An Effective Transmission Network Expansion Cost Allocation Based on Game Theory

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***Abstract*—The expansion of transmission systems impacts many entities in the market environment. Each entity may fare better or worse as a result of congestion relief in the presence of new investments. Negatively affected firms exert their influence to prevent the expansion from taking place. The opposition of these firms and the lack of appropriate incentives results in insufficient investments in transmission assets. The network is being frequently used at its maximum limits, leading to economic inefficiencies and reduced reliability. Hence, there is a need for effective incentive schemes for network expansion.**

In this paper, we propose a game theory-based scheme for the allocation of transmission expansion costs among market entities. The allocation takes into account both the physical and economic impacts of the new transmission assets and the influence of each firm on the expansion decision. This is the first scheme designed to give all market participants explicit incentives to support the expansion. The application of the allocation solution to the Garver six-bus system is presented to illustrate the capabilities of the proposed method.

***Index Terms*—Cooperative game theory, cost allocation, investment incentives, Shapley value, social welfare, transmission expansion.**

I. Introduction

UNDER the conventional vertically integrated structure, each utility is granted a *franchise territory* to provide electric service. In return, the utility has the *obligation to serve* all customers on a nondiscriminatory basis at tariff rates. Utilities serve the loads in their territories mainly using their own generation and transmission facilities. Network reliability is the primary driver for investing in interconnections between utilities. Transmission investments usually impact few utilities. If approved by regulators, the utility recovers the investments costs and earns the regulated rate of return on them.

In the restructured market environment, utilities are required to have independent generation, transmission, and distribution business units. In addition, there are new players, such as independent power producers, new structures are created, such as the independent grid operator (IGO), and the ownership of the network is separated from the control and operation of the network. In this multi-player setting, the lack of appropriate incentives has resulted in investments in transmission not keeping pace with load growth and investments in generation [1], [2]. As a result, the network is being frequently used at its maximum limits, leading to economic inefficiencies and reduced reliability. Hence, new, effective incentive schemes are needed for transmission network expansion. The incentives have to take into account both the prospective investors and the prospective users of the new assets. In this paper, we focus on the allocation of transmission expansion costs such that all market entities have incentives to support the expansion.

Several allocation methods have been proposed for the network expansion costs [3]–[5]. They are basically extensions of allocation methods for existing transmission network costs, such as the ones discussed in [4], [6], and [7]. Some allocate the costs according to the contribution of each entity to the network flows. Other methods allocate the costs according to the economic benefits each entity receives from the network expansion. The latter are preferred [5], [8], since the economic impacts of the expansion are taken into account more explicitly. Negative allocations, or reimbursements, are not allowed on the basis that the cost allocation should not compensate firms for profits gained due to congestion [5].

The objectives of increased market efficiency and social welfare maximization may compete with those of the individual firms in the planning of transmission asset additions. Each firm may be differently affected, faring better or worse as a result of congestion relief with the new investments [9]. Firms that are hurt if the expansion takes place exert their influence so as to prevent the expansion from happening. If the number of firms that oppose the expansion is significant, it is likely that the transmission network will not be expanded and the economic and reliability benefits from the expansion will not be realized.

The focus of this paper is on controversial cases, where other network expansion cost allocations do not succeed in giving each firm an incentive to support the expansion. A scheme based on cooperative game theory is designed so that all market participants support the expansion. The physical and economic impacts of the new transmission assets are taken into account. The expansion decision is explicitly modeled, in contrast to all existing schemes. The payments made by each participant are based on the increase in social welfare brought about by the new assets, the pre-investment surpluses of the players, and the influence the different market participants may have on the expansion decision.

Section II presents the transmission investment problem formulation. Section III contains the design of the proposed cost allocation scheme. Section IV discusses representative quantitative results using the Garver six-bus system. Section V summarizes the results and shows the need for further work.

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II. Transmission Investment Problem Formulation

The transmission expansion problem has been studied in [10]–[16], and various solution methods have been proposed. This section mostly follows the formulation in [16] due to its generality and flexibility. However, the cost allocation scheme can be applied to any transmission expansion problem formulation.

To formulate the transmission investment problem in a restructured environment, the electricity market is modeled as a pool-based system. The double-auction pool-based market mechanism maximizes the social welfare, so as to determine the maximum net benefit for society, measuring the overall impacts of both producers and consumers. It is assumed that transmission expansion planning and coordination is the responsibility of the IGO.

Without loss of generality, assume a single producer and a single consumer at each node $n = 1, 2, \ldots, N$ of the network, where $\mathcal{L} \triangleq \{1, 2, \ldots, L\}$ is the set of lines that connect the buses of the network. The node n selling entity's marginal offer in hour h is integrated and denoted by $\beta^s_{n,h}(p^s_{n,h})$. Similarly, the node n buying entity's marginal bid is integrated and denoted by $\beta^b_{n,h}(p^b_{n,h})$. The vectors $\boldsymbol{p}^s$ and $\boldsymbol{p}^b$ are constructed, consisting of the amounts sold and bought by the pool entities, respectively.

The set of candidate lines to be built is denoted by $\mathcal{L}^C \triangleq \{1, 2, \ldots, L^C\}$. The integer variable n^c takes the value of 1 if the investment in line asset ℓ^c is made, and 0 otherwise. The IGO estimates a cost Q^c for the investment in line asset ℓ^c.

