Sensitivity-Based Security-Constrained OPF Market Clearing Model

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***Abstract*—This paper proposes a novel technique for representing system security constraints that properly include voltage stability limits in the operation of competitive electricity markets. The market-clearing algorithm is modeled as a voltage stability constrained optimal power flow (OPF) problem, while the distance to the closest critical power flow solution is represented by means of a loading parameter and evaluated using a continuation power flow (CPF) technique. Sensitivities obtained at the OPF step are used to estimate power directions for the CPF method, while the CPF analysis provides the loading parameter to be used in the OPF problem based on an $N-1$ contingency criterion. The OPF and the CPF steps are repeated until the maximum loading parameter is found, thus providing optimal solutions considering both proper market conditions and security margins. Two benchmark systems with both supply and demand bidding are used to illustrate and test the proposed technique.**

***Index Terms*—Continuation power flow (CPF), electric energy markets, optimal power flow (OPF), security, sensitivity analysis, voltage stability.**

I. Introduction

In THE last decade, the electricity industry has undergone worldwide restructuring, which has significantly changed the energy market place. This fact, combined with the fact that in most industrialized countries, it is difficult to build new transmission lines while demand is constantly increasing, is leading market participants and system operators to look for adequate and practical ways of evaluating, maintaining, and pricing system security in order to allow secure and "fair" market transactions. In this context, system stability and associated sensitivity should be evaluated in a simple but effective way, so that market participants can obtain clear market signals. However, pricing security through proper system constraints requires a variety of assumptions as well as complex models and simulations. In the various market models that have been proposed and implemented, how to properly include system security is still an open question.

This paper proposes an optimal power flow (OPF)-based market-clearing algorithm that properly accounts for voltage stability limits. This constitutes a different setting than a traditional "centralized" OPF, where significant contributions has been made regarding security modeling that had led to well-established formulations for the security-constrained OPF, although using different approaches to the one proposed here, as discussed in more detail below. The proposed market-clearing algorithm embodies a complex tradeoff: its computational burden should fit the requirements of daily or hourly markets (below a few CPU minutes), it should be sufficiently complex to model relevant stability limits (which constitutes a novel contribution with respect to currently available market-clearing procedures), and sufficiently clear to be meaningful for market operators.

Nonlinear optimization techniques are reliable tools when applied to power system markets [1]–[4]. Moreover, a variety of OPF-based models have been successfully used for addressing voltage stability issues, such as the maximization of the loading parameter in voltage collapse studies, as discussed in [5] and [6]. The main advantage of optimization procedures is their inherent ability to provide, along with the optimal accepted bids, the costs associated with system security, since Lagrangian multipliers associated with the power flow equations can be used to determine locational marginal prices (LMPs) and identify cost components due to different power system constraints [7]–[11]. Cost components associated with the voltage stability constraints that are proposed in this paper can, thus, be used as price signals for security; furthermore, we also show that Lagrangian multipliers at the optimal solution lead to sensitivity formulas of the power bids with respect to the stability margin of the system.

In [12], the authors proposed an OPF-based market representation with voltage stability constraints, so that system security is not simply modeled through the use of voltage and power transfer limits, typically determined offline. A multiobjective optimization method is presented, so that the social benefit and the distance to a maximum loading condition are simultaneously maximized, considering both elastic and inelastic demand bidding. However, the multiobjective approach has the drawback of not directly providing a "pure" market solution, as the objective function explicitly depends on the stability margin and on the weighting factors. Furthermore, the values of the weighting factors, which play a significant role in the optimization process and the solution, are not known *a priori*, and hence, additional studies are necessary to determine adequate values for these weighting factors. Finally, the technique that was proposed in [12] does not take into consideration system contingencies, which is rather important in the proper representation of system security.

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An attempt to include an $N-1$ contingency criterion in the multiobjective VSC-OPF-based auction is described in [13], where two different techniques based on multiple VSC-OPF solutions together with a contingency ranking methodology are proposed and studied. The contingency ranking proved to be an effective method to find a "reasonable" worst case, although it did not always guarantee an optimal solution to the problem.

The present paper proposes an iterative CPF-OPF technique to avoid the multiobjective optimization approach while providing market solutions as a function of a security margin determined using an $N-1$ contingency criterion. The objective is to maximize the social welfare while maintaining an "adequate" distance to a maximum loading condition associated with bus voltage limits, equipment thermal limits, and/or the system voltage stability limits. It is relevant to observe that, since the objective function is a pure social welfare, the Lagrangian multipliers associated with the active power equations are the LMPs. Furthermore, the proposed technique allows controlling the level of security, while in [12] and [13], the security margin was an output of the multiobjective OPF-based market-clearing procedure.

This paper is organized as follows: Section II describes the standard and the VSC-OPF-based market models, as well as the sensitivity analysis used in the proposed CPF-OPF technique, which is described in detail in Section III. In Section IV, the results of applying the proposed methodology to a simple six-bus test system with both elastic and inelastic demand bidding, and a benchmark 24-bus test systems with elastic demand model, are presented and discussed in detail. Finally, Section V summarizes the contributions of this paper and proposes possible future research directions.

II. OPF Market-Clearing Models and Sensitivities

A. Standard OPF Market-Clearing Model

The OPF-based market-clearing procedure is a nonlinear constrained optimization problem and consists of an objective function and a set of equality and inequality constraints, as follows:

$$\begin{aligned} \text{Maximize}_{(x,p)} \quad & f(p) \\ \text{subject to} \quad & g(x,p)=0 \\ & h_{\min} \le h(x,p) \\ & h(x,p) \le h_{\max} \\ & p_{\min} \le p \\ & p \le p_{\max} \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ are the dependent variables, such as bus voltage phasors, and $p \in \mathbb{R}^m$ are the control variables, i.e., power demand and supply bids P_D and P_S, respectively. The functions $f : \mathbb{R}^m \mapsto \mathbb{R}$, $g : \mathbb{R}^n \times \mathbb{R}^m \mapsto \mathbb{R}^n$, and $h : \mathbb{R}^n \mapsto \mathbb{R}^\ell$ are defined as follows.

1) Objective Function: The objective function f is defined as

$$f = \sum_i C_{D_i}(P_{D_i}) - \sum_i C_{S_i}(P_{S_i}). \tag{2}$$

Equation (2) represents consumer surplus plus the producer surplus, i.e., the net social welfare, which is computed as the difference of two terms. The first term is the sum of accepted demand bids P_{D_i} times their corresponding bid prices C_{D_i} in \$/MWh. (In the case of inelastic demand, demand powers P_D are known, which can be represented in (2) by setting $C_D = 0$.) The second term is the sum of accepted production bids P_{S_i} times their corresponding bid prices C_{S_i} in \$/MWh.

2) Equality Constraints: The set g represents the standard power flow equations

$$g(x,p) = g(\theta, V, k_G, P_S, P_D) = 0 \tag{3}$$

where $x = (\theta, V, k_G)$ and $p = (P_S, P_D)$. The variables θ and V are the bus voltage phases and magnitudes, respectively, while k_G is a scalar variable used to account for system losses by means of either a unique or distributed slack bus. Observe that x represents the set of dependent variables to be optimized, i.e., $x \in \mathbb{R}^n$. Generator reactive powers Q_G are not included in this set and are expressed as a function of x and p.

