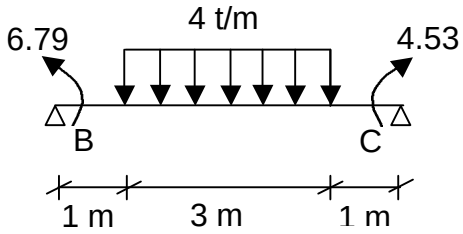


4º Diagrama de momentos

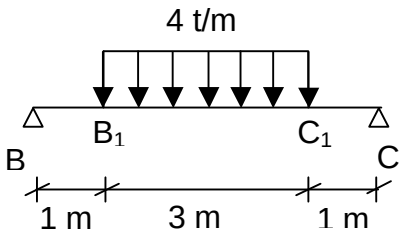


Obtención de los momentos en el centro del vano

Tramo BC

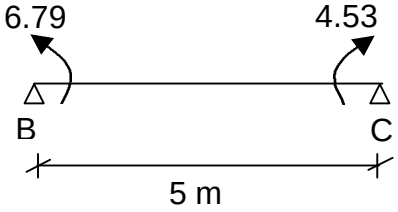


[1]



+

[2]



[1]

$$M_{BB_1} = \frac{q \cdot b \cdot c}{l} \cdot x = \frac{4 \cdot 2.5 \cdot 3}{5} \cdot x = 6 \cdot x$$

$$M_{B_1C_1} = \frac{q \cdot b \cdot c}{l} \cdot x - \frac{q}{2} \cdot \left[x - \left(a - \frac{c}{2} \right) \right]^2$$

$$M_{B_1C_1} = \frac{4 \cdot 2.5 \cdot 3}{5} \cdot x - \frac{4}{2} \cdot \left[x - \left(2.5 - \frac{3}{2} \right) \right]^2$$

$$M_{B_1\,C_1} = 6 \cdot x - 2 \cdot (x-1)^2 = -2 \cdot x^2 + 10 \cdot x - 2$$

$$M_{C_1C} = \frac{q \cdot a \cdot c}{l} \cdot (l - x)$$

$$M_{C_1C} = \frac{4 \cdot 2.5 \cdot 3}{5} \cdot (5 - x) = 30 - 6 \cdot x$$

[2]

$$M_{BC} = -\frac{M_B}{l} \cdot (l - x) - \frac{M_C}{l} \cdot x$$

$$M_{BC} = -\frac{6.79}{5} \cdot (5 - x) - \frac{4.53}{5} \cdot x$$

$$M_{BC} = -6.79 + 1.36 \cdot x - 0.91 \cdot x = 0.45 \cdot x - 6.79$$

Superponiendo:

$$M_{BB_1} + M_{BC} = 6 \cdot x + 0.45 \cdot x - 6.79 = 6.45 \cdot x - 6.79$$

$$M_{B_1C_1} + M_{BC} = -2 \cdot x^2 + 10 \cdot x - 2 + 0.45 \cdot x - 6.79 = -2 \cdot x^2 + 10.45 \cdot x - 8.79$$

$$M_{C_1C} + M_{BC} = 30 - 6 \cdot x + 0.45 \cdot x - 6.79 = 23.21 - 5.55 \cdot x$$

Puntos de corte:

$$6.45 \cdot x - 6.79 = 0$$

$$x = 1.05\,\text{m} \rightarrow \notin B_1C_1$$

$$2 \cdot x^2 - 10.45 \cdot x + 8.79 = 0$$

$$x = \frac{10.45 \pm \sqrt{10.45^2 - 4 \cdot 2 \cdot 8.79}}{4}$$

$$\left.\begin{array}{l} x_1 = 1.05\,m \\ x_2 = 4.17\,m \notin B_1C_1 \end{array}\right\} \quad x = 1.05\,m$$

$$23.21 - 5.55 \cdot x = 0$$

$$x = 4.18\,m \notin B_1C_1$$

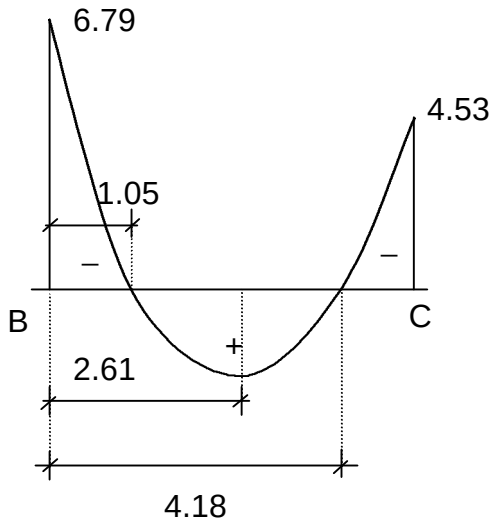
Momento máximo:

$$\frac{dM}{dx} = 0$$

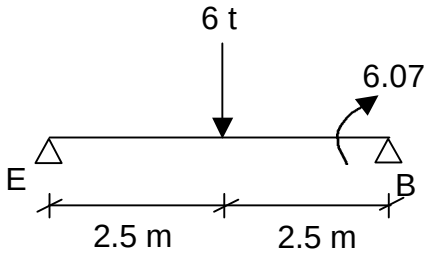
$$-4 \cdot x + 10.45 = 0$$

x =2.61 m

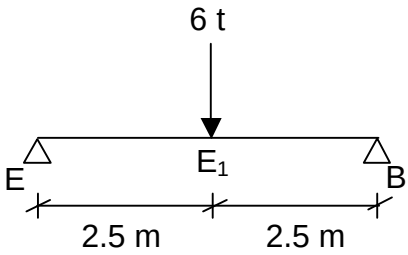
$$M_{x=2.61} = -2 \cdot (2.61)^2 + 10.45 \cdot 2.61 - 8.79 = 4.86\,t \cdot m$$



Tramo BE

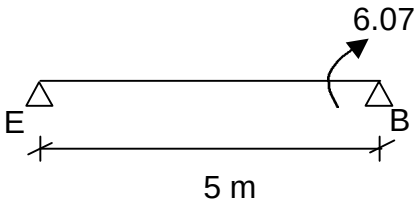


[1]



+

[2]



[1]

$$M_{EE_1} = \frac{p}{2} \cdot x = \frac{6}{2} \cdot x = 3 \cdot x$$

$$M_{E_1B} = \frac{p}{2} \cdot (l - x) = \frac{6}{2} \cdot (5 - x) = 15 - 3 \cdot x$$

[2]

$$M_{EB} = -\frac{M_e}{l} \cdot (l - x) - \frac{M_b}{l} \cdot x = -\frac{6.07}{5} \cdot x = 1.21 \cdot x$$

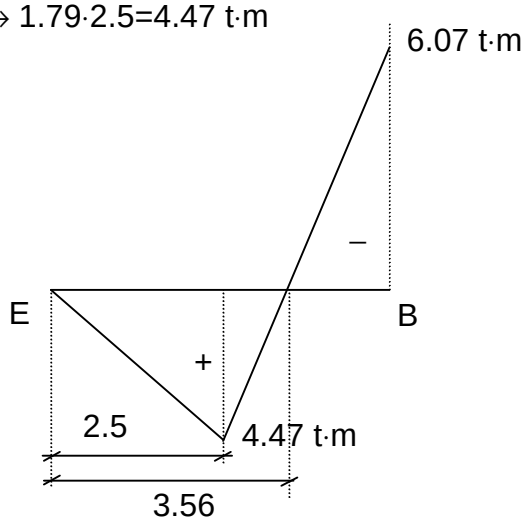
Superponiendo:

$$M_{EE_1} + M_{EB} = 3 \cdot x - 1.21 \cdot x = +1.79 \cdot x$$
$$M_{E_1B} + M_{EB} = 15 - 3 \cdot x - 1.21 \cdot x = 15 - 4.21 \cdot x$$

Puntos de corte

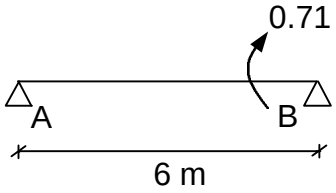
$$15 - 4.21 \cdot x = 0$$
$$x = 3.56 \text{ m}$$

En x =2.5 m → 1.79·2.5=4.47 t·m



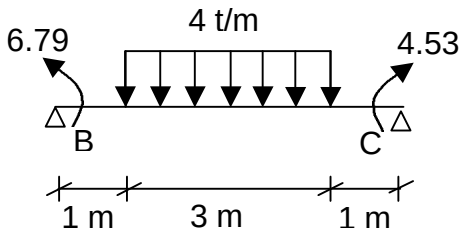
5º Reacciones y diagrama de esfuerzo cortante

Tramo AB



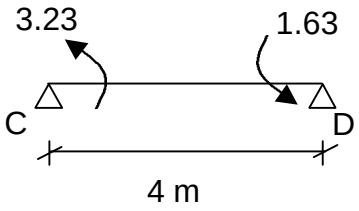
$$\sum M_A = 0$$
$$R_B \cdot 6 - 0.71 = 0$$
$$R_B = +0.12\ t$$
$$\sum M_B = 0$$
$$R_A \cdot 6 + 0.71 = 0$$
$$R_A = -0.12\ t$$

Tramo BC



$$\sum M_B = 0$$
$$R_C \cdot 5 + 6.79 - 4.53 - 4 \cdot 3 \cdot 2.5 = 0$$
$$R_C = +5.55\ t$$
$$\sum M_C = 0$$
$$R_B \cdot 5 + 4.53 - 6.79 - 4 \cdot 3 \cdot 2.5 = 0$$
$$R_B = +6.45\ t$$