The IGO's process to determine the successful bids/offers of the pool players on an hourly basis is stated as the *generalized transmission scheduling problem* (GTSP) [17]. The IGO needs this hourly information to maximize the aggregate social welfare minus the investment costs subject to the network constraints for a certain period of study. Therefore, the IGO calculates the social welfare over a specified planning horizon $\mathcal{H} \triangleq \{h : 1, 2, \ldots, H\}$, where h represents one hour of the planning horizon. The GTSP problem can be formulated as follows:

$$\max S_H \triangleq \sum_{h\in\mathcal{H}} \sum_{n=1}^{N} \left[\beta^b_{n,h}\left(p^b_{n,h}\right) - \beta^s_{n,h}\left(p^s_{n,h}\right)\right] - \sum_{\ell^c=1}^{L^C} n^c Q^c \quad (1)$$

s.t.

$$g_{n,h}(\boldsymbol{p}^s, \boldsymbol{p}^b, n^c) = 0 \leftrightarrow \mu_{n,h} \quad \forall n = 1, 2, \ldots, N, \forall h \in \mathcal{H} \quad (2)$$

$$f_{\ell,h}(\boldsymbol{p}^s, \boldsymbol{p}^b, n^c) \leqslant f^{\max}_{\ell} \leftrightarrow \lambda_{\ell,h} \quad \forall \ell \in \mathcal{L}, \forall h \in \mathcal{H} \quad (3)$$

$$f_{\ell^c,h}(\boldsymbol{p}^s, \boldsymbol{p}^b, n^c) \leqslant f^{\max}_{\ell^c} \leftrightarrow \lambda_{\ell^c,h} \quad \forall \ell^c \in \mathcal{L}^C, \forall h \in \mathcal{H} \quad (4)$$

$$n^c \in \{0, 1\} \quad \forall \ell^c \in \mathcal{L}^C \quad (5)$$

where $g_{n,h}(\cdot)$ is the power flow balance equation at node n for hour h, $f_{\ell,h}(\cdot)$, and $f_{\ell^c,h}(\cdot)$ are the expressions of the power flow in line ℓ and candidate line ℓ^c, respectively, for hour h, and $f^{\max}_{\ell}$ is the maximum permissible flow on line ℓ. For every constraint set, there is a set of dual variables: $\{\mu_{n,h} : n = 1, 2, \ldots, N; h \in \mathcal{H}\}$ for the power flow balance equations, and $\{\lambda_{\ell,h} : \ell \in \mathcal{L}; h \in \mathcal{H}\}$ and $\{\lambda_{\ell^c,h} : \ell^c \in \mathcal{L}^C; h \in \mathcal{H}\}$ for the power flow in line ℓ and candidate line ℓ^c, respectively.

The optimization problem solved in (1)–(5) is formulated as a mixed-integer linear programming problem, following the model in [13], where we obtain a linear approximation for the cosine function (power losses) and the sine functions (lossless power flow). We also transform the model in such a way that the products of discrete and continuous variables are eliminated. The solution of (1)–(5) determines the social welfare S^*_H, the amounts $\boldsymbol{p}^{s*}$ and $\boldsymbol{p}^{b*}$ sold and bought by the pool players, the new lines to be built n^{c*}, and the cost of the investment in new lines Q^{c*}. In addition, the dual variables $\mu^*_{n,h}$, $\lambda^*_{\ell,h}$ and $\lambda^*_{\ell^c,h}$ provide the locational marginal prices (LMP) at each node n, and the marginal values of a change in the line limit for each line ℓ and candidate line ℓ^c, respectively.

The LMP and the quantities sold and bought are used to compute each firm's surplus for each hour h. The individual *producer surplus* $r^s_{n,h}$ during hour h for the producer in node n is defined as the difference between the revenues from the sale and the monetary offer that the producer makes for the sale

$$r^s_{n,h} = \mu^*_{n,h} p^{s*}_{n,h} - \beta^s_{n,h}\left(p^{s*}_{n,h}\right). \quad (6)$$

The individual *consumer surplus* $r^b_{n,h}$ during hour h for the consumer in node n is defined as the difference between what the consumer is willing to pay and the actual payment [18]

$$r^b_{n,h} = \beta^b_{n,h}\left(p^{b*}_{n,h}\right) - \mu^*_{n,h} p^{b*}_{n,h}. \quad (7)$$

The individual producer and consumer surpluses during the H -hour period for the producer and consumer in node n are

$$r^s_n = \sum_{h\in\mathcal{H}} r^s_{n,h} \quad (8)$$

$$r^b_n = \sum_{h\in\mathcal{H}} r^b_{n,h}. \quad (9)$$

The producer surplus r^S and the consumer surplus r^B are the sum of all the individual producer and consumer surpluses, respectively

$$r^S = \sum_{n=0}^{N} r^s_n, \quad (10)$$

$$r^B = \sum_{n=0}^{N} r^b_n. \quad (11)$$

Let $\bar{S}_H$ be the optimal social welfare without the payments for line investments

$$\bar{S}_H = S^*_H + \sum_{\ell^c=1}^{L^C} n^c Q^c = \sum_{h\in\mathcal{H}} \sum_{n=1}^{N} \left[\beta^b_n\left(p^{b*}_n\right) - \beta^s_n\left(p^{s*}_n\right)\right]. \quad (12)$$