The generator and load powers are defined as follows:

$$\begin{aligned} P_G &= P_{G_0} + P_S \\ P_L &= P_{L_0} + P_D \end{aligned} \tag{4}$$

where P_{G_0} and P_{L_0} stand for generator and load powers that are not part of the market trading (e.g., must-run generators, inelastic loads). Loads are assumed to have constant power factor; thus

$$Q_L = P_L \tan(\phi_L). \tag{5}$$

3) Inequality Constraints: In (1), the set of inequality constraints has been split into h, which represents the physical and security limits of the system, and the bid blocks for control variables p.

The physical and security limits considered in this paper are similar to what are used in [11] and take into account transmission line thermal limits

$$\begin{aligned} I_{ij}(\theta, V) &\le I_{ij_{\max}} \\ I_{ji}(\theta, V) &\le I_{ij_{\max}} \end{aligned} \tag{6}$$

generator reactive power limits

$$Q_{G_{\min}} \le Q_G(x,p) \le Q_{G_{\max}} \tag{7}$$

voltage "security" limits

$$V_{\min} \le V \le V_{\max} \tag{8}$$

and power limits on transmission lines

$$|P_{ij}| \le P_{ij\,\max} \tag{9}$$

which are used to represent security limits of the system [11], based typically on an $N-1$ contingency criterion. These limits are typically "conservative" and can in general lead to low transaction levels, higher costs, and lower security margins as shown in [12]. Thus, $h = [I_{ij}, I_{ji}, Q_G, V, P_{ij}]$.

The limits in p are represented as follows:

$$\begin{aligned} P_{S_{\min}} &\le P_S \le P_{S_{\max}} \\ P_{D_{\min}} &\le P_D \le P_{D_{\max}}. \end{aligned} \tag{10}$$

B. VSC-OPF Market-Clearing Model

In this paper, the following optimization problem is used to represent an OPF market-clearing model with inclusion of voltage stability constraints:

$$
\begin{aligned}
\text{Maximize}_{(x,p,\hat{x})} \quad & f(p) \\
\text{subject to} \quad & g(x,p)=0 \\
& \hat{g}(\hat{x},p,\lambda)=0 \\
& \lambda=\hat{\lambda} \\
& h_{\min} \le h(x,p) \\
& h(x,p) \le h_{\max} \\
& \hat{h}_{\min} \le h(\hat{x},p) \\
& h(\hat{x},p) \le \hat{h}_{\max} \\
& p_{\min} \le p \\
& p \le p_{\max}.
\end{aligned} \tag{11}
$$

In (11), a second set of power flow variables $\hat{x} \in \mathbb{R}^n$ and equations $\hat{g} : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R} \mapsto \mathbb{R}^n$, together with their associated constraints $h(\hat{x}) : \mathbb{R}^n \mapsto \mathbb{R}^\ell$, are introduced to represent the solution associated with a loading parameter λ, where λ represents an increase in generator and load powers, as follows:

$$
\begin{aligned}
\hat{P}_G &= (1+\lambda+\hat{k}_G)P_G \\
\hat{P}_L &= (1+\lambda)P_L.
\end{aligned} \tag{12}
$$

In (12), the scalar variable $\hat{k}_G$ allocates losses, assuming a distributed slack bus model. Observe that (11) is somewhat similar to the optimization problems proposed in [6] and [12] but is not the same, as λ is a fixed value $\hat{\lambda}$ and, hence, is assumed to be an input. Furthermore, $\lambda \le \lambda_{\max}$, where $\lambda_{\max}$ stands for the maximum loading margin. In this paper, it is assumed that (11) has a solution for $\lambda = 0$, i.e., the base loading conditions do not exceed the maximum system loading.

An iterative CPF-based technique is proposed here based on the sensitivity analysis described below to guarantee that $\lambda \le \lambda_{\max}$ in (11). It is important to stress the fact that the value of $\lambda_{\max}$ can be associated with thermal or bus voltage limits or a voltage stability limit due to a singularity of the power flow Jacobian or a controller limit such as a generator reactive power limit.

Notice also that in this case, $h = [I_{ij}, I_{ji}, Q_G, V]$, i.e., no limits on the active power flowing through transmission lines are represented, since the stability limits are already accounted for in (11) through the maximization of λ. The representation of stability limits is more accurate in (11) than in (1), as the stability situation is evaluated at the current loading condition, not offline as in the case of (9). However, physical constraints such as (6)–(8) have still to be taken into account, as they represent physical/security constraints of the system and are, thus, included in (11).

Observe that the limit case $\lambda = 0$ corresponds to no security. If $\lambda = 0$, (11) does not reduce to (1), and its solution does not guarantee any security level. To ensure a security level in the solution, the loading parameter should be $\hat{\lambda} > 0$.

C. Sensitivity Analysis

In addition to the optimal operating point, the OPF-based market-clearing problem provides, with almost no additional computational effort, a set of sensitivity variables, namely, dual variables or Lagrangian multipliers, which are LMPs.

LMPs can be directly deduced from the Lagrangian function of (11), i.e.,

$$
\begin{aligned}
\mathcal{L} = f(p) &- \rho_g^T g(x,p) - \rho_{\hat{g}}^T \hat{g}(x_c,\lambda,p) - \rho_\lambda(\lambda-\hat{\lambda}) \\
&- \mu_{g\max}^T(h_{\max} - h(x,p) - s_{g\max}) \\
&- \mu_{g\min}^T(h(x,p) - h_{\min} - s_{g\min}) \\
&- \mu_{\hat{g}\max}^T(\hat{h}_{\max} - h(\hat{x},p) - s_{\hat{g}\max}) \\
&- \mu_{\hat{g}\min}^T(h(\hat{x},p) - \hat{h}_{\min} - s_{\hat{g}\min}) \\
&- \mu_{p\max}^T(p_{\max} - p - s_{p\max}) \\
&- \mu_{p\min}^T(p - p_{\min} - s_{p\min})
\end{aligned} \tag{13}
$$

where the ρ variables are the Lagrangian multipliers associated with the equality constraints, the μ variables $(\mu_i \ge 0 \; \forall\, i)$ correspond to the dual variables associated with the inequality constraints, and the s variables $(s_i \ge 0 \; \forall\, i)$ form the slack vector that has to be non-negative.

