Auction Design in Day-Ahead Electricity Markets (Republished)

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***Abstract*—Competition in day-ahead electricity markets has been established through auctions where generators and loads bid prices and quantities. Different approaches have been discussed regarding the market auction design. Multi-round auctions, despite its implementation complexity, allow market participants to adapt their successive bids to market prices considering their operational and economic constraints. However, most of the day-ahead electricity market implementations use noniterative single-round auctions. This paper presents a market simulator to compare both auction models. Different auction alternatives, such as the Spanish single-round auction that takes into account special conditions included in the generator bids, and multi-round auctions with different stopping rules, are analyzed. The results and acquired experience in the simulation of the Spanish market, started in January 1998, are presented. Hourly market prices, average daily price, price/demand correlation and several economic efficiency indicators, such as generator surplus, consumer surplus and social welfare, are compared to derive conclusions regarding the performance of the auction alternatives.**

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***Index Terms*—Auctions, bidding strategies, electricity markets, market clearing price.**

I. INTRODUCTION

FOR ELECTRIC power systems all over the world, the transition from regulation to competition, has resulted in very profound changes in the procedures applied to guarantee an economic and secure system operation.

Generation competition has replaced traditional centralized cost minimization dispatch algorithms by market auctions where market participants bid prices and quantities.

Presently, there is an open discussion about the advantages and shortcomings of several proposed and implemented auction models [1].

In particular, whether iterative multi-round auctions are better than noniterative single-round auctions, is still under discussion. On one hand, a multi-round approach [2], [3] should allow different market participants to reach an adequate final dispatch from an operational (ramp rates, minimum unit's output, etc.) and profitability (recovering of start-up cost, nonload costs, etc.) points of view. This is the type of auction used in the National Electricity Market (NEM) in Australia, and the New Zealand Electricity Market (NZEM). On the other hand, the difficulty of the implementation associated with the multiple iterations needed to reach a market equilibrium and the risk of collusion, seem to be the main drawbacks of this approach. Specific activity and closing rules have been designed to implement a multi-round auction in the day-ahead market in California [3].

However, most of the current implementations of day-ahead electricity markets have chosen a single daily auction. This auction receives the day-ahead generator and demand hourly bids and produces the hourly schedules and market prices in one round, i.e., without carrying out an iterative procedure demanding new bids from market participants. This is the case of England and Wales, Norway, the current market implementation in California and Spain, where with different variations from case to case single daily auctions have been implemented. In the day-ahead Spanish market, a single-round approach with complex bids has been adopted to take into account the individual operation conditions of the generators. Generation units are allowed to bid using additional conditions besides simple quantity/price bids. These additional conditions include: nondivisible quantity bids, minimum daily income, and up and down ramp rates.

The aim of this paper is to compare the market results obtained under several auction models when ideal competitive conditions[1] are assumed. A market simulator tool has been developed to implement the auction alternatives and the bidding strategies of the market participants. Two main aspects have been studied using this simulator. First, the impact on market results of the extra conditions included in the generator bids in the Spanish single-round auction are studied. Secondly, a multi-round auction where market participants adapt their consecutive bids to resulting market prices until the market equilibrium is reached, is simulated. The market equilibrium condition depends on the selected stopping rule. Two alternative stopping rules have been implemented in the simulation tool. Note that all these simulations reflect a simplified competitive model that should be used as a benchmark to test auction methods.

The paper is organized as follows. Section II presents a description of the single-round auction model distinguishing between simple and complex bids. Section III describes the multi-round auction model and its implementation in the market simulator. Section IV details the bidding strategies implemented for

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[1] Under ideal competitive conditions all generation units are independent bidders regardless of current ownership. They bid using their variable, fixed operation costs and start-up costs.

thermal and hydro generation units. Section V presents the two alternative stopping rules to reach the market equilibrium in the multi-round auction. Section VI shows the results obtained by the simulation of the different auction models in the Spanish market. Finally, several conclusions derived from the results are presented in Section VII.

II. SINGLE-ROUND AUCTIONS

In the single-round auction, the supply bids are matched with the demand bids by the market clearing algorithm without any further iteration with market participants. In the day-ahead electricity market each participant presents bids for each one of the day ahead 24 hours.

The "simple" bid format consists of a pair of (hourly) values: quantity (MWh) and price (pta/kWh).[2] Each selling or buying participant can present several pairs of values for the same generation or demand unit. If there are extra operational and/or economic conditions that are added to the simple bid, the bid is called "complex."

In case of simple bids, the market clearing algorithm implemented in the simulation tool works as follows. For each hour, a supply curve is built up considering the selling bids for that hour ordered by increasing prices, and a demand curve is built up considering the buying bids for that hour ordered by decreasing prices. The intersection of the supply and demand curves determines the selling and buying bids that are accepted and the hourly market price obtained as the price of the last accepted selling bid.