Note that

$$\bar{S}_H \geqslant S_H^* \tag{13}$$

and the equality holds if and only if there are no new line assets. The social welfare $\bar{S}_H$ can be expressed as the sum of the producer surplus r^S, the consumer surplus r^B, and the *congestion rents* or *merchandising surplus* κ [19], [20]

$$\bar{S}_H = r^S + r^B + \kappa. \tag{14}$$

In many jurisdictions, the IGO is a non-for-profit organization, and so it is assumed that the congestion rents are allocated among the market participants.[1] Let κ_n^b and κ_n^s be the share of the congestion rents allocated to the consumer and producer at node n, respectively, with

$$\sum_n \left(\kappa_n^b + \kappa_n^s\right) = \kappa. \tag{15}$$

The congestion rents allocation rules are a key element of market design. The allocation may depend on the use of the network by each player, the location of each player in the network, the ownership of the lines, the ownership of certain financial rights, and so on [20]–[22]. The consumer and producer surpluses considering the apportioning of the congestion rents are defined as

$$x_n^s = r_n^s + \kappa_n^s, 0 \leqslant n \leqslant N \tag{16}$$

and

$$x_n^b = r_n^b + \kappa_n^b, 0 \leqslant n \leqslant N. \tag{17}$$

Let x^b and x^s be the vectors consisting of the surpluses x_n^b and x_n^s, respectively. Let $x = [\boldsymbol{x}^{b'} \boldsymbol{x}^{s'}]'$, where $\boldsymbol{x}'$ is the transpose of x. From (14)–(17)

$$\sum_n \left(x_n^s + x_n^b\right) = \bar{S}_H. \tag{18}$$

The social welfare, the surplus, and the congestion rents in the pre-investment scenario, where no new transmission assets are considered, are denoted by S_H^0, κ^0, and $\boldsymbol{x}^0$, respectively. As the objective of (1)–(5) is to increase the social welfare with the network expansion

$$S_H^* \geqslant S_H^0. \tag{19}$$

If $S_H^* > S_H^0$, the solution of (1)–(5) is to build at least one new line. Since the social welfare is increased, from a societal point of view, it makes sense to build the lines. However, it is very likely that for some $n, x_n^b < x_n^{b\,0}$ and/or $x_n^s < x_n^{s\,0}$, meaning that some players may fare worse after the network expansion.

Each consumer and each producer makes a payment a_n^b and a_n^s, respectively, for the investment in new lines. Thus, the consumer and the producer at node n obtain a *net surplus* of $(x_n^b - a_n^b)$ and $(x_n^s - a_n^s)$, respectively, after the expansion, during the H-hour period. It is assumed that the objective of each firm is to maximize its net surplus for the period.[2] Let $\boldsymbol{a}$ be the vector of payments, consisting of the values of a_n^b and a_n^s for each node n. In the following section, we propose a scheme for the construction of $\boldsymbol{a}$. In the remainder of the paper, $\boldsymbol{x}, \boldsymbol{x}^0, n^{c*}$ and Q^{c*} as obtained from the solution of (1)–(5) are assumed known.

III. Proposed Scheme for Payment Allocation

Since the market participants usually can influence the expansion of the network, it is assumed that the network expansion is decided by the result of a poll. The poll takes place before the H-hour period. Each consumer and each producer is assigned a weight, $w_n^b, w_n^s \in [0,1]$, with $\sum_n (w_n^b + w_n^s) = 1$. These weights, assumed known, measure the influence of each firm on the expansion decision. The network is expanded if the total weight of the firms that favor the expansion is larger than a known parameter q, with $0.5 \leqslant q \leqslant 1$, and not expanded otherwise.

Denote the consumer at node n by the two-tuple $\{b, n\}$ and the producer at node n by the two-tuple $\{s, n\}$.[3] Let $\mathcal{F}^b$ and $\mathcal{F}^s$ be the sets of consumers and producers in favor of the expansion, i.e., the consumer at node n favors the expansion whenever $\{b, n\}$ is in $\mathcal{F}^b$; the producer at node n favors the expansion whenever $\{s, n\}$ is in $\mathcal{F}^s$. In mathematical terms, the network is expanded if

$$\sum_{n:\{b,n\}\in\mathcal{F}^b} w_n^b + \sum_{n:\{s,n\}\in\mathcal{F}^s} w_n^s > q. \tag{20}$$

The first two terms are the sums of the voting weights of the consumers and producers in favor of the expansion, respectively. Let O^b and O^s be the sets of consumers and producers that have their surpluses decreased with the expansion, i.e., $\{b, n\}$ is in O^b whenever $x_n^b < x_n^{b\,0}$, and $\{s, n\}$ is in O^s whenever $x_n^s < x_n^{s\,0}$. These players fare worse after the expansion if negative payments are not allowed by the payment scheme or if the payments are ignored. Under such conditions, these players oppose the expansion. The focus of this paper is on cases where the voting weight of these players is enough to block the expansion, i.e.,

$$\sum_{n:\{b,n\}\in O^b} w_n^b + \sum_{n:\{s,n\}\in O^s} w_n^s \geqslant 1 - q. \tag{21}$$

In these cases, a payment scheme without reimbursements would result in the expansion not taking place, as the opposition to the expansion has at least $1 - q$ voting weight.

The goal is to construct a vector of payments $\boldsymbol{a}$ so that all the firms want to have the transmission network expanded. In this problem setting, the firms can be induced to cooperate. Cooperative game theory[4] has been successfully applied to solve other problems in power systems [24]–[26], and so we make use of it to approach our problem.

[1]If the IGO is a profit-making organization, we can consider it as a market participant who receives the congestion rents as a surplus.

[2]This assumption can be interpreted as assuming that each firm is a profit maximizer, and that the market rules are incentive-compatible, i.e., the sellers' offers and the buyers' bids represent their actual costs and benefits.

[3]The letter b stands for buyer, and the letter s stands for seller.

[4]The essential background needed to follow this paper is given in the Appendix. For a more in-depth treatment of cooperative game theory, see [23].

Consider a cooperative voting game played by all the producers and all the consumers. Any subset of firms can form a coalition in the poll. Since there are $2N$ firms, the number of different possible coalitions is 2^{2N}, including the (trivial) empty one. A *winning coalition* is one that has a total voting weight larger than q. The complement of a winning coalition, i.e., the set of all firms that do not belong to the coalition, forms a *losing coalition*. The rules of the game are as follows.