The LMPs are the Lagrangian multipliers ρ_g associated with the power flow equation g; furthermore, LMPs depend on the loading parameter λ and the Lagrangian multipliers of the equations $\hat{g}$, as it can be deduced from the KKT conditions for supply and demand powers, i.e.,

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial P_{S_i}} &= \frac{dC_{S_i}}{dP_{S_i}} - \rho_{g_{P_i}} + \mu_{p\max_i} - \mu_{p\min_i} \\
&\quad - \rho_{\hat{g}_{P_i}}(1+\lambda+\hat{k}_G) = 0 \\
\frac{\partial \mathcal{L}}{\partial P_{D_i}} &= -\frac{dC_{D_i}}{dP_{D_i}} + \rho_{g_{P_i}} + \rho_{g_{Q_i}} \tan(\phi_{L_i}) \\
&\quad + \mu_{p\max_i} - \mu_{p\min_i} + \rho_{\hat{g}_{P_i}}(1+\lambda) \\
&\quad + \rho_{\hat{g}_{Q_i}}(1+\lambda)\tan(\phi_{L_i}) = 0
\end{aligned} \tag{14}
$$

which lead to the following expressions for the LMPs [12]:

$$
\begin{aligned}
\text{LMP}_{S_i} = \rho_{g_{P_i}} &= \frac{dC_{S_i}}{dP_{S_i}} + \mu_{p\max_i} - \mu_{p\min_i} \\
&\quad - \rho_{\hat{g}_{P_i}}(1+\lambda+\hat{k}_G) \\
\text{LMP}_{D_i} = \rho_{g_{P_i}} &= \frac{dC_{D_i}}{dP_{D_i}} + \mu_{p\min_i} - \mu_{p\max_i} \\
&\quad - \rho_{\hat{g}_{P_i}}(1+\lambda) - \rho_{\hat{g}_{Q_i}}(1+\lambda)\tan(\phi_{L_i}) \\
&\quad - \rho_{g_{Q_i}} \tan(\phi_{L_i})
\end{aligned} \tag{15}
$$

where subindexes P and Q of the dual variables ρ_g and $\rho_{\hat{g}}$ indicate the active and reactive power flow equations, respectively, at bus i. Observe that (15) contains terms that depend on the loading parameter λ. These terms can, thus, be viewed as costs associated with the security constraints of the system, and thus, (15) shows how system security affects market prices.

These Lagrangian multipliers and LMPs can be used to determine the sensitivities of the parameters p with respect to the parameter λ, i.e., $dp/d\lambda$, which can then be used to determine the load and generation directions for computing $\lambda_{\max}$ with the help of a CPF. To determine $dp/d\lambda$, one can use the following optimization problem [14], which is directly related to problem (11):

$$
\begin{aligned}
\text{Maximize}_{(x,\hat{x})} \quad & f(p) \\
\text{subject to} \quad & g(x,p)=0 \\
& \hat{g}_x(\hat{x},\lambda)=\hat{g}_p^* \\
& \lambda=\lambda^* \\
& p=p^* \\
& h_{\min} \le h(x,p) \\
& h(x,p) \le h_{\max} \\
& \hat{h}_{\min} \le h(\hat{x},p) \\
& h(\hat{x},p) \le \hat{h}_{\max} \\
& p_{\min} \le p \\
& p \le p_{\max}
\end{aligned} \tag{16}
$$

where λ^* and p^* are the optimal solution of (11), $\hat{g}_{p_i}^*$ are the right-hand side constant powers injected at buses

$$
\begin{aligned}
\hat{g}_{p_i}^* = \left(1+\lambda^*+\hat{k}_G^*\right)\left(P_{G_{0i}}+P_{S_i}^*\right) \\
-(1+\lambda^*)\left(P_{L_{0i}}+P_{D_i}^*\right)
\end{aligned} \tag{17}
$$

and $\hat{g}_x$ are the equations representing bus power injections, as follows:

$$\hat{g}_x(\hat{x},\lambda)=\hat{g}(\hat{x},\lambda,p)+\hat{g}_p^*. \tag{18}$$

For the sake of simplicity and without loss of generality, observe that (17) is written for the case of one supplier and one consumer connected at the same bus i.

It is important to observe that (16) is introduced here only to derive the sensitivity formulas for $dp/d\lambda$, which are obtained below. However, in practice, there is no need to solve (16). As a matter of fact, (11) and (16) have the same optimal solution, and their KKT conditions are identical. From (16) and from the definition of dual variables [15], one has

$$\left.\frac{df}{d\lambda}\right|_* = \frac{df}{d\lambda^*} = -\rho_\lambda \tag{19}$$

$$\left.\frac{df}{d\hat{g}_{pi}}\right|_* = \frac{df}{d\hat{g}_{pi}^*} = -\rho_{\hat{g}_{pi}} \tag{20}$$

where $\rho_{\hat{g}_{pi}}$ is the Lagrangian multipliers associated with the ith equation of (18). This yields the sensitivities of the objective function f with respect to the actual values of the loading parameter and the total power injections at the network buses. From (17) and (20), one has that

$$\left.\frac{df}{dp_i}\right|_* = \left.\frac{df}{d\hat{g}_{pi}}\right|_* \left.\frac{d\hat{g}_{pi}}{dp_i}\right|_* = -\rho_{\hat{g}_{pi}} \nabla_{p_i} \hat{g}_{pi}. \tag{21}$$

From (21) and (19), one obtains

$$\left.\frac{dp_i}{d\lambda}\right|_* = \left.\frac{dp_i}{df}\right|_* \left.\frac{df}{d\lambda}\right|_* = \frac{\rho_\lambda}{\rho_{\hat{g}_{pi}} \nabla_{p_i} \hat{g}_{pi}}. \tag{22}$$

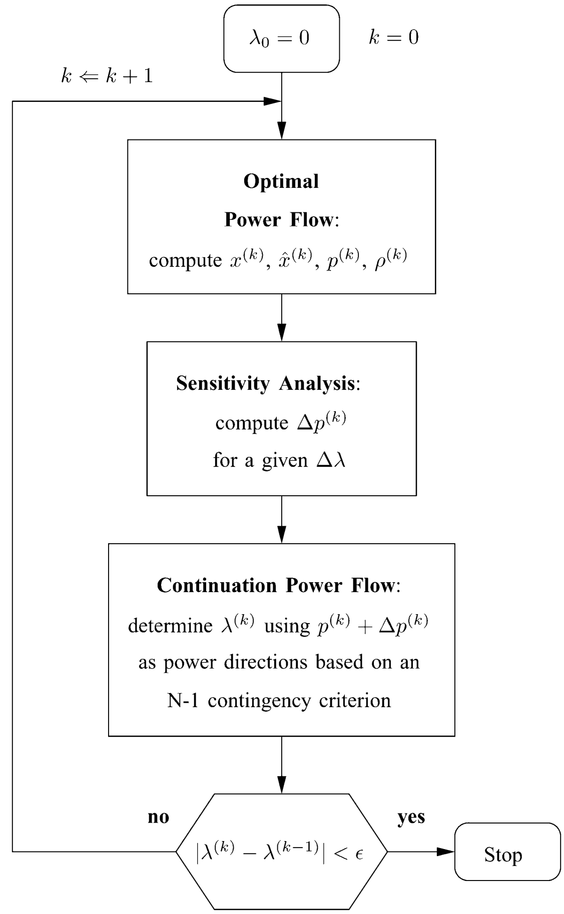

Fig. 1. Flowchart of the proposed CPF-OPF technique.

