A Stochastic Programming Approach to Electric Energy Procurement for Large Consumers

Miguel Carrión, *Student Member, IEEE*, Andy B. Philpott, *Member, IEEE*, Antonio J. Conejo, *Fellow, IEEE*, and José M. Arroyo, *Senior Member, IEEE*

***Abstract*—This paper provides a technique based on stochastic programming to optimally solve the electricity procurement problem faced by a large consumer. Supply sources include bilateral contracts, a limited amount of self-production and the pool. Risk aversion is explicitly modeled using the conditional value-at-risk methodology. Results from a realistic case study are provided and analyzed.**

***Index Terms*—Conditional value-at-risk (CVaR), electricity procurement, large consumer, stochastic programming.**

NOTATION

The notation used throughout the paper is stated below for quick reference.

Real Variables:

$C_t(\omega)$	Production cost in period t and scenario ω [Euro].
$P_t^{\mathrm{A}}(\omega)$	Self-produced energy consumed during period t and scenario ω [MWh].
$P_{b,t}^{\mathrm{C}}(\omega)$	Energy purchased from contract b in period t and scenario ω [MWh].
$P_t^{\mathrm{G}}(\omega)$	Energy self-produced by the consumer in period t and scenario ω [MWh].
$P_{i,t}^{\mathrm{G}}(\omega)$	Energy self-produced in block i of the piecewise linear production cost function in period t and scenario ω [MWh].
$P_t^{\mathrm{P}}(\omega)$	Energy purchased from the pool in period t and scenario ω [MWh].
$P_t^{\mathrm{S}}(\omega)$	Self-produced energy sold in the pool in period t and scenario ω [MWh].
ξ	Auxiliary variable used to calculate CVaR [Euro].
$\eta(\cdot)$	Auxiliary variable used to calculate the Kantorovich distance between two probability distributions.
$\mu(\omega)$	Auxiliary variable related to scenario ω and used to calculate CVaR [Euro].

Binary Variables:

$h_b(\omega)$	Binary variable which is equal to 1 if contract b is selected in scenario ω, and 0 otherwise.

Random Variables:

ϵ_t	White noise process which represents the error term in the ARIMA model of the pool price in period t [ln(Euro/MWh)].
λ_t^{P}	Price of energy in the pool in period t [Euro/MWh].

Constants:

$A(\omega,k)$	0/1 constant which is equal to 1 if scenarios ω and $\omega+1$ are equal up to stage k, being 0 otherwise.
F_i	Slope of block i of the piecewise linear production cost function [Euro/MWh].
$H(b)$	Stage in which the decision on the selection of contract b is made.
$P_b^{\mathrm{C,max}}$	Upper limit of energy that can be purchased from contract b in one period [MWh].
P_t^{D}	Demand in period t [MWh].
P_i^{G}	Upper limit of the energy produced in block i of the piecewise linear production cost function [MWh].
$P^{\mathrm{G,max}}$	Upper limit of energy that can be produced by the self-production unit in one period [MWh].
$P_{b,e}^{\max}$	Upper limit of energy that can be purchased from contract b in subset of periods $T_{b,e}$ [MWh].
$P_{b,e}^{\min}$	Lower limit of energy that can be purchased from contract b in subset of periods $T_{b,e}$ [MWh].
$S(t)$	Stage in which the decisions of purchases from contracts are taken in period t.
α	Confidence level used in the calculation of CVaR.
β	Weighting factor [1/Euro].
ϵ_{ini}	First error term considered in the ARIMA model [ln(Euro/MWh)].

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M. Carrión, A. J. Conejo and J. M. Arroyo are with Universidad de Castilla-La Mancha, Ciudad Real, Spain (e-mail: Miguel.Carrion@uclm.es; Antonio.Conejo@uclm.es; JoseManuel.Arroyo@uclm.es).

A. B. Philpott is with Auckland University, Auckland, New Zealand (e-mail: a.philpott@auckland.ac.nz).

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$\epsilon_t(\omega)$ Error term in period t and scenario ω [ln(Euro/MWh)].

ϕ_u Parameter related to delay u used in the ARIMA model.

$\lambda^{\mathrm{C}}_{b,t}(\omega)$ Price of energy for contract b in period t and scenario ω [Euro/MWh].

$\lambda^{\mathrm{C,D}}_{b,t}$ Deterministic price of energy for contract b in period t [Euro/MWh].

$\lambda^{\mathrm{P}}_{\mathrm{ini}}$ First pool price considered in the ARIMA model [Euro/MWh].

$\lambda^{\mathrm{P}}_t(\omega)$ Price of energy in the pool in period t and scenario ω [Euro/MWh].

$\pi(\omega)$ Probability of scenario ω.