- Players in a losing coalition obtain their pre-expansion surplus, i.e., they are not affected by the expansion.
- Players in a winning coalition can choose whether or not they want to expand; if they decide to expand, they obtain the net gain in social welfare due to the expansion, plus their pre-expansion surplus.

Since the net gain in social welfare is never negative (19), a winning coalition will always choose to merge. Players in a winning coalition fare better than players in a losing coalition, and so the objective of the players is to be in a winning coalition. Thus, under this game, all players agree on expanding. The rules of the game can be expressed using the characteristic function $u : \mathcal{P} \rightarrow \mathbb{R}$ that gives the total utility (in terms of the surplus achieved by the coalition) that the coalition players receive (see the Appendix) in (22) at the bottom of the page, where $\mathcal{P} \triangleq \{\{b,n\},\{s,n\} : n = 1,2,\ldots,N\}$ is the set of the N buyers and the N sellers, and $\mathcal{C}$ is a coalition of firms (producers and/or consumers). Note that a coalition $\mathcal{C}$ can be formed exclusively by producers (or consumers). We see in (22) that the net gain of a winning coalition is the fixed term $(S_H^* - S_H^0)$.

The objective of a cooperative game analysis is to obtain a set of allocations that satisfy certain criteria given by the *solution concept*. Without loss of generality, the Shapley value solution concept is chosen due to its uniqueness and fairness properties [27]. The Shapley value of a player in a game can be interpreted as the expected increase to the coalition surplus brought by the player to a random coalition. Its expression for the producer at bus n is given by

$$\varphi_n^s = \sum_{\substack{\mathcal{C}\subseteq\mathcal{P}\\ \{s,n\}\in\mathcal{C}}} \frac{(c-1)!(2N-c)!}{2N!} \times[u(\mathcal{C}) - u(\mathcal{C}-\{s,n\})] \quad (23)$$

where c is the number of players in C, and $2N$ is the total number of players. Analogously, the Shapley value for the consumer at bus n is given by

$$\varphi_n^b = \sum_{\substack{\mathcal{C}\subseteq\mathcal{P}\\ \{b,n\}\in\mathcal{C}}} \frac{(c-1)!(2N-c)!}{2N!} \times[u(\mathcal{C}) - u(\mathcal{C}-\{b,n\})]. \quad (24)$$

For any solution concept, the sum of the allocations has to be equal to the value of the characteristic function for the coalition of all players, $\mathcal{P}$ [23]. Thus

$$\sum_n \left(\varphi_n^b + \varphi_n^s\right) = u(\mathcal{P}) = S_H^*. \quad (25)$$

Using (19) and (22)–(24), it can be shown that

$$\varphi_n^s \geqslant x_n^{s0}, \varphi_n^b \geqslant x_n^{b0}. \quad (26)$$

The following payment allocation is proposed:

$$a_n^s = x_n^s - \varphi_n^s, a_n^b = x_n^b - \varphi_n^b. \quad (27)$$

Using (12), (18), (25), and (27), the sum of the payments is

$$\begin{aligned}\sum_n \left(a_n^s + a_n^b\right) &= \sum_n \left(x_n^s + x_n^b\right) - \sum_n \left(\varphi_n^s + \varphi_n^b\right)\\ &= \overline{S}_H - S_H^*\\ &= \sum_{\ell^c=1}^{L^C} n^c Q^c. \end{aligned} \quad (28)$$

Thus, the sum of the payments made by the players is equal to the investment costs. From (27), it can be seen that the players' net surplus after the expansion is equal to the Shapley value of the game u. Hence, using (26), it is concluded that no player fares worse than in the pre-investment scenario, and so all players have explicit incentives to support the expansion.

The payments given by (27) may be positive or negative. Consider the producer at node n. If $x_n^s > x_n^{s0}$, then $a_n^s > 0$, and the firm makes a positive payment. If $x_n^s < x_n^{s0}$, then $a_n^s < 0$, and the firm is reimbursed for its surplus losses by the firms benefited by the expansion. Thus, the scheme effectively redistributes the social welfare gains such that all firms agree on the expansion. This redistribution can lead to the subsidy of inefficient players, who benefited from congestion, by the more efficient players, who increase their surplus with the transmission upgrade. Although these subsidies may seem "unfair" at first sight, this scheme attains its goal pragmatically by making all players agree on the expansion. Other schemes, which may seem fairer, fail to do so.

Three special cases deserve attention: 1) the expansion can only take place if all the players agree ($q = 1$), 2) all players have the same voting weight ($w_n^s = w_n^b = 1/2N$ for all n), and 3) there is a dominant firm who can decide the expansion without support from any other player ($\exists\{s,n\}, w_n^s > q$ or $\exists\{b,n\}, w_n^b > q$). Consider the case where all players need to agree. If any player disagrees, the expansion is not carried out. Then all the players are equally powerful in the poll regardless

$$u(\mathcal{C}) = \begin{cases} \sum_{n:\{b,n\}\in\mathcal{C}} x_n^{b0} + \sum_{n:\{s,n\}\in\mathcal{C}} x_n^{s0}, & \text{if } \sum_{n:\{b,n\}\in\mathcal{C}} w_n^b + \sum_{n:\{s,n\}\in\mathcal{C}} w_n^s \leq q \\ \sum_{n:\{b,n\}\in\mathcal{C}} x_n^{b0} + \sum_{n:\{s,n\}\in\mathcal{C}} x_n^{s0} + S_H^* - S_H^0, & \text{otherwise} \end{cases} \quad (22)$$

of their voting weights. Hence, by symmetry, the players will distribute the social welfare gains in equal parts. That is, each producer receives