Observe that from (22), $\rho_{\hat{g}_{p_i}} \neq 0$; this cannot be guaranteed, but it would be "unusual" in practice to have a zero Lagrangian multiplier of a binding equality constraint. These sensitivity expressions based on Lagrangian multipliers constitute one of the main contributions of this paper and can be shown to be equal to the sensitivities proposed in [16], which are based on bifurcation analyses.

For example, particularizing (22) for the power bids $P_{S_i}^*$ and $P_{D_i}^*$, respectively, one can obtain the following sensitivities:

$$
\begin{aligned}
\left.\frac{dP_{S_i}}{d\lambda}\right|_* &= \frac{1}{\left(1+\lambda^*+\hat{k}_G^*\right)} \frac{\rho_\lambda}{\rho_{\hat{g}_{P_i}}} \\
\left.\frac{dP_{D_i}}{d\lambda}\right|_* &= -\frac{1}{(1+\lambda^*)} \frac{\rho_\lambda}{\rho_{\hat{g}_{P_i}}}.
\end{aligned} \tag{23}
$$

III. Mixed CPF-OPF Technique

Fig. 1 depicts the flowchart of the proposed technique for determining the maximum loading parameter value $\lambda_{\max}$ associated with an optimal market solution. This technique works as follows.

Step 0) The loading parameter is initialized to $\lambda 0 = 0$, which is the "base case" or pure market-clearing problem. Observe that (11) must have a feasible solution for this base case condition.

Step 1) The VSC-OPF problem (11) is solved using the current value of the loading parameter.

Step 2) The current OPF solution and the associated dual variables are used to determine the variations of power supplies and demands $\Delta p^{(k)}$ as follows:

$$\Delta p^{(k)} = \frac{\left.\frac{dp}{d\lambda}\right|_k}{\left\| \left.\frac{dp}{d\lambda}\right|_k \right\|} \Delta\lambda \tag{24}$$

where $\Delta\lambda$ is a desired increment in the loading parameter (e.g., $\Delta\lambda = 0.05$). Observe that $|\Delta p^{(k)}|$ must not exceed the values of the slack variables s_p in order to avoid violations on bid block limits (10). Notice also that it is assumed that $\Delta p^{(k)} = 0$ if $\rho_\lambda^{(k)} = 0$, as no inequality constraint is active for the current critical power flow solution, and thus, λ can be increased without changing the actual bids. Observe also that the sensitivities $dp/d\lambda$ are normalized to avoid small or large steps if $\|dp/d\lambda\|$ is low or high, respectively.

Step 3) Given the power directions $p^{(k)} + \Delta p^{(k)}$, a standard CPF technique is used to find a new value of the loading parameter $\lambda \leq \lambda_{\max}$ considering an $N-1$ contingency criterion [17]. Observe that using the CPF analysis, it is possible to find limit-induced bifurcations and saddle-node bifurcations while taking into account all security constraints that are used in problem (11) (transmission line thermal limits, generator reactive power limits, and voltage limits). Furthermore, contingencies are considered in this step, i.e., a CPF is run for each "critical" transmission line outage (typically a small number of line outages have to considered [13]). If no better solution is found, i.e., the CPF analysis does not return a loading parameter $\lambda^{(k)}$ such that $\lambda^{(k)} \geq \lambda^{(k-1)}$, $\Delta p^{(k)}$ is reduced by a given factor (e.g., by half) and the CPF is carried out again. Observe that the solution of (11) ensures that there exists at least a solution for ($p^{(k)}$, $\lambda^{(k)}$); thus, the CPF analysis step always ends successfully.

Step 4) If $|\lambda^{(k)} - \lambda^{(k-1)}| < \epsilon$, then the algorithm stops; otherwise, $k \Leftarrow k+1$ and the procedure returns to *Step 1)*. Notice that (11) always presents a feasible solution for the increased loading parameter value found at *Step 3)*, as the CPF analysis is performed using the same constraints as (11).

Step 3) ensures that the increment of λ, obtained by means of the CPF analysis, will always lead to a feasible OPF problem. It would be certainly possible to use a fixed step for the loading parameter, say, $\Delta\lambda$, and stop the process when the OPF problem fails to converge. However, this approach presents the inconvenience that $\Delta\lambda$ cannot be determined *a priori*. If $\Delta\lambda$ is too small, the number of OPF steps will be high, thus resulting in an unnecessary computational effort. On the other hand, if $\Delta\lambda$ is too high, the information about the maximum loading condition and critical points will not be accurate.

It is important to highlight the fact that the proposed CPF-OPF method yields not only one solution for a given value of λ but a series of OPF solutions for $0 \leq \lambda \leq \lambda_{\max}$. If no contingencies are considered, this technique yields similar results as those obtained using the methodology proposed in [12], as discussed in Section IV.

Summarizing, the proposed technique is more versatile than the multiobjective techniques proposed in [12] and [13], for the following reasons.

1) Contingencies can be taken into account directly so that system security can be properly handled, which was not possible with the techniques described in [12] and [13].
2) Thus, the iterative process proposed in this paper allows controlling the value of the loading parameter λ, while in [12], λ was an output of the multiobjective OPF-based market-clearing procedure.
3) In [12], the Lagrangian multipliers are not the LMPs, as the multiobjective function is not a pure social welfare. Thus, additional computational effort was required to compute the LMPs, which is not an issue in the proposed methodology.

IV. Case Studies

The proposed technique is applied to the six-bus test system used in [12] for comparison purposes and to a benchmark 24-bus test system [18]. All the results discussed here were obtained using the MATLAB-based program PSAT [19], which makes use of a primal-dual IP method based on a Mehrotra's predictor-corrector technique and a CPF routine (OPF and CPF results were double-checked with GAMS-CONOPT [20], [21], and UWPFLOW [22], respectively.)

On a Pentium 4, 2.66 GHz, with 1 GB of RAM, the six-bus test cases took 9 s of CPU time for the elastic load case and 7 s for the inelastic load case, whereas the 24-bus test case needed about 49 s. The proposed technique was also tested on a 129-bus model of the 400-kV Italian grid, with 32 generators and 82 consumers [13]; for this network, the desired results were obtained in about 6 min. These results show that the computational burden of the proposed technique can readily fit the requirements of realistic daily or hourly markets, since interior point solution methods used to solve the proposed optimization problems, which make use of matrix sparsity, have been shown to be efficient for large problems, with CPU times increasing roughly linearly with network size. Furthermore, the CPU times mentioned above correspond to a MATLAB implementation of the proposed method; performances would certainly improve if a compiled language, such as Fortran or C, were used.

A. Six-Bus Test Case

Fig. 2 depicts the six-bus test case used in [12], representing three generation companies (GENCOs) and three energy service companies (ESCOs) that provide linear supply and demand bids, respectively. The complete set of data for this system is provided in Appendix A.