In the day-ahead Spanish market, that uses a complex bid, each production unit, thermal generation unit or subsystem of hydro power plant in the same river basin, is allowed to bid, in addition to the simple price/quantity bids, with three extra conditions:

- *A Non-Divisible Quantity*: The cheapest bid quantity can be designated as nondivisible. That is, in case that the designated bid is accepted, it should be for the total quantity and not for a fraction of it. This condition has been implemented to take into account that some thermal units have to run above a specified minimum operating level.
- *A Minimum Daily Income Amount*: It can be set as a prerequisite for the scheduling of the considered generating unit. This condition is specified with a fixed term (in pta) and a variable term (in pta/kWh). The specified minimum income value (the fixed term plus the variable term times the sum of the energy dispatched for a 24 hour period) must be lower than the income that would result by multiplying the specified quantities and prices in the "simple" format of the considered generating unit's bid. This condition has been implemented in the market clearing algorithm to minimize generators' risks when they have to internalize start-up and no-load fuel costs in their simple bids.
- *Up and Down Ramp Rates*: The maximum variations of the unit output in two consecutive hours can be specified. That is, for the considered production unit, the energy scheduled by the matching algorithm in two consecutive hours must meet the maximum variation condition specified as MW/min for increasing output and/or MW/min for decreasing output. This condition has been implemented in the matching algorithm to take into account ramp rate limits of thermal units.

In case of complex bids, the market clearing algorithm is a modification of the simple matching algorithm that can be expressed in two steps as follows.

1) A feasible matching solution, known as the "first feasible solution," that meets all the extra bidding constraints is determined in the following steps:
    - A simple matching solution is obtained assuming all bids are just "simple" bids.
    - The ramp rate limits are enforced in the scheduled quantities for each two consecutive hour periods.
    - The minimum daily income condition for those production units that have submitted it is verified, units that do not meet that condition are not scheduled.
2) An additional procedure is carried out to check whether the first solution found in the previous steps can be improved. The check rule is to find at least one unit that was not matched in the first solution for which the difference between its potential daily income derived from the resulting market prices and its required minimum income is positive. The improvement criterion is to minimize the sum of the differences between the unit daily incomes derived from the market prices and the minimum incomes specified for the units that were not matched and had a positive difference. A combinatorial heuristic search algorithm has been implemented to explore the space of feasible solutions. A detailed presentation of the market clearing algorithm in case of complex bidding can be found in [4]–[6].

III. MULTI-ROUND AUCTIONS

Multi-round auctions are based on an iterative procedure, where the generation agents update their simple price/quantity bids at every iteration or round, as seen in Fig. 1. This iterative multi-round auction allows generating agents to minimize the risk of cost under recovery or infeasibility of dispatch. The implemented approach mainly follows the one proposed by Otero-Novas *et al.* [2] with some differences regarding the stopping rules to finish the iterative bidding process. At every iteration, price bids are either frozen, or decreased by a fixed price decrement, to avoid clearing prices reaching high values, as Wilson proposed in [3]. Note that when the agents decrease their price bids they become more competitive, but they risk losing the recovery of their generation costs. In that sense, the model simulates perfect competition conditions among the generation agents.

The implemented simulation model has a modular design, receiving (iterative) simple bids from the market agents (generators and consumers), matching all these bids with a clearing algorithm, and showing a final market price and dispatch (see Fig. 1). The bids' update and the stopping rules are heuristic, as will be described in Sections IV and V.

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[2] Pta is the Spanish currency unit until the year 2002; 150 pta were approximately equivalent to $1 during 1998.

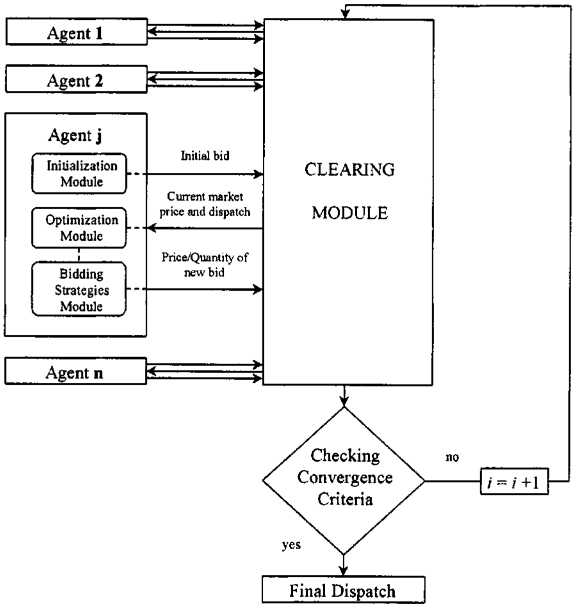

Fig. 1. Overall iterative algorithm scheme.

The four modules of the model are:

- Initialization module
- Clearing module
- Optimization module
- Bidding strategies module

The initialization module reads the auxiliary files containing data for the model: demand, generator costs, hydro data, etc., and the options file containing: number of iterations, stopping criteria, hydro bidding options, etc.