$$\varphi_n^s = x_n^{s0} + \frac{S_H^* - S_H^0}{2N} \quad (29)$$

and each consumer receives

$$\varphi_n^b = x_n^{b0} + \frac{S_H^* - S_H^0}{2N}. \quad (30)$$

The results for case 2) are also given by (29) and (30). This is due to symmetry: no player has an advantage in the poll. In case 3), no player except the dominant firm plays any role in the poll, and the dominant firm receives all the social welfare gains. If, for example, this firm is the producer at node n

$$\varphi_n^s = x_n^{s0} + S_H^* - S_H^0. \quad (31)$$

Voting weights allocation is discussed next. These weights need to be decided by the firms in the market. However, there are some guidelines for their allocation. We make the remark that the effectiveness of the proposed scheme does not depend on the voting weight allocation. Since the weights indicate each firm's influence on the expansion decision, the allocation can depend on the impacts of the expansion on the firms, installed capacity of the producers, load served by the consumers, geographic location of the players, and so on. Using the utility function concept [18], the *impact* on a firm can be defined as the change in the value taken by the utility function of the firm due to the expansion. The larger the losses or gains with the expansion, the larger the absolute value of the impact, and the more influence the firm will try to exert on the expansion decision.

One possible weight allocation is to assign the weights proportionally to the absolute value of the impact on each firm. Such a weight allocation takes into account the system's physical characteristics described in (2)–(5). If all firms have the same utility function that depends only on the firm's surplus, the impact on each firm is an increasing function of $x_n^s - x_n^{s0}$ and $x_n^b - x_n^{b0}$, for a seller and a buyer, respectively. If a linear utility function is used, the weights are assigned in proportion to $|x_n^s - x_n^{s0}|$ and $|x_n^b - x_n^{b0}|$, for a seller and a buyer, respectively. With these weights, and using (13), (18), and (19), it can be shown that the set of firms who have increased surpluses $\boldsymbol{x}$ with the expansion has more voting weight than the set of firms who have decreased surpluses $\boldsymbol{x}$ with the expansion.

Another possibility is to allocate the weights so that the voting *power* [23] of a firm, instead of the weight, is proportional to the absolute value of the impact on the firm.[5] Such a weight allocation is discussed in [28] and [29].

The minimum weight q needed to decide the expansion is decided *a priori* of any transmission expansion plans by the market participants. The value of q used in different organizations normally varies from 0.5 to 0.85, with most of the decisions requiring only a simple majority $(q = 0.5)$ [28], [29]. Larger q's are used for important decisions requiring a good degree of consensus. As q increases, the voting power of the different voters tends to equalize, since the veto power of low weight voters increases [28]. As q decreases, decisions are made more easily. The values of q adopted may depend on the type and magnitude of the expansion being decided.

[5]If the Shapley value is used for measuring the voting power [30], then the social welfare gains would be distributed proportional to the impacts of the expansion of the firms.

The following algorithm describes the procedure for the application of the proposed allocation.

Step 1) The IGO solves (1)–(5) to obtain $\boldsymbol{x}, \boldsymbol{x}^0, n^{c*}$ and Q^{c*}. If the n^{c*} are all zero, stop.

Step 2) The weights w_n^b and w_n^s are decided by the market participants.

Step 3) The transmission expansion cost allocated to each player is computed using (23), (24), and (27).

The only significant computational work required by the proposed scheme is the computation of the Shapley value for each firm. If the number of players is small, this is a trivial task. For systems with hundreds of players, the computation of the Shapley value, in general, is intractable. However, if there are a few groups of players, with all players in each group having the same voting weight, (23) and (24) can be simplified and the Shapley value computation can be made extremely efficient. If most players have different voting weights, any computationally efficient solution concept, such as the bilateral Shapley value [31], [32], can be used. The results in (26) hold for any solution concept, and so the effectiveness of the allocation scheme to ensure the expansion does not depend on the Shapley value properties.

IV. Case Study

This section discusses the application of the proposed scheme to a case study based on the Garver six-bus system [10]. The system contains five nodes and six lines connecting them; a sixth node is considered at which some generation is placed. This node is not initially connected to the other five nodes, but lines to connect it to the system could be built if necessary. The one-line diagram of the system is shown in Fig. 1. The market structure of the system considered consists of three generating units and five loads. Table I shows the line structure of the system considered. The first two columns provide the nodes of origin and destination of the lines, the third and fourth columns show the electric parameters of the lines, and the fifth column shows (in p.u.) the lines' transmission capacities. The investment cost is shown in the sixth column for all the lines. It is assumed that the new lines will be operative for at least 25 years; thus, a 25-year investment return period has been considered. A 10% interest discount rate is assumed as the cost of capital. With these values, the capital recovery factor is approximately 10%. This means that, for the next 25 years, the investment cost in new lines is yearly repaid at a rate of approximately 10% of the total initial investment. This is also known as the annualized cost. Up to a total of three parallel lines can be built for every possible connection between the nodes. A time span of one year is assumed. Although this is a short time span for investments, the proposed cost allocation scheme can be run for several years by changing the bidding patterns and parameters such as the capital recovery factor.

Table II shows the location of generators and demands in the network, along with other relevant information. For the generators, the total amount of energy offered, evenly split according

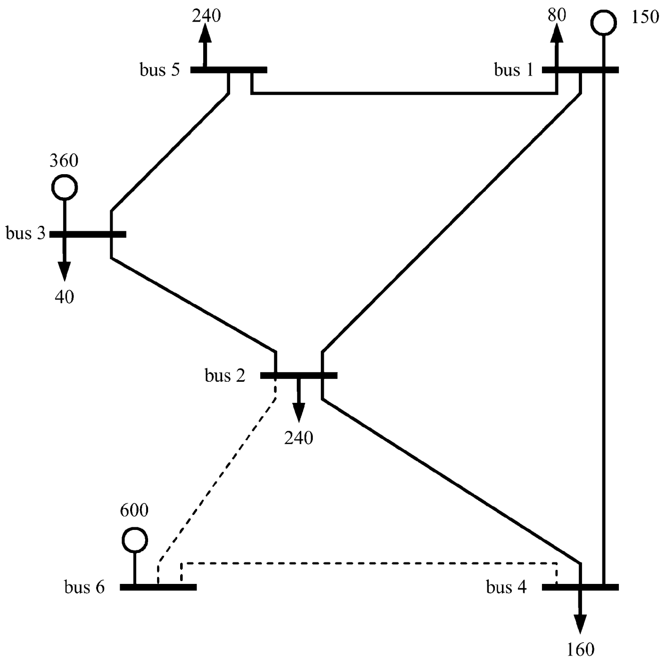

Fig. 1. One-line diagram of the Garver six-bus system. The dashed lines indicate the lines to be built.