Table I depicts the results for the VSC-OPF-based market problem (11) with $\lambda = 0$, i.e., for the base case solution; these results are in accordance with results presented in [12], as expected. The initial solution with $\lambda = 0$ is then used as the first point of the CPF-OPF algorithm. Fig. 3 depicts the total transaction level T $(T = \sum_i P_{L_i})$ for the six-bus system as a function

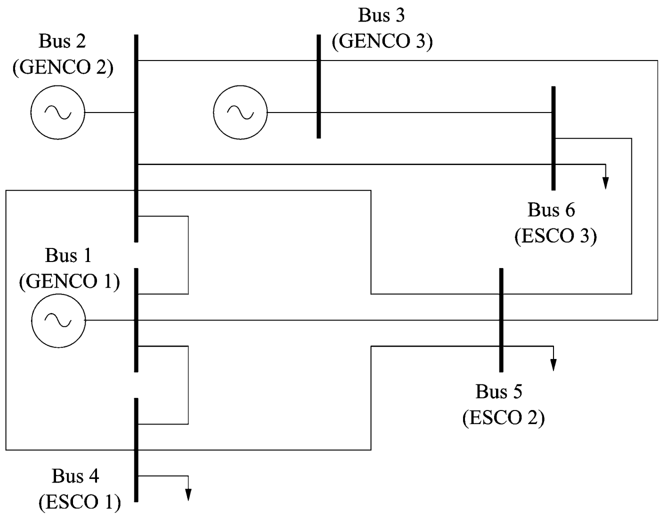

Fig. 2. Six-bus test system.

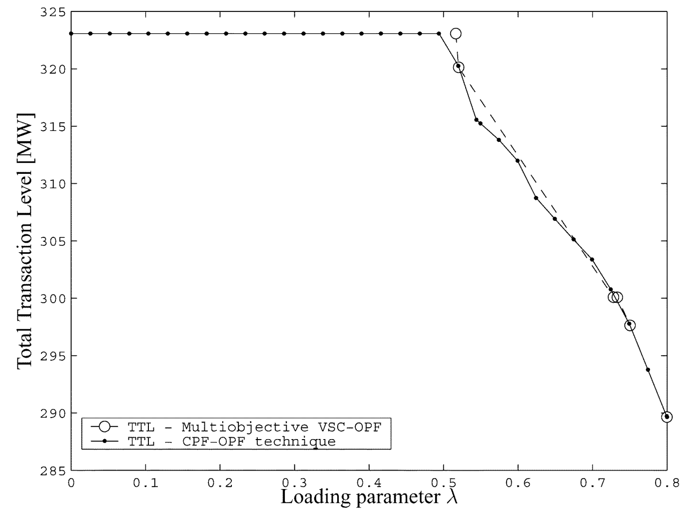

Fig. 3. Total transaction level for the six-bus test system with elastic demand; no contingencies.

TABLE I
SIX-BUS TEST SYSTEM: OPF SOLUTION FOR $\lambda = 0$

Participant	V [p.u.]	LMP [\$/MWh]	$P_{\rm BID}$ [MW]	P_0 [MW]	Pay [\$/h]
GENCO 1	1.100	8.94	0.0	90	-805
GENCO 2	1.100	8.91	25.0	140	-1470
GENCO 3	1.100	9.07	20.0	60	-726
ESCO 1	1.021	9.49	25.0	90	1091
ESCO 2	1.013	9.57	10.0	100	1053
ESCO 3	1.039	9.35	8.0	90	916
TOTALS	T = 323 MW		Losses = 12.0 MW		

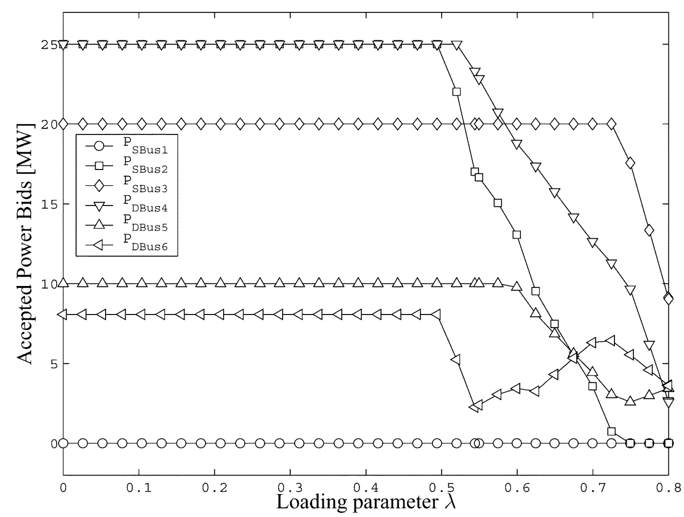

Fig. 4. Accepted power bids for the six-bus test system with elastic demand; no contingencies.

of the loading parameter λ obtained with the proposed technique as well as those obtained with the multiobjective VSC-OPF method proposed [12]. Notice that in this example, no $N-1$ contingency criterion has been considered in the CPF analysis to allow a comparison with the multiobjective VSC-OPF method. The two methods provide consistent results, as the constraints used in the two methods are similar. However, the proposed method allows to control the values of the loading parameter λ, while the algorithm that was proposed in [12] does not. Observe, for example, that in Fig. 3, the curve obtained with the multiobjective VSC-OPF presents a large gap between 0.5 and 0.7, which is a problem, since results in this region are likely to present the right compromise of security and acceptable transaction levels.

Fig. 4 illustrates the accepted power bids for the six-bus example with elastic demand with respect to system demand changes represented by the parameter λ, illustrating the effect of security limits (system congestion) on market conditions. Observe that the overall total transaction level decreases, which is to be expected, since as the load increases, the system gets closer to its security margins, i.e., gets more congested, and hence, transactions levels decrease to meet the security constraints due to the elasticity of the loads; the power bids at each bus, on the other hand, may increase or decrease as the load increases, depending on the active security constraints. It is interesting to observe in Fig. 5 that the LMPs decrease as the system demand, and hence, congestion levels increase; this is due to the load elasticity, which allows market participants to properly respond to increased system congestion, which is not the case for inelastic demand, as discussed below. Observe also that at the loading parameter value $\lambda \approx 0.73$, LMPs decrease below the minimum power supply price bid of 7 \$/MWh (see Table II in Appendix A); this is due to the OPF constraints forcing the system to work at the power levels needed to maintain the required loading margin, regardless of the social benefit (this behavior was observed in [12] as well). Thus, market solutions for $\lambda > 0.73$ are likely to be discarded by the market participants, as the LMPs are smaller than the cheapest supply bid. The algorithm was stopped at $\lambda = 0.8$, as further increases in the loading parameter values are not really relevant for market operations.