The clearing module consists of a simple clearing algorithm, where the bids are added up to match the hourly demand. The intersection of supply and demand curves determines the hourly market prices in each iteration.

The optimization module receives the clearing prices from the previous iteration and maximizes each generation unit profit individually. The output is the amount of generation that every unit will offer in the next iteration. It is a decentralized module, where each generation unit decides its production based on market prices and its own costs, additionally the ramp rate limits are also considered. Thermal unit's costs are composed of variable operation costs, fixed operation costs, and startup costs. Total generation quantity is offered in two blocks: the first block or minimum generating power (the minimum generation that is necessary to run the unit), and the second block (between the minimum and the maximum power). Moreover, hydro bids take into account the opportunity costs of water.

The bidding strategies module receives the output of the clearing and optimization modules at every iteration and decides what will be the next offer in price and quantity for every generator. Hydro and thermal units have different quantity and price bidding policies as it will be explained in Section IV.

Finally, several convergence conditions will determine that the iterative process has finished. The stopping rules that determine the convergence of the algorithm will be treated in detail in Section V.

IV. ITERATIVE BIDDING STRATEGIES

As explained in Section III, once the results are obtained from the clearing and optimization modules, each generation unit is ready to bid prices and quantities using the bidding strategy module.

Before the first iteration, the initial values for the quantity and price bids of all units are set. Initial bids start from highly conservative prices in all 24 daily hours, because competition through the iterative algorithm will gradually decrease bid prices. Thermal gas units, commonly used during peak hours, internalize their startup costs in their initial bids. Thermal nongas units, for instance coal plants, only internalize part of their higher start-up costs, because they cannot recover the whole start-up costs in just a 24 hour period. Initial bids are presented in the Appendix, Section B, in mathematical notation.

A. Thermal Units

The quantities that are offered by the generation units are always provided by the optimization module, based on [2]. The price strategy uses a heuristic rule that takes into account the current price, the generator cost structure, and the market ee price decrement, see [3], at every iteration.

In the simple clearing algorithm used, whenever a price bid is lower than thefinal clearing price, the whole quantity bid is accepted. Therefore, a unit whose quantity bid was fully accepted at iteration i can repeat its price bid for the next iteration. Otherwise, the unit must change its price bid. Units modify their price bids with decrements that are controlled with a parameter [$a(g, h)$ in the Appendix]. This parameter limits excessively low price bids in the trough hours, where the units run the risk of not recovering their costs.

There might be convergence problems, as can be seen in Section V, where the reason for oscillations for consecutive market prices is analyzed. These oscillations can be smoothened with adequate stopping rules. Note that the first block is offered at zero price, to guarantee that the minimum power is always accepted in the auction. The mathematical formulation of the quantity and price bidding strategies is shown in Section C of the Appendix.

B. Hydro Units

The hydro bidding strategy considers an amount of energy in MWh that is available daily for each bidding unit. This amount is distributed among daily hours depending on the level of demand. That is, the higher the demand, the larger the amount of energy offered, subject to capacity and daily energy constraints. Hydro bidding does not require two bidding blocks because run-of-river constraints are included by modifying the demand curve.

The price strategy depends on three terms: the matched quantity and the previous iteration offered quantity, and the specified band value. In the Spanish market, a hydro river basin can offer different blocks of energy at different prices. In the proposed simulation model, each hydro unit is equivalent to each one of the energy/price blocks that compose the bid of each hydro river basin. The specified band value represents the minimum price

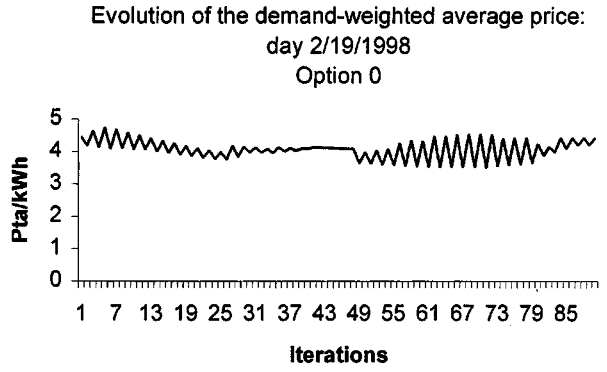

Fig. 2. Option 0 demand-weighted average price.

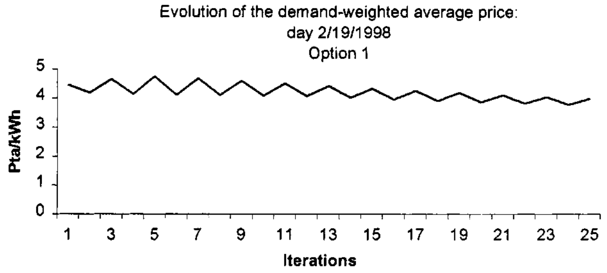

Fig. 3. Option 1 demand-weighted average price.

at which each hydro unit would bid its associated energy block. Since, for hydro units, competition can not be established based on variable costs, an accepted/offered quantity parameter called $b(g, h)$ in the Appendix is used.