TABLE I
GARVER SIX-BUS SYSTEM. LINE STRUCTURE

from	to	R (p.u.)	X (p.u.)	limit (p.u.)	cost (M$)	already built
1	2	0.10	0.40	1.00	40	1
1	3	0.09	0.38	1.00	38	0
1	4	0.15	0.60	0.80	60	1
1	5	0.05	0.20	1.00	20	1
1	6	0.17	0.68	0.70	68	0
2	3	0.05	0.20	1.00	20	1
2	4	0.10	0.40	1.00	40	1
2	5	0.08	0.31	1.00	31	0
2	6	0.08	0.30	1.00	30	0
3	4	0.15	0.59	0.82	59	0
3	5	0.05	0.20	1.00	20	1
3	6	0.12	0.48	1.00	48	0
4	5	0.16	0.63	0.75	63	0
4	6	0.08	0.30	1.00	30	0
5	6	0.15	0.61	0.78	61	0

TABLE II
GARVER SIX-BUS SYSTEM. PRODUCERS AND CONSUMERS LOCATION

Node	Generators			Demands		
	Name	MWh offer	Offer price [$/MWh]	Name	MWh bid	Bid price [$/MWh]
1	G1	150	10	D1	80	30, 28, 26, 24, 20
2	-	-	-	D2	240	30, 27, 24, 22, 19
3	G2	360	15, 19, 23	D3	40	20, 16, 14, 12, 10
4	-	-	-	D4	160	30, 27, 24, 21, 17
5	-	-	-	D5	240	34, 30, 26, 24, 18
6	G3	600	8, 12, 15, 17, 19, 21	-	-	-

to the offer prices, is shown. For the demands, the total amount of energy bid in the market is shown; this total is evenly divided according to the bid prices.

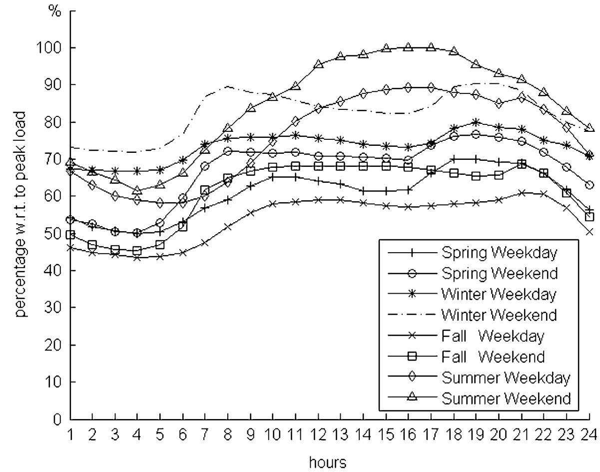

Fig. 2. Load curves for each of the representative days of the year.

TABLE III
GARVER SIX-BUS SYSTEM. PRODUCER AND CONSUMER SURPLUSES BEFORE AND AFTER THE EXPANSION

Node	Producer surplus ($$10^6$)		Consumer surplus ($$10^6$)		Mean price ($/MWh)	
	Before	After	Before	After	Before	After
1	16.58	11.76	2.30	4.11	21.83	17.96
2	-	-	3.88	9.55	23.32	18.68
3	5.59	3.96	0.09	0.17	18.99	17.68
4	-	-	2.11	6.94	24.11	17.58
5	-	-	8.02	10.99	22.33	19.95
6	0.00	7.13	-	-	8.00	13.08
Total	22.17	16.40	21.53	31.77	19.76	17.49

The year under study is decomposed into four seasons to describe possible bidding patterns. For each season, a representative working day and a weekend day is selected, as shown in Fig. 2. To represent the changing demand patterns, the size of the blocks bid by the demand are proportional to the load pattern of every season of the year. That means, for example, that when the demand is 50% of its peak value, all the block sizes bid by the demand reach only 50% of their maximum values shown in Table II. The transmission expansion model has been solved using the software CPLEX [33] within the GAMS optimization suite, with CPU running times below 1 min.

The solution obtained by the model presented in (1)–(5) is next described; two new lines are proposed: one from node 2 to node 6 and the other from node 4 to node 6. The annualized cost of these lines is \$6 000 000 (1/10 of the total cost). The total social welfare and congestion rents before the expansion are: $S_H^0 = \$38\,574\,529$ and $\kappa^0 = \$4\,880\,107$, respectively. After the expansion, the social welfare without the payments for line investments is $\bar{S}_H = \$54\,620\,758$, and the congestion rents are $\kappa = \$8\,228\,411$. Finally, the social welfare after the expansion is $S_H^* = \$54\,620\,758 - \$6\,000\,000 = \$48\,620\,758$. Table III shows the pre- and post-expansion values of the surpluses (16) and (17), including the corresponding congestion rents terms, allocated to all the firms. The congestion rents for each representative day are allocated to each firm in proportion to its daily

TABLE IV
GARVER SIX-BUS SYSTEM (EXPANSION COST ALLOCATION IN $\$10^6$)