θ_u Parameter related to delay u used in the ARIMA model.

Numbers:

n_{B} Number of bilateral contracts.

n_{E} Number of sets of periods used in the modeling of the contracts.

n_{I} Number of blocks of the piecewise linear approximation of the production cost function.

n_{T} Number of periods.

n_{W} Number of periods comprising a week.

$n_{\Omega'}$ Number of scenarios after the scenario reduction process.

$n_{\Omega',n}$ Number of coincident scenarios in node n.

Sets:

B Set of bilateral contracts.

B^{M} Set of monthly bilateral contracts.

B_n Set of contracts selected in node n.

N Set of nodes.

T Set of periods.

T_b Set of periods in which contract b is defined.

$T_{b,e}$ Set of periods belonging to subset e in contract b.

Ω Set of scenarios initially generated.

Ω' Set of preserved scenarios after the scenario reduction process.

Others:

$c(\omega,\omega')$ Auxiliary function used to calculate the distance between scenarios ω and ω' of a random variable.

$D_K(\cdot)$ Kantorovich distance of two probability distributions.

Q Probability distribution of a random variable.

I. Introduction

THIS paper considers an electricity market that includes a pool and in which bilateral contracts among producers and consumers can be freely arranged. The pool consists of a day-ahead auction as well as auctions with shorter time horizons, such as control, reserve and balancing auctions. Bilateral contracts can be agreed upon on a daily, weekly or monthly basis, but contracts embracing longer time horizons generally provide more effective hedging against pool price volatility than those spanning shorter time periods. An example of such a market arrangement is the electricity market of mainland Spain [1].

This paper considers the perspective of a large consumer that owns a limited self-production facility (e.g., a cogeneration unit), and derives a methodology that allows the consumer to optimally decide its involvement in bilateral contracts, self-production and its participation in the pool. Uncertainty is treated in detail through a stochastic programming framework [2].

The objective pursued is minimizing the expected value of the procurement cost while limiting its volatility (risk) by incorporating risk aversion through the conditional value-at-risk (CVaR) methodology [3], [4].

A. Motivation, Aim and Solution

A large consumer has the opportunity to procure its electric energy needs through bilateral contracts, self-production and the pool. Signing bilateral contracts reduces the risk associated with the volatility of pool prices usually at the cost of high average prices for the signed contracts. Self-producing also reduces the risk related to pool price. On the other hand, relying mostly on the pool might result in an unacceptable volatility of procurement cost. Hence, the consumer faces a tradeoff between its level of involvement in bilateral contracts, its self-production and its participation in the pool. To resolve such a tradeoff, this paper describes and develops a model that is a multistage stochastic integer programming problem with recourse [2]. This problem is made tractable using scenario-reduction techniques and solved using a commercially available branch-and-cut software. The solution to this problem determines which contracts should be signed among the set of available ones, and the amounts of energy to be purchased from each of the selected contracts. The solution to the problem also provides the optimal strategy (policy) of pool purchases for each realization of pool prices.

B. Literature Review and Contributions

Although the technical literature is rich on papers addressing the point of view of the producer, i.e., addressing the self-scheduling and bidding problems of generating companies, e.g., [5], [6] and [7], few references are found on how large consumers should procure their electricity consumption. The pioneering work of Daryanian *et al.* [8], and the recent work of Kirschen [9] deserve special attention. Within a centralized decision framework, in [8] the optimal response of a large consumer to varying electricity spot prices is derived in terms of consumptions and consumption rescheduling. In [9] a detailed analysis and characterization of the decision-making tools that consumers and retailers need to participate in an electricity market are presented.

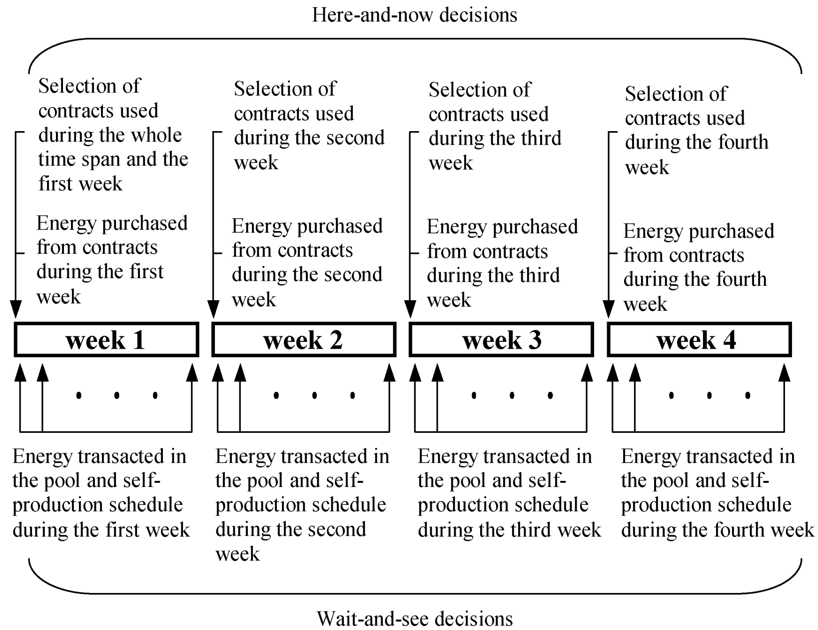

Fig. 1. Decision framework.