The mathematical formulation of the quantity and price bidding strategies for hydro units is shown in Section D of the Appendix.

V. STOPPING RULES AND ITERATIVE ALGORITHM CONVERGENCE

The bidding rules presented in Section IV produce market prices whose values may oscillate from one iteration to the next, as seen in Figs. 2 and 3. This oscillatory behavior comes from the fact that the primal and dual solutions of the theoretical optimization problem (which satisfies the market equilibrium conditions) never intersect (the "duality gap" problem in LaGrangian relaxation, as pointed out in [7]). Therefore, to the authors' knowledge, even sophisticated heuristic bidding rules may produce oscillatory price behavior.

Two alternative stopping rules (options) have been implemented in order to fix price oscillations, achieving different market equilibrium points. Both options have been designed to study the market equilibrium price impact over generator surplus,[3] consumer surplus,[4] and social welfare (consumer surplus plus generator surplus) when the market is cleared. See Fig. 4 for a graphic representation.

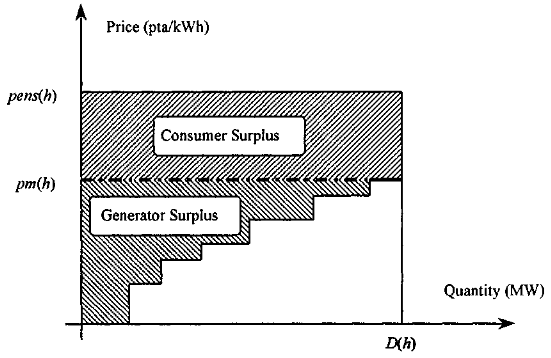

Fig. 4. Studied economic variables.

The first option, called "option 0," allows generators to compete against each other so that the resulting prices eliminate noncompetitive units from the auction during the iterative process. The iterative process continues until prices are so low that the demand is not sufficiently covered. At that iteration, some needed units leave the auction because low market prices do not allow them to recover their costs. Then the algorithm goes back two iterations, providing a final market equilibrium that matches the demand and a final price that still allows for a (minimum) profit for all the generators. Thus, final iteration prices show an increasingly unstable pattern due to the lack of a control parameter to dampen oscillations.

The main advantage of "option 0" (and "option 1") is the high correlation between price and demand, as seen in Table I. The main disadvantage of "option 0" (for the generators) is that they obtain the minimum profits that are feasible (minimum generator surplus); for a more detailed explanation see Section VI.

The second stopping rule option, called "option 1" in Fig. 3, uses two control parameters for checking the convergence: demand-weighted average price[5] and total financial loss.[6] This option is characterized by a continuously decreasing demand-weighted average price maintaining profits for all generators at every iteration. This option calculates the weighted average price and the total loss, and it compares the current average price with the average price obtained from previous iterations. When the ith iteration average price is higher than the two previous iteration prices, and the total loss in iteration $i-2$ is zero (i.e., there is no unit with losses in iteration $i-2$), then, the algorithm stops, and the final solution is the one resulting from iteration $i-2$.

The main advantage of "option 1" is to consider both generator and consumer surpluses in its calculations. Its main disadvantage is the smoother price profile as compared to "option 0," see Section VI for more details.

For both options, iteration $i-1$ profits are calculated using iteration i prices and iteration $i-1$ quantities. However, iteration i prices do not comply with the stopping rules of any option. Thus, it is necessary to go back one more iteration to find a solution where prices meet the stopping rules criteria.

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[3] Generator surplus is the area above the generating unit supply curve and below the market price. That area represents the revenue over variable production costs in a perfectly competitive market.

[4] Consumer surplus is the area under the consumer demand curve and above the market price. Fig. 4 $pens(h)$ represents the nonserved energy price, $D(h)$ the hourly demand and $pm(h)$ the hourly market clearing price at which the generator and consumer surplus are measured.

[5] Demand-weighted average price is the average of 24 hourly prices weighted by the corresponding 24 hourly demands.

[6] Total financial loss is the sum of all daily losses (negative generator surpluses) of all the bidding units that are also dispatched.

To test both stopping rules under ideal competitive conditions, the Spanish market demand curve of February 19, 1998 was selected. The iterative process was monitored until convergence criteria were met, as seen in Figs. 2 and 3. "Option 0" price oscillations almost flattened, but, after a while, increased. "Option 0" and "option 1" shared the first 26 iterations; at that point the stopping criterion of "option 1" stopped the simulation. "Option 0" continued until its own stopping criterion stopped the process at the 90th iteration. Note that both options showed a sawtooth shape before stopping, since Fig. 3 is just a magnification of Fig. 2 for the first 26 iterations. Generator units entering and leaving the auction through iterations produced this sawtooth curve. Note that in the Spanish market there is almost no bidding from the demand side, that is, the demand is represented by 24 hourly inelastic curves (see Fig. 4).