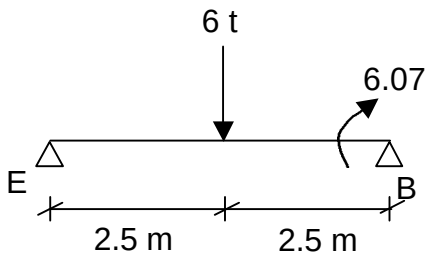
Tramo CD



$$\sum M_C = 0$$
$$R_D \cdot 4 + 3.23 + 1.63 = 0$$
$$R_D = -1.21\,t$$

$$\sum M_D = 0$$
$$R_C \cdot 4 - 3.23 - 1.63 = 0$$
$$R_C = +1.21\,t$$

Tramo EB



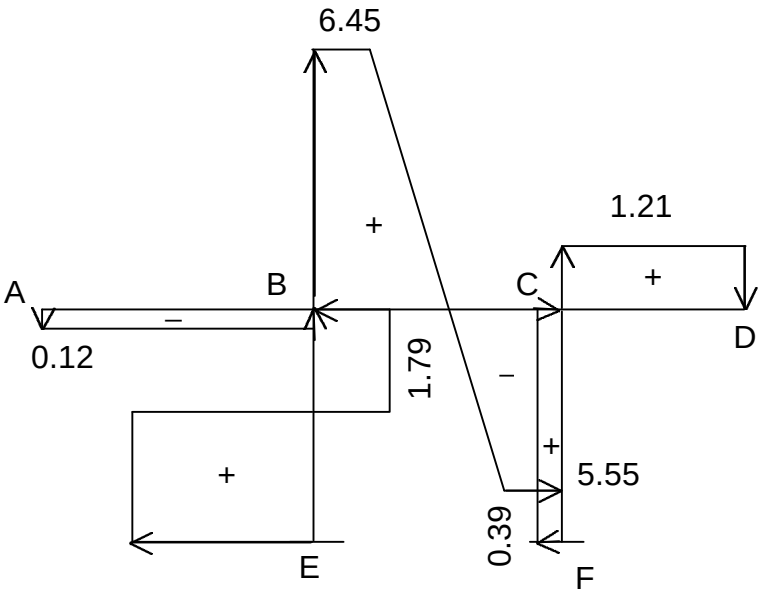
$$\sum M_E = 0$$
$$R_B \cdot 5 - 6 \cdot 2.5 - 6.07 = 0$$
$$R_B = +4.21\,t$$

$$\sum M_B = 0$$
$$R_E \cdot 5 - 6 \cdot 2.5 + 6.07 = 0$$
$$R_E = +1.79\,t$$

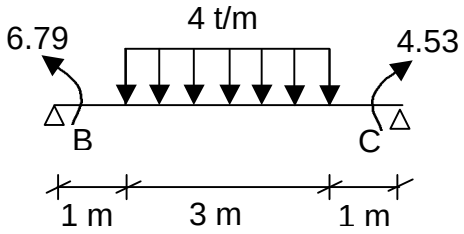
Tramo FC

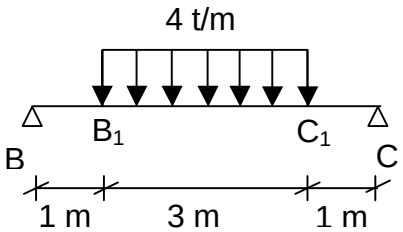
$$\sum M_F = 0$$
$$R_C \cdot 5 + 0.65 + 1.30 = 0$$
$$R_C = -0.39\,t$$

$$\sum M_C = 0$$
$$R_F \cdot 5 - 0.65 + 1.30 = 0$$
$$R_F = +0.39\,t$$



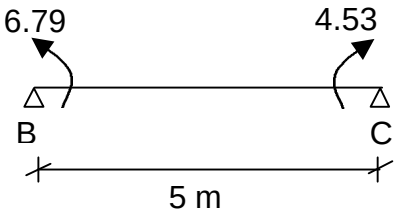
6º Angulo girado en C por la barra BC





$$\varphi_C = -\frac{q \cdot b \cdot a \cdot c}{6 \cdot E \cdot I \cdot l} \cdot \left(l + a - \frac{c^2}{4 \cdot b} \right)$$

$$\varphi_C = -\frac{4 \cdot 2.5 \cdot 2.5 \cdot 3}{6 \cdot E \cdot I \cdot 5} \cdot \left(5 + 2.5 - \frac{3^2}{4 \cdot 2.5} \right) = -\frac{16.5}{E \cdot I}$$



$$\varphi_C = \frac{l}{6 \cdot E \cdot I} \cdot \left(M_b + 2 \cdot M_c \right)$$

$$\varphi_C = \frac{5}{6 \cdot E \cdot I} \cdot \left(6.79 + 2 \cdot 4.53 \right) = \frac{13.21}{E \cdot I}$$

$$\varphi_C = -\frac{16.5}{E \cdot I} + \frac{13.21}{E \cdot I} = -\frac{3.29}{E \cdot I} \text{ rad}$$

(E, I en t y m)