Firm	Weight	Surplus before	Net surplus after	Payment
G1	0.17	16.58	18.06	-6.30
G2	0.06	5.59	6.41	-2.45
G3	0.25	0.00	3.16	3.97
D1	0.06	2.30	3.12	0.99
D2	0.20	3.88	5.36	4.19
D3	0.00	0.09	0.09	0.07
D4	0.17	2.11	3.59	3.36
D5	0.10	8.02	8.83	2.16
Sum	1.00	38.57	48.62	6.00

MW sale or purchase.[6] The generators in nodes 1 and 3, G1 and G2, respectively, have their surpluses decreased with the expansion. The mean prices[7] in nodes 1 and 3 decrease with the new lines: G1 and G2 would oppose the expansion. The rest of the firms benefit from the expansion due to the mean price decrease in the nodes with demands and the increase in node 6's price. It is not always the case that all consumers benefit from the expansion as in this example. For instance, a consumer at bus 6 would be hurt by the expansion, since prices increase at bus 6.

We assign the weights to each firm in proportion to $|x_n^s - x_n^{s0}|$ and $|x_n^b - x_n^{b0}|$, for a seller and a buyer, respectively. The weights are indicated in Table IV. The total weight of the firms that increase their surplus with the expansion, G3, D1, D2, D3, D4, and D5, is 0.77. Hence, for $q < 0.77$, an allocation that does not allow negative payments (or reimbursements) may work for the expansion to take place. However, if $q \geqslant 0.77$, the votes of G1 or G2 are needed to obtain the total weight necessary for the expansion to occur. The focus of this paper is on such cases. Schemes without reimbursements to G1 and G2 would not succeed in giving sufficient incentives for the expansion to occur. This is in spite of a net increase in social welfare, from S_H^0 to S_H^*, of more than 26%. Note that a very small quantitative change in q at 0.77 may lead to a very large qualitative result if an allocation without reimbursements is used: the expansion may or may not take place.

Consider the case $q = 0.80 > 0.77$: at least 80% favorable votes are needed to decide the expansion. The results with the proposed scheme are shown in Table IV. G1 and G2 are paid by the rest of the firms to compensate for the losses so that they would agree on the expansion. G1 receives a larger payment than G2 due to G1's larger voting weight and surplus reduction. Note that all firms have their net surpluses increased with the expansion. Thus, each firm has incentives to support the expansion. The net surplus increase can be interpreted as an increase in the mean price for the generation firms and a decrease in the mean price for the demands.

[6]This is done arbitrarily, as the purpose of this section is to illustrate the application of the expansion cost allocation scheme and not to advocate the use of a particular allocation of the congestion rents. The variation of the congestion rents allocation, studied at the end of this section, yields different quantitative results in the expansion cost allocation. However, the main qualities of the proposed expansion cost allocation, such as the effectiveness to bring about the expansion, remain unchanged.

[7]Mean prices are calculated as the weighted average of the representative days' nodal prices, with the weights being proportional to the number of days per year that the representative day accounts for.

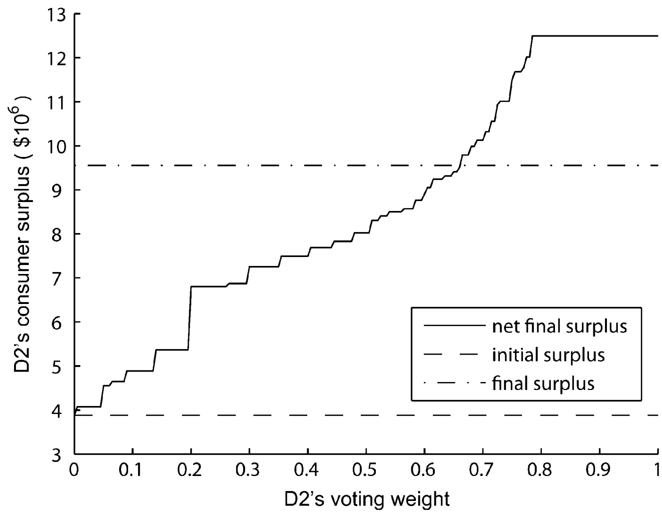

Fig. 3. Net final surplus of the demand at node 2 as a function of its voting weight.

The dependence of the net final surplus on the voting weight of a firm, D2, is studied next. The voting weights of the remaining firms are changed proportionally. The numerical results are displayed in Fig. 3. When the voting weight is zero, D2 receives the initial surplus x_2^{b0}, i.e., the firm does not receive any portion of the surplus gains $S_H^* - S_H^0$. As the voting weight increases, the surplus allocated to the player increases until the voting power reaches q and the player gets all the surplus gains. Note that D2 may receive a net surplus smaller or larger than x_2^b, i.e., make a payment or receive a reimbursement, depending on its voting weight. The large jump in D2's surplus for a voting weight of 0.2 is due to the fact that for $w_2^b \geqslant 0.2$, any winning coalition has to have D2 as a member, since $q = 0.8$. Also demonstrated in Fig. 3 is the piecewise constant nature of the net surplus as a function of the player's voting weight. This is due to the fact that the Shapley value is a weighted sum of the extra value brought by the player to a coalition C, i.e., $u(C) - u(C - \{b, n\})$. These extra values are clearly piecewise constant functions of the voting weight of the player.