VI. CASE RESULTS

Different auction models are compared in this section. The Spanish day-ahead market was simulated assuming ideal competitive conditions among the generation units as seen in Section E of the Appendix. The single-round auction, considering simple and complex bids, and the multi-round iterative auction with two alternative stopping rules, were simulated with the real market demand from October 1, 1998.

The single-round market clearing algorithm was implemented using MATLAB® and the multi-round auction market simulator was developed in C using CPLEX® 5.0 in implementation of the optimization module.

Thermal unit bids were generated taking into account generation costs and well known operational constraints under the previous Spanish regulatory framework [8]. Hydro unit bids were generated according to the actual band values that were dispatched during that day. The hydro bands were distributed according to hourly demands.

Several economic efficiency indicators were obtained to evaluate the market clearing prices provided by each auction model. The chosen indicators were consumer surplus, generator surplus, and social welfare. In order to carry out this analysis, the demand was considered inelastic, and the price of nonserved energy was set at 30 pta/kWh.

A. Analysis of the Spanish Market Prices

Fig. 5 shows the market prices obtained by the different auction models: single-round with simple and complex bids (simple and complex), and iterative multiround with the two alternative stopping rules (option 0 and option 1). Weighted average market prices (pta/kWh) for every case are shown in Table I.

In the single-round auction with simple bids, market prices were extremely low in the valley hours because bids were at variable cost and the bidding strategy did not allow for cost recovery during the peak hours.

In the single-round auction with complex bids, generation units were able to recover much of their costs by using the additional conditions instead of using iterative bids.

In the multi-round auction that used the "option 0" stopping rule there was a significant difference between peak hour prices and valley prices. The reason was that the generation units internalized their costs in their bids so that they could earn in the peak hours what they were losing in the valley hours.

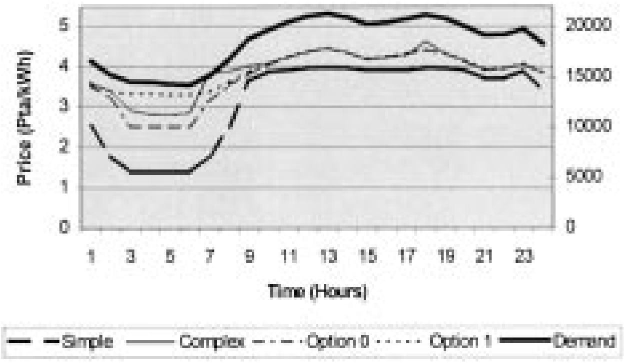

Fig. 5. Comparison of hourly market prices (10/1/1998).

TABLE I
WEIGHTED AVERAGE PRICES

AUCTION	PRICE (pta/kWh)
Simple	3.3011
Complex	3.9535
Option 0	3.8449
Option 1	3.9808

TABLE II
CORRELATION INDEXES

AUCTION	CORRELATION
Simple	96.54 %
Complex	87.66 %
Option 0	95.62 %
Option 1	98.59 %

"Option 1" price curves were smoother than "simple," "complex" and "option 0" price curves. There were no significant differences between peak and valley hour prices. This option had a controlling effect over the demand-weighted average price that avoided significant differences among hourly prices.

Additionally, a price/demand correlation index was obtained from a linear price/demand correlation equation, as summarized in Table II. It was observed that the worst correlation corresponded to the single-round auction with complex bids because the additional bid conditions distorted the high correlation associated with simple matching algorithms.

B. Economic Analysis of the Results

Fig. 6 shows the generator surplus obtained with the different auction models presented. In this graph, bars represent the generator surplus in millions of pta for October 1, 1998. "Option 0" provided lower values than "option 1" due to the design of the stopping rules. Single-round auctions with simple and complex bids showed an intermediate surplus between "option 0" and "option 1" values. Note that simple bids showed a reasonable global generator surplus because price bids were at

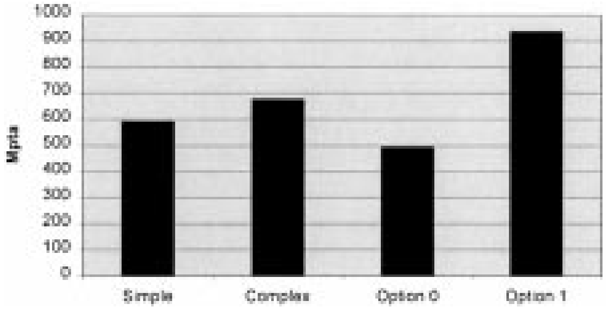

Fig. 6. Comparison of generator surplus.

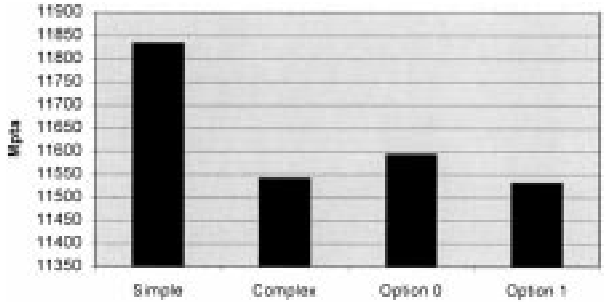

Fig. 7. Comparison of consumer surplus.