Additionally, in [10], a relevant method for purchase allocation and demand bidding is provided.

The electrical energy procurement problem by a large consumer is treated in [11] without considering risk accruing from uncertainty in the pool prices.

The related problem faced by an industrial consumer managing both electricity and heat (emphasizing heat issues) is addressed by [12] and [13].

The contributions of this paper are:

1) The electricity procurement decision problem faced by a large consumer is formulated as a stochastic programming problem with recourse.
2) Risk aversion is explicitly considered through the CVaR methodology.
3) The stochastic programming problem is made tractable through scenario reduction techniques and solved using a commercial branch-and-cut code.

C. *Paper Organization*

The rest of this paper is organized as follows. Section II characterizes the uncertainty associated with the considered decision-making problem. It provides a description of the multistage decision framework under uncertainty, specifying both *here-and-now* and *wait-and-see* decisions. Additionally, it characterizes the stochastic variables involved through ARIMA models, builds a scenario tree and reduces the size of this tree to make the associated problem tractable.

Section III formulates mathematically the stochastic decision-making problem as a stochastic programming problem with recourse. The solution technique to tackle this problem is also discussed.

Section IV provides and analyzes results from a realistic case study based on the electricity market of mainland Spain.

Finally, Section V provides several relevant conclusions obtained from the study reported in this paper.

II. Stochastic Programming Framework

A. *Decision Framework*

A time span of one month divided into four weeks is considered. Within this time span, the consumer has 4 procurement options, the decisions on which are made at different times: i) monthly bilateral contracts, that are selected at the beginning of the month and whose weekly consumption levels are decided at the beginning of each week, ii) weekly bilateral contracts, that are decided at the beginning of each week, iii) pool purchases or sales, that are decided one day-ahead, and iv) self-production, that is also a day-ahead decision. Fig. 1 shows the decision framework of the procurement problem faced by the consumer. The different levels of information available to the consumer when it has to make the purchases and sales of energy motivate the distinction between here-and-now decisions and wait-and-see decisions. Decisions on bilateral contracts are made under uncertainty of pool prices, i.e., before knowing the realization of these random variables. These decisions are made with imperfect information about prices and they are denoted as here-and-now decisions (Fig. 1). In contrast, pool purchases and sales, and self-production are deferred in time with respect to the here-and-now decisions, and we consider that they are made with perfect price information, i.e., prices are considered known one-day ahead. These decisions are referred to as wait-and-see decisions (Fig. 1). Wait-and-see decisions are thus used to properly model short-term decisions (pool purchases and sales, and self-production), so that optimal longer-term decisions (monthly and weekly bilateral contracts) can be derived.

In order to account for the impact of pool price uncertainty on bilateral contracting decisions we propose a multi-stage stochastic programming approach wherein each stage represents the beginning or end of each week. Therefore, a 4-week planning horizon comprises 5 stages. Under this stochastic programming framework, pool prices are considered as random variables and their probability distribution is represented by price scenarios.

The sequence of decisions is as follows:

1) Choose a level of contracting for the next month and first week of the month.
2) Choose quantities to buy from each of the contracts in each period in week 1.
3) For each scenario considered and for each period choose quantities to buy from the spot market and to self generate (accounting for the contracted quantities) so as to meet demand at least cost.
4) Choose a level of contracting for the second week of the month.
5) Choose quantities to buy from each of the contracts in each period in week 2.
6) The above sequence continues for the considered four weeks.

Observe that in our model we assume for simplicity that the selection of monthly contracts is decided on the first day of each month and is not revisited until the beginning of the next month. An alternative but much more complicated model would allow a "portfolio" of such contracts expiring and being updated (possibly) every week.

It should be emphasized that the user of the proposed tool would implement the optimal values of the here-and-now decisions for the first week. By the time decisions for the second week have to be made, pool prices of the first week will be already known and the price scenarios for the remaining weeks may be different from the original estimation. A similar situation arises at the end of the second and third weeks. Therefore, in order to make optimal decisions for weekly contracts for the second, third and fourth weeks, the proposed approach should also be run at the beginning of those weeks with an updated set of price scenarios and keeping the selection of monthly contracts determined at the beginning of the month.