Fig. 6 depicts the normalized sensitivities $dp/d\lambda$ for the elastic demand case; these sensitivities are zero for $\lambda = 0$ and remain at zero up to $\lambda \approx 0.50$, which is basically the loading margin of the initial solution depicted in Table I. This means that as the system demand increases from $\lambda = 0$ to $\lambda = 0.50$ $(0 \leq \lambda \leq 0.50)$, no constraints in $h(\hat{x})$ are active; for

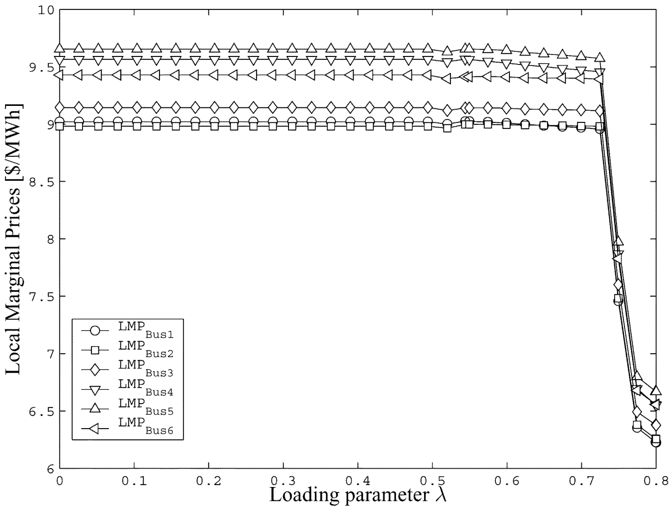

Fig. 5. LMPs for the six-bus test system with elastic demand; no contingencies.

TABLE II
GENCO AND ESCO BIDS AND BUS DATA FOR THE SIX-BUS TEST SYSTEM

Part.	C [\$/MWh]	$P_{\max}^{\text{bid}}$ [MW]	P_{L_0} [MW]	Q_{L_0} [MVAr]	P_{G_0} [MW]
GENCO 1	9.7	20	0	0	90
GENCO 2	8.8	25	0	0	140
GENCO 3	7.0	20	0	0	60
ESCO 1	12.0	25	90	60	0
ESCO 2	10.5	10	100	70	0
ESCO 3	9.5	20	90	60	0

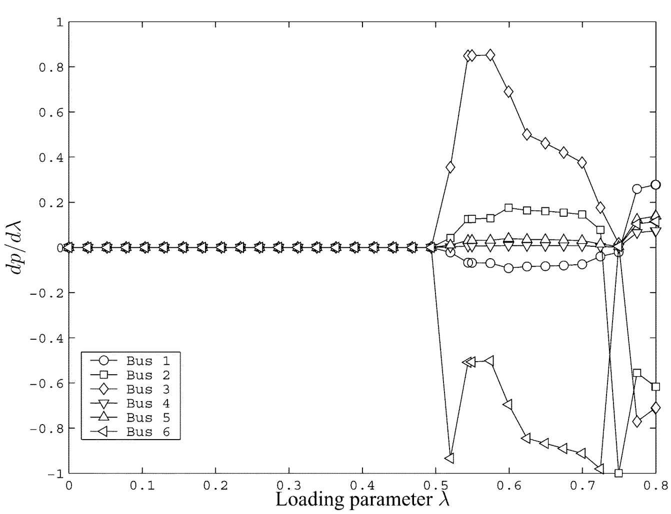

Fig. 6. Sensitivities $dp/d\lambda$ for the six-bus test system with elastic demand; no contingencies.

$\lambda > 0.50$, the sensitivities are not zero ($dp/d\lambda \neq 0$), since at least one constraint $h(\hat{x})$ is active. Observe that the signs of the sensitivities are generally consistent with the variations of supply and demand powers. However, in some cases, the OPF problem may force the power levels to vary in ways different to their corresponding sensitivity values in order to maximize the overall social benefit while meeting all security constraints.

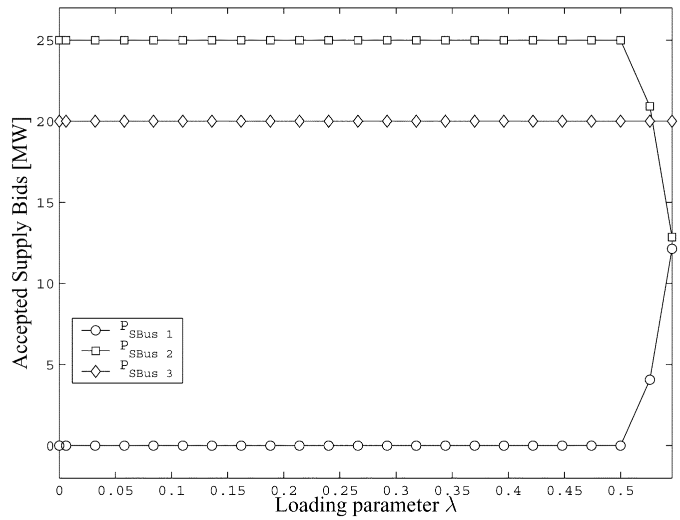

Fig. 7. Accepted supply bids for the six-bus test system with inelastic demand; no contingencies.

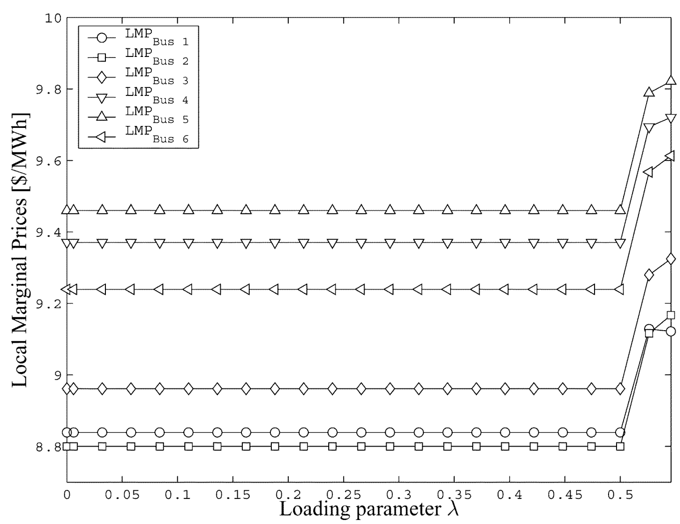

Fig. 8. LMPs for the six-bus test system with inelastic demand; no contingencies.

Figs. 7 and 8 depict the accepted bids and the LMPs, respectively, for the six-bus example with inelastic demand. Notice that the demand bids were fixed at the values illustrated in Table I, which lead to a constant total transaction level $T = 323$ MW. Since the demand is constant, the increase in the loading margin λ leads to a redistribution of generated powers and, thus, to more expensive transactions, as one would expect. Furthermore, the variation of power levels are consistent with the signs of the sensitivities $dp/d\lambda$, as depicted in Fig. 9.

B. Twenty-Four-Bus Test Case

Fig. 10 depicts the 24-bus test system used here to illustrate a more realistic application of the proposed CPF-OPF technique; this system corresponds to the IEEE One Area RTS-96 benchmark network and is fully described in [18]. This system was chosen due to the relatively large number of possible supply and demand bidders, since the system has 32 generators and 17 consumers. In this example, elastic linear demand-side bids have been assumed, whereas the line-wise generator cost curves

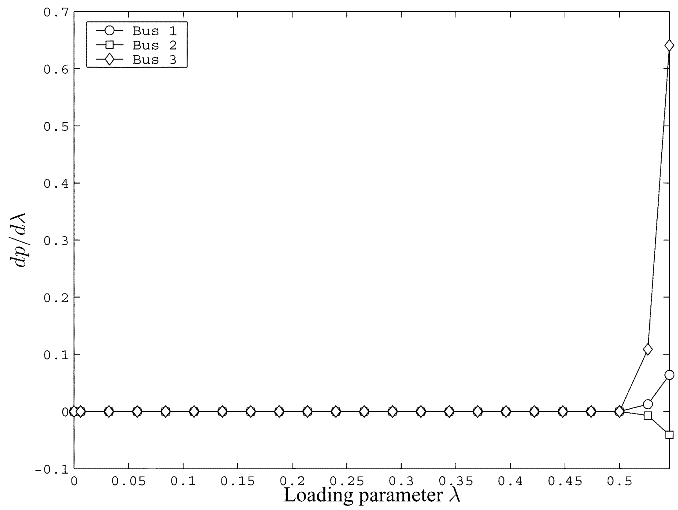

Fig. 9. Sensitivities $dp/d\lambda$ for the six-bus test system with inelastic demand; no contingencies.