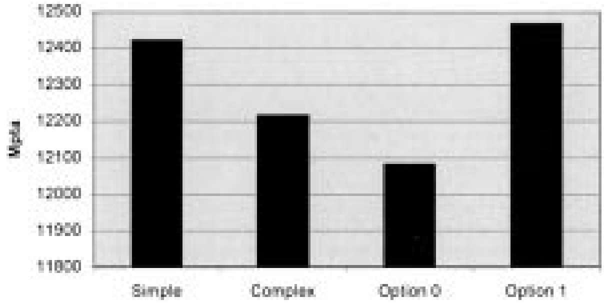

Fig. 8. Comparison of social welfare.

variable cost. However, some of the generators did not recover the total incurred cost, i.e., start-up cost, operational cost and fixed cost. For that reason, simple bidding was not adopted in the Spanish market, and a complex bidding process was chosen instead. Complex bids showed higher price bidding curves and also higher final prices, therefore higher generator surplus.

The same bar analysis was made with the results of the consumer surplus, using 30 pta/kWh as reference for nonserved energy price. Consumer surplus is calculated as a function of the difference between the nonserved energy price and the market price in each hour.

According to Fig. 7, "option 0" provided a higher consumer surplus than "option 1" due to the stopping rule that favored higher generation competition and results in lower market prices. The single-round auction with simple bids was extremely good for the consumers, as expected, because prices went down, specially during the valley hours. Complex bids results were intermediate between options 0 and 1.

Finally, the social welfare is represented in Fig. 8, i.e., the addition of the previous economic variables: generator and consumer surpluses.

The social welfare obtained with "option 0" was the lowest one. "Option 0" selected the necessary competitive generators to cover the demand, providing a price profile that just allowed for cost recovery, thus, the generator surplus was low. On the other hand, consumer surplus was reasonably high, but not high enough to guarantee an adequate social welfare. Single-round auctions results were intermediate between options 0 and 1 showing that either simple bids or additional constraints can also lead to a reasonable social welfare, although worse than "option 1."

"Option 1" provided the best result. The resulting price profile allowed not only the highest income level for the companies (highest generator surplus), but also a reasonable consumer surplus. Note that "option 1" convergence criteria (demand-weighted average price and total loss) were designed to reach a compromise between a decreasing price pattern and adequate generator profits. In exchange, both "option 0" and the complex bidding used in Spain had different targets (minimum generator profits and minimum distortion in the required revenues expected by the nondispatched generators, respectively) that did not lead to the best overall results.

VII. CONCLUSION

Several conclusions can be drawn regarding day-ahead market auction designs from our case studies of the Spanish electricity market:

- Multi-round auctions with iterative bidding require much higher computational resources than their simple-round counterparts to reach an adequate market equilibrium. Although they can be implemented in practice, the number of iterations is more than 25 for the presented Spanish generation case, for example.
- In this paper, two stopping rules have been proposed and tested with a market simulation tool. However, some kind of oscillatory behavior can be expected, even if converging, due to duality gap convergence problems.
- The highest social welfare was obtained by the iterative market simulation using the stopping rule "option 1." However, the single-round auction with simple bids provided an almost equal result, and, regarding consumer surplus, it was much better. However, due to the lack of recovery of total costs of some generators, the single-round option with simple bids was not adopted in Spain.
- The current single-round auction with complex bids used in the Spanish market provided a social welfare value lower than the one obtained by the same single-round auction with just simple bids. In addition, with the current Spanish market clearing algorithm, because of the treatment of complex bids, it is sometimes difficult to distinguish between competitive and noncompetitive generators that are finally matched.
- Market prices produced by a single-round auction with complex bids are less correlated with demand than those obtained with matching algorithms that only consider simple bids, even if multi-round auctions take place.
- Iterative bidding does not allow generating units to have losses because they can recover total production costs in

the bidding process simply by internalizing total costs in their bids. In contrast, simple bidding at variable costs generates losses, as expected.

- The accumulated experience with multi-round auction simulations does not recommend the implementation of iterative bidding in day-ahead markets. However, the use of this kind of simulation model as a benchmark to compare the performance of single-round auction designs would be very useful. The present single-round auction with complex bids used in the Spanish market is an attempt to minimize generation risks associated to intrinsic operational and economic characteristics. However, several improvements should be introduced to achieve better economic performance, such as the one presented by the multi-round auction with stopping rule "option 1."

APPENDIX

A. Notation

i: number of iteration.

h: hour of day.

g: hydro or thermal generator unit.

$D(h)$: hourly demand in MW.