The congestion rents allocation impacts the consumer and producer surpluses, and these in turn affect the characteristic function (22) and the voting weights. Thus, the numerical results of the expansion costs allocation depend on the particular congestion rents allocation method used in the market. To study the quantitative impacts of the congestion rents allocation on the expansion costs allocation, we vary the congestion rents allocated to a firm, D2, from 0 to κ. The congestion rents allocated to the remaining firms are changed keeping their initial proportions and such that the total congestion rents are allocated. The results are displayed in Fig. 4. As κ_2^b increases, x_2^{b0} increases and so the net final surplus has the tendency to increase.[8] However, for κ_2^b between 0 and 0.41 p.u., an increase in κ_2^b results in a decrease in the difference between the initial and the final surplus. Thus, the incentives to exert an influence in the expansion decision decreases for an increase of κ_2^b in this range, and the voting weight decreases. As the voting weight decreases,

[8]Note that, at the points of continuity, the slope of the net final surplus is the same as that of the initial surplus.

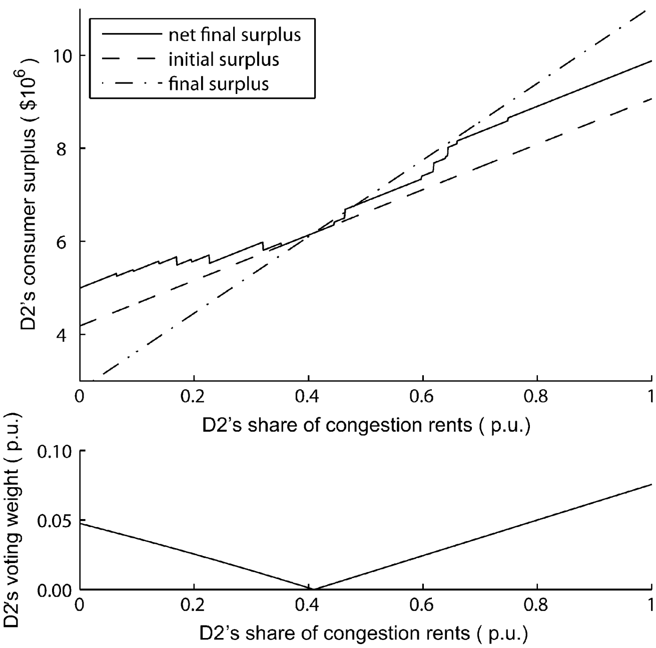

Fig. 4. Impacts of a change in the congestion rents allocated to the demand at node 2. (a) Surplus. (b) Voting weight.

the portion of the surplus gains $S_H^* - S_H^0$ allocated to D2 decreases in a piecewise constant fashion (see Fig. 3). For κ_2^b between 0.41 and 1 p.u., as κ_2^b increases, the difference between the final and initial surplus increases and so the voting weight increases. Thus, the portion of the surplus gains allocated to D2 increases in a piecewise constant manner, and both mechanisms contribute to the increase in the net final surplus. We conclude that an increase in the congestion rents allocated to a firm does not imply an increase in the firm's net final surplus in general, unless the increase results in an increase of the voting weight. Note in Fig. 4 that the net final surplus curve lies over the initial surplus curve, meaning that the firm supports the expansion regardless of the congestion rents allocation method employed. The reason for this is that the firms are guaranteed to receive at least the initial surplus $\boldsymbol{x}^0$, which takes into account the congestion rents allocation.

V. Summary and Further Work

A scheme for the allocation of the transmission network expansion costs among market participants has been presented. Based on cooperative game theory, the allocation takes into account the physical and economic impacts of the new transmission assets. The payments made by each participant are calculated using the Shapley value formula and are based on the increase in social welfare brought about by the new assets, the pre-investment surpluses of the players, and the influence the different market participants may have on the expansion decision. Reimbursements are provided where necessary, so that all firms have incentives to support the expansion.

Further research is needed in this area on how to handle long-term issues due to uncertainty, such as departures from the assumed load patterns, entrance and exit of market players, more investments in the transmission network, and so on. Also, there is a need for developing other temporary allocation schemes that would ensure the expansion. The proposed scheme guarantees each player will increase his net surplus, but this may be more than is needed to ensure the expansion. For example, if there are dominant players, the only group that needs to benefit from and approve the expansion is the group of dominant players.

Appendix
Cooperative Game Theory Background

A cooperative game is determined by a real-valued function u called the *characteristic function* [23]. The function u assigns to each subset $\mathcal{C}$ of $\mathcal{P}$ the maximum value of a game played between $\mathcal{C}$ and $\mathcal{P}$–$\mathcal{C}$, i.e., $u(\mathcal{C})$ is the best total utility that the coalition $\mathcal{C}$ can obtain under the worst scenario induced by the actions of the remaining players.

There are many solution concepts for cooperative games, the "best" being the core. The core is a solution concept that requires that no set of players be able to break away and take a joint action that results in a better outcome for all its members [34]. The game (22) does not have a core unless either there is a dominant player who can decide the expansion without support from any other player ($\exists\{s,n\}, w_n^s > q$ or $\exists\{b,n\}, w_n^b > q$) [23], or all players have to agree for the expansion to take place. If those conditions do not hold, any coalition of $2N-1$ players can decide the election and obtain all its benefits, giving the remaining player no benefit at all. Other well-known solution concepts are the Shapley value, the Kernel, the nucleolus, and the Banzhaf–Coleman index of power [23]. The Shapley value of a game u for player i is given by (23) and (24) and is the unique value vector that satisfies the following,

- The set of players receive all the resources available:

$$\sum_{i \in \mathcal{P}} \varphi_i[u] = u(\mathcal{P}).$$

- If i is a dummy, i.e., $u(\mathcal{C}) - u(\mathcal{C} - \{i\}) = u(\{i\})$ for each coalition $\mathcal{C}$ in $\mathcal{P}$, then

$$\varphi_i[u] = u(\{i\}).$$

- The value assigned to player i does not depend on the position of the player in the set of players.
- If u and v are two games,

$$\varphi_i[u+v] = \varphi_i[u] + \varphi_i[v].$$

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