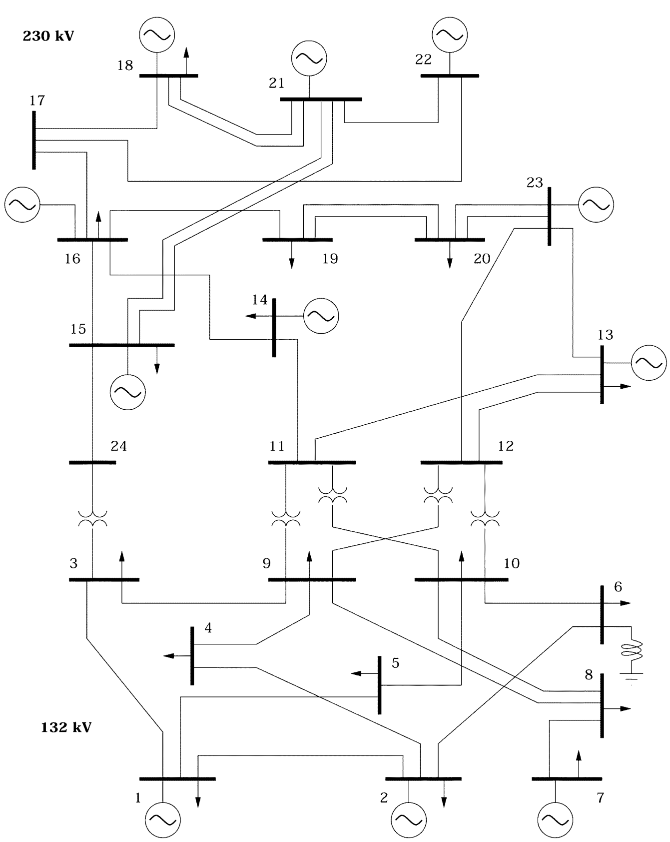

Fig. 10. Twenty-four-bus test system (IEEE one area RTS-96 [18]).

given in [18] have been approximated with a quadratic function, as follows:

$$C_{S_i}(P_{S_i}) = c_{0_i} + c_{1_i} P_{S_i} + c_{2_i} P2_{S_i}. \tag{25}$$

The coefficients c_{0_i}, c_{1_i}, and c_{2_i}, as well as demand bids, are given in Appendix B.

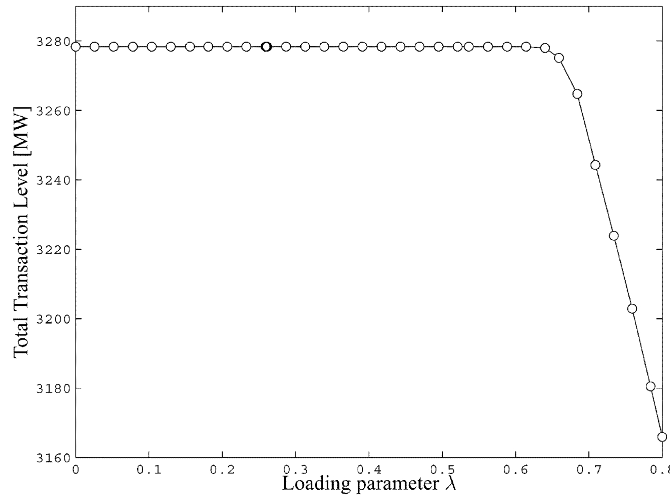

Fig. 11. Total transaction level for the 24-bus test system with elastic demand.

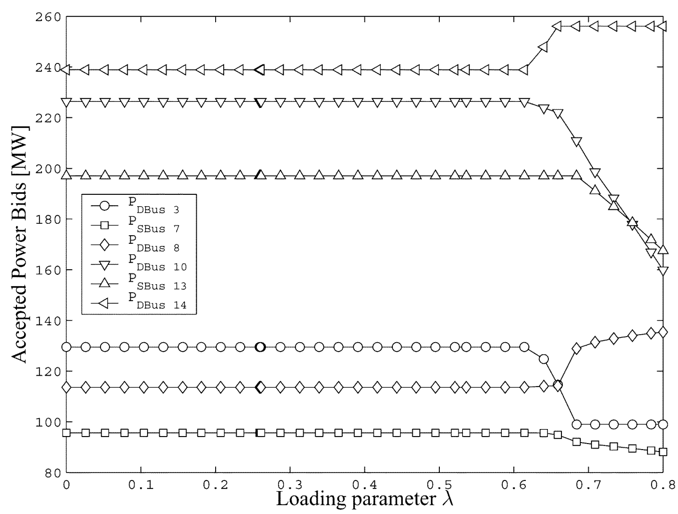

Fig. 12. Significant accepted power bids for the 24-bus test system with elastic demand.

Fig. 11 depicts the total transaction level with respect to system load changes obtained with the proposed CPF-OPF technique using an $N-1$ contingency criterion during the CPF analysis. The proposed CPF-OPF iteration process is stopped at the loading level $\lambda = 0.8$, as the market conditions at this point have changed significantly with respect to the base load. The worst-case condition, i.e., the lowest total transaction level and the lowest stability margin, is obtained for a contingency on the transformer that connects buses 3 and 24. This is to be expected, since most of the generation is located in the 230-kV region, while the loads are mostly concentrated in the 132-kV region; hence, reducing the transmission system capacity between the two regions will reduce the transaction level and increase system congestion. Observe that, in this case, a direct comparison of the solutions for the proposed method with similar solutions for the multiobjective VSC-OPF cannot be performed here since the latter does not allow for $N-1$ contingency analyses.

Figs. 12 and 13 depict some significant accepted power bids and the associated LMPs for the 24-bus example. Observe that

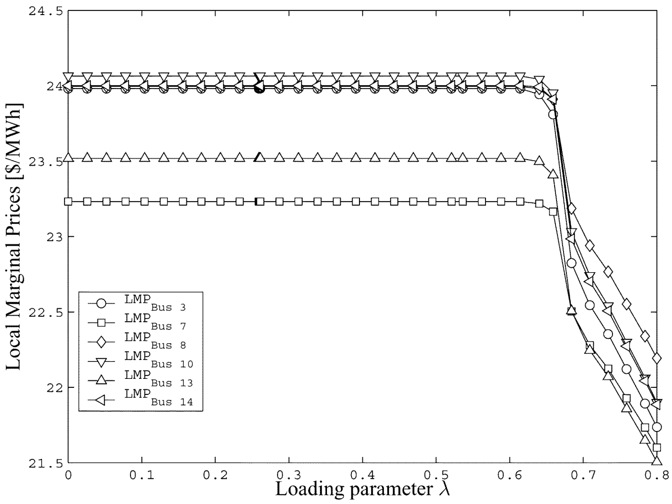

Fig. 13. LMPs at significant buses for the 24-bus test system with elastic demand.