$q_{i\_1}(h)$: quantity in MW that the group presents to the clearing module in the hth hour of the ith iteration, corresponding to the first bidding block.

$q_{i\_2}(h)$: quantity in MW that the group presents to the clearing module in the hth hour of the ith iteration, corresponding to the second bidding block.

$q_i(h)$: quantity in MW that the group presents to the clearing module in the hth hour of the ith iteration, such that $q_{i\_1}(h) + q_{i\_2}(h) = q_i(h)$.

$q_{io\_1}(h)$: quantity in MW that the group presents to the clearing module in the hth hour of the initial iteration $(i = 1)$, corresponding to the first bidding block.

$q_{io\_2}(h)$: quantity in MW that the group presents to the clearing module in the hth hour of the initial iteration $(i = 1)$, corresponding to the second bidding block.

$q_{io}(h)$: quantity in MW that the group presents to the clearing module in the hth hour of the initial iteration $(i = 1)$, such that $q_{io\_1}(h) + q_{io\_2}(h) = q_{i0}(h)$.

$qd_{i\_1}(h)$: quantity in MW from the first bidding block that is accepted by the clearing algorithm in the hth hour of the ith iteration.

$qd_{i\_2}(h)$: quantity in MW from the second bidding block that is accepted by the clearing algorithm in the hth hour of the ith iteration.

$qd_i(h)$: quantity in MW that is accepted by the clearing algorithm in the hth hour of the ith iteration, such that $qd_{i\_1}(h) + qd_{i\_2}(h) = qd_i(h)$.

$p_{i\_1}(h)$: bid price in pta/kWh at which the quantity $q_{i\_1}(h)$ is offered.

$p_{io\_1}(h)$: bid price in pta/kWh at which the quantity $q_{io\_1}(h)$ is offered.

$p_{i\_2}(h)$: bid price in pta/kWh at which the quantity $q_{i_2}(h)$is offered.

$p_{io\_2}(h)$: bid price in pta/kWh at which the quantity $q_{io\_2}(h)$ is offered.

$pm_i(h)$: marginal market price of hour h in pta/kWh when clearing the market at the ith iteration.

$pm(h)$: real market marginal market price of hour h in pta/kWh when clearing the market.

$pens(h)$: nonserved energy price in pta/kWh.

$P_{\max}(g)$: maximum power generation in MW.

$P_{\min}(g)$: minimum power generation in MW.

$cv(g)$: generator variable cost in pta/kWh.

$cac(g)$: generator fixed operation cost in pta/h.

$carr(g)$: generator start-up cost in pta.

ee: price decrement in pta that a generator applies to its current offered price to decrease it for the next iteration (according to Wilson's rules [3]). Typical values are lower than 1.

pre: initial bid price in pta/kWh of the hydro units.

$valof(g)$: lowest bid price in pta/kWh of a hydro unit.

$E(g)$: available energy in MWh for the hydro unit g.

$E_{total}(g)$: sum of all the accepted energy bids of thermal unit g for a 24 hour period in kWh.

$E_{acc}(g, h)$: accepted quantity bid of thermal unit g in the ith hour in kW.

$a(g, h)$: hourly variable cost/marginal price ratio: $cv(g)/pm_i(h)$.

$b(g, h)$: hourly accepted/offered second hydro quantity block ratio: $qd_{i\_2}(h)/q_{i\_2}(h)$.

α: hydro bidding heuristic coefficient.

$T_f(g)$: fixed term of the minimum daily income condition of the Spanish clearing algorithm in pta.

$T_v(g)$: variable term of the minimum daily income condition of the Spanish clearing algorithm in pta/kWh.

B. Bid Initialization

B.1 Thermal conventional units:

$$\begin{aligned} q_{io\_1}(h) &= P_{\min} \\ q_{io\_2}(h) &= P_{\max} - P_{\min} \\ p_{io\_1}(h) &= 0 \\ p_{io\_2}(h) &= cv(g) + cac(g). \end{aligned}$$

B.2 Thermal gas units:

$$\begin{aligned} q_{io\_1}(h) &= P_{\min} \\ q_{io\_2}(h) &= P_{\max} - P_{\min} \\ p_{io\_1}(h) &= cv(g) + cac(g)/P_{\max}(g) \\ &\qquad + carr(g)/[24^* P_{\max}(g)] \\ p_{io\_2}(h) &= cv(g) + cac(g)/P_{\max}(g) \\ &\qquad + carr(g)/[24^* P_{\max}(g)]. \end{aligned}$$

B.3 Hydro units:

$$\begin{aligned} q_{io\_1}(h) &= 0 \\ q_{io\_2}(h) &= \text{depending on hydro bidding} \\ &\qquad \text{strategies (see Section IV-B).} \end{aligned}$$

$$p_{io\_1}(h) = 0$$
$$p_{io\_2}(h) = pre.$$

C. Thermal Bidding Strategies

The quantity bidding algorithm is as follows:

```
C.1 if $q_{(i+1)}(h) \geq p_{\min}$
        then
        $q_{(i+1)\_1}(h) = P_{\min}$
        $q_{(i+1)\_2}(h) = q_{(i+1)}(h) - P_{\min}.$
C.2 else
        $q_{(i+1)\_1}(h) = 0$
        $q_{(i+1)\_2}(h) = 0.$
```

Note that the quantities for the next iteration $(i + 1)$ are based on the optimization based on the previous clearing price. See [2] for details.

Thermal bidding price strategies, in mathematical notation, are as follows:

```
C.3 if $q_{(i+1)}(h) \geq P_{\min}$
    a) if $pm_i(h) - ee \geq p_{i\_2}(h)$
       then
       $p_{(i+1)\_1}(h) = 0$
       $p_{(i+1)\_2}(h) = p_{i\_2}(h)$
    b) else
       b.1) if $cv(g) < pm_i(h)$ & $1 - ee \geq a(g, h)$
            then
            $p_{(i+1)\_1}(h) = 0$
            $p_{(i+1)\_2}(h) = pm_i(h) - ee$
       b.2) if $cv(g) < pm_i(h)$ & $1 - ee < a(g, h)$
            then
            $p_{(i+1)\_1}(h) = 0$
            $p_{(i+1)\_2}(h) = pm_i(h) - [1 - a(g, h)]$
       b.3) if $cv(g) = pm_i(h)$
            then
            $p_{(i+1)\_1}(h) = 0$
            $p_{(i+1)\_2}(h) = cv(g)$
       b.4) if $cv(g) > pm_i(h)$
            then
            $p_{(i+1)\_1}(h) = 0$
            $p_{(i+1)\_2}(h) = pm_i(h) - [1 - 1/a(g, h)]$
C.4 else
        $p_{(i+1)\_1}(h) = 0$
        $p_{(i+1)\_2}(h) = 0.$
```

In C.3.b.1, the variable cost is lower than the current price by a factor of $(1 - ee)$, and the decrement for the next bid is ee. In C.3.b.2, the variable cost is lower than the current price, but higher than $(1 - ee)^* pm_i(h)$. This heuristic decrement is just a threshold such that the decrement in the next price bid is $[1 - a(g, h)]$, which is lower than ee. Therefore, the price bid decrement diminishes to gradually smoothen the process of decommitment of the units. Only in C.3.b.3, where the next price bid is at variable cost (which is lower than the previous price bid $+ee$) there can be a very limited price increment. Note that a dimensionless ratio $a(g, h)$ is added to a price. Although not a correct procedure, it serves the purpose of smoothing the price decrement process.

D. Hydro Bidding Strategies

The quantity bidding algorithm is as follows:

$$q_{(i+1)\_1}(h) = 0$$
$$q_{(i+1)\_2}(h) = \frac{E(g) * D(h)}{\sum_{h=1}^{24} D(h)}$$

Hydro bidding price strategies, in mathematical notation, are as follows:

```
D.1 if $b(g, h) = 1$
    then
    $P_{(i+1)\_1}(h) = 0$
    $P_{(i+1)\_2}(h) = p_{i\_2}(h).$
D.2 if $p_{i\_2}(h) - \{ee + \alpha[1 - b(g, h)]\} \geq valof(g)$ & $b(g, h) < 1$
    then
    $p_{(i+1)\_1}(h) = 0$
    $p_{(i+1)\_2}(h) = p_{i\_2}(h) - \{ee + \alpha[1 - b(g, h)]\}.$
D.3 if $pm_i(h) - ee < valof(g)$
    then
    $p_{(i+1)\_1}(h) = 0$
    $p_{(i+1)\_2}(h) = valof(g).$
```

This parameter $b(g, h)$ permits more competitive bids according to the matched quantities. The price bid decrement is increased using an empirical parameter α (0.005 in the simulations). Parameter α is used to avoid instability in price decrements, due to highly volatile values of $b(g, h)$ as compared to $a(g, h)$. Note that a dimensionless ratio $b(g, h)$ is added to a price. Although not a correct procedure, it serves the purpose of smoothing the price decrement process.

E. Competitive Bidding Values

Competitive bids in the real Spanish market were only applied to thermal generation units, whilst hydro units used real quantity/price bids that occurred in that day in the real market. For thermal units, competitive bids were as follows:

$$q_{i\_1}(h) = P_{\min}$$
$$q_{i\_2}(h) = P_{\max} - P_{\min}$$
$$p_{i\_1}(h) = 0$$
$$p_{i\_2}(h) = cv(g).$$

Minimum daily income condition implementation-was implemented as follows:

$$T_f(g) = cac(g)^* 24 + 0.2^* carr(g)$$
$$T_v(g) = cv(g).$$

Therefore, start-up costs would be recovered in a period of 5 days. Note that it is considered that the minimum daily income condition is violated if:

$$T_f(g) + T_v(g)^* E_{total}(g) < \sum_{h=1}^{24} E_{acc}(g,\, h)^* pm(h).$$

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