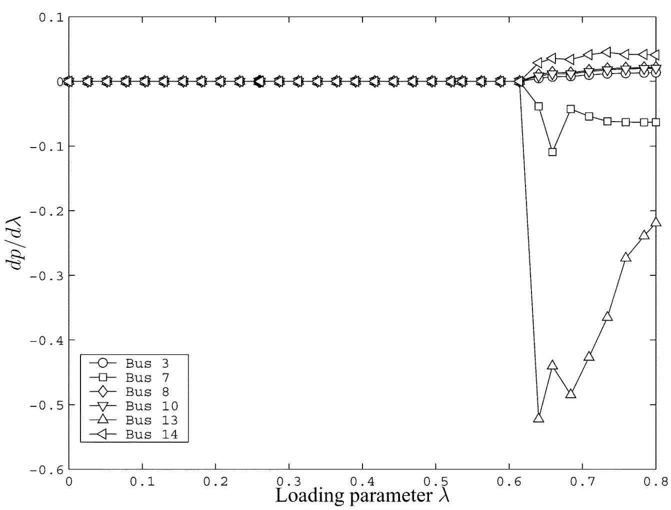

Fig. 14. Sensitivities $dp/d\lambda$ for the 24-bus test system with elastic demand.

also in this case, power bids may increase or decrease, while the LMPs decrease as the loading parameter increases, confirming that an elastic demand properly reacts to congestion problems. In this example, the transaction level is constant up to $\lambda \approx 0.62$, which is also the loading margin of the base case solution ($\lambda = 0$), as confirmed by the values of the sensitivities $dp/d\lambda$ depicted in Fig. 14. For $\lambda > 0.62$, $dp/d\lambda$ is not zero, as some system security constraints become active, and thus, the total transaction level begins to decrease. Observe that also in this example, the signs of the sensitivities $dp/d\lambda$ are in good accordance with the variations of supply and demand powers.

V. Conclusions

In this paper, an iterative CPF-OPF technique based on a sensitivity analysis for managing and pricing system security are proposed and tested on a simple test system as well as on a more realistic test system. The results demonstrate the advantages of a proposed technique that provides a set of optimal market solutions as a function of system security or congestion levels; thus, the proposed CPF-OPF method allows system operators and market participants to investigate the effects of system security on the market-clearing mechanism.

The proposed technique also presents promising features for future research. First, since the determination of the system security limits is basically decoupled from the OPF-based market-clearing problem, the proposed CPF-OPF technique could be, in principle, extended to take into account dynamic system models and their associated security limits. Second, and also based on the decoupling of the market problem and the CPF analysis, it should be possible to simplify the OPF problem (e.g., using linear approximations of certain constraints such as a dc power flow model) while still performing full nonlinear security analyses to properly represent system security limits in market operations.

TABLE III
Line Data for the Six-Bus Test System

Line i-j	R_{ij} [p.u.]	X_{ij} [p.u.]	$B_i/2$ [p.u.]	$I_{\max}$ [A]
1-2	0.1	0.2	0.02	37
1-4	0.05	0.2	0.02	133
1-5	0.08	0.3	0.03	122
2-3	0.05	0.25	0.03	46
2-4	0.05	0.1	0.01	200
2-5	0.1	0.3	0.02	103
2-6	0.07	0.2	0.025	132
3-5	0.12	0.26	0.025	95
3-6	0.02	0.1	0.01	200
4-5	0.2	0.4	0.04	26
5-6	0.1	0.3	0.03	29

TABLE IV
Supply Costs Coefficients for the 24-Bus Test System

Bus	Generator	c_0 [\$/h]	c_1 [\$/p.u.h]	c_2 [\$/p.u.^{2}h]
1, 2	1, 2, 5, 6	0.017	24.842	36.505
1, 2	3, 4, 7, 8	0.035	10.239	3.840
7	9-11	0.00	17.974	2.748
13	12-14	0.0058	18.470	1.011
15	15-19	0.011	21.227	37.937
15, 16, 23	20, 21, 30, 31	0.011	9.537	0.559
18, 21	22, 23	0.058	5.230	0.007
22	24-29	0	1	0
23	32	0.016	9.587	0.315

Appendix A
Six-Bus Test System Data

This appendix depicts the complete data set for the six-bus test system of Fig. 2. Table II shows supply and demand bids and the bus data for the market participants, whereas Table III shows the line data. Thermal limits were assumed to be twice the values of the line currents at base load conditions for a 400-kV voltage rating; it is also assumed that $I_{ij_{\max}} = I_{ji_{\max}} = I_{\max}$. Finally, maximum and minimum voltage limits are considered to be 1.1 p.u. and 0.9 p.u. and reactive power limits for all three GENCOs are ± 150 MVAr.

Appendix B
24-Bus Test System Supply and Demand Bids

This appendix depicts supply and demand bids for the 24-bus test system of Fig. 10. Table IV shows the coefficient of the

TABLE V
DEMAND BIDS DATA FOR THE 24-BUS TEST SYSTEM

Bus	P_{D_0} [p.u.]	$P_{D_{\max}}$ [p.u.]	$P_{D_{\min}}$ [p.u.]
1	1.188	1.4256	0.9504
2	1.067	1.2804	0.8536
3	1.98	2.376	1.584
4	0.814	0.9768	0.6512
5	0.781	0.9372	0.6248
6	1.496	1.7952	1.1968
7	1.375	1.65	1.1
8	1.881	2.2572	1.5048
9	1.925	2.31	1.54
10	2.145	2.574	1.716
13	2.915	3.498	2.332
14	2.134	2.5608	1.7072
15	3.847	4.6164	3.0776
16	1.1	1.32	0.88
18	3.663	4.3956	2.9304
19	1.991	2.3892	1.5928
20	1.408	1.6896	1.1264

quadratic approximation of supply bids for all generators, whereas demand data are illustrated in Table V, where P_{D_0} stands for the initial solution of (11) with $\lambda = 0$. Demand bid limits $P_{D_{\max}}$ and $P_{D_{\min}}$ have been chosen to be 120% and 80% of P_{D_0}, respectively. A demand costs C_D equal to 24 $/p.u.h was chosen for all consumers, while voltage limits are considered to be 1.1 p.u. and 0.9 p.u. Observe that, although demand costs do affect OPF solutions, the aim of using the 24-bus system example was only to properly test the proposed technique when applied to a realistic network and should not be considered representative of a real market scenario. Finally, the short-time emergency rating data reported in [18] were used to compute maximum current limits in transmission lines, assuming nominal voltages.

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