B. Random Variables

Each week has been divided into seven days, and each day into three periods, thus yielding a time span of 84 periods. Each daily period comprises eight hours defined as follows:

$$\text{Day d:}\begin{aligned}&\text{Period } 1=\{1,2,3,4,5,6,7,8\}\\&\text{Period } 2=\{9,10,15,16,17,18,23,24\}\\&\text{Period } 3=\{11,12,13,14,19,20,21,22\}.\end{aligned}$$

The hours in each period have been selected depending on their pool price level. Thus, periods 1, 2 and 3 include the hours with low, medium and high pool prices, respectively. Each 8-hour period is characterized by a pool price equal to the average of the hourly pool prices during those 8 hours. Uncertainty of pool prices is handled by treating them as stochastic variables that are characterized by an ARIMA model [14], [15]. Time series models such as ARIMA, allow an adequate representation of the probability distribution of a stochastic variable through the generation of multiple realizations of it. Thus, we characterize the month by 84 price values and we use the ARIMA model not to forecast the most likely scenario but to generate a large enough number of equiprobable price scenarios that adequately represent the probability distribution of pool prices over the month. Other stochastic models (heuristic or otherwise) could also be used to produce monthly price scenarios. However, this paper does not focus on accurately generating price scenarios but on providing an appropriate decision framework for the procurement of energy to a power consumer.

TABLE I
PARAMETERS OF THE ARIMA MODEL (1)

$\phi_1 = 0.7257$	$\phi_{21} = -0.1134$	$\phi_{42} = -0.0976$
$\phi_2 = -0.1603$	$\phi_{24} = 0.0661$	$\theta_3 = 0.7491$
$\phi_3 = 0.1206$	$\phi_{27} = -0.0852$	$\theta_{21} = 0.8304$

The ARIMA model below is used to generate a scenario tree to represent the market-clearing prices of December 2004 of the electricity market of mainland Spain [1]:

$$\begin{aligned}&\left(1-\phi_1 B^1-\phi_2 B^2\right)\left(1-\phi_3 B^3-\phi_{24} B^{24}-\phi_{27} B^{27}\right)\\&\left(1-\phi_{21} B^{21}-\phi_{42} B^{42}\right)\left(1-B^3\right)\left(1-B^{21}\right)\ln\left(\lambda_t^{\mathrm{P}}\right)=\\&\qquad\left(1-\theta_3 B^3\right)\left(1-\theta_{21} B^{21}\right)\epsilon_t;\ \forall t\in T\end{aligned}\tag{1}$$

where B^u is the backshift operator that if applied to λ_t^{P} renders $B^u\lambda_t^{\mathrm{P}}=\lambda_{t-u}^{\mathrm{P}}$. The standard deviation of the error term ϵ_t for the next period, σ, is obtained at the end of the estimation phase of the ARIMA model (for model (1), $\sigma = 0.1186$). To generate scenarios we make the simplifying assumption that σ is constant for all considered time periods. The parameters of model (1) are listed in Table I. Note that model (1) shows that λ_t^{P} depends on $\left\{\lambda_{\mathrm{ini}}^{\mathrm{P}},\ldots,\lambda_{t-1}^{\mathrm{P}}\right\}$ and $\{\epsilon_{\mathrm{ini}},\ldots,\epsilon_t\}$, where $\lambda_{\mathrm{ini}}^{\mathrm{P}}$ and ϵ_{ini} are, respectively, the first pool price and error term considered in the estimation phase of the construction of the ARIMA model.

Moreover, we consider that a large consumer has a precise knowledge of its own demand that allows it to accurately forecast that demand. Note that this is not the case with the price that the large consumer faces while buying from or selling to the pool (electricity prices do not depend on the consumer). This drastic difference on uncertainty level makes us consider the consumer own demand deterministic (i.e., very accurately forecastable). Nevertheless, we point out that demand can be treated similarly to prices as a stochastic variable.

C. Scenario Tree

A scenario tree is a set of nodes and branches used in models of decision making under uncertainty. The nodes represent states of the "world" at a particular instant, being the points where decisions are taken. Each node has only one predecessor and can have several successors. The first node is called the root node and, in this work, it corresponds to the beginning of the first week of the planning horizon. In the root node, the first-stage decisions are taken. The nodes connected to the root node are the second-stage nodes and represent the points where the second-stage decisions are taken. The scenario tree branches to include all stages. The number of nodes in the last stage equals the number of scenarios. These nodes are denominated leaves. In a stochastic scenario tree, the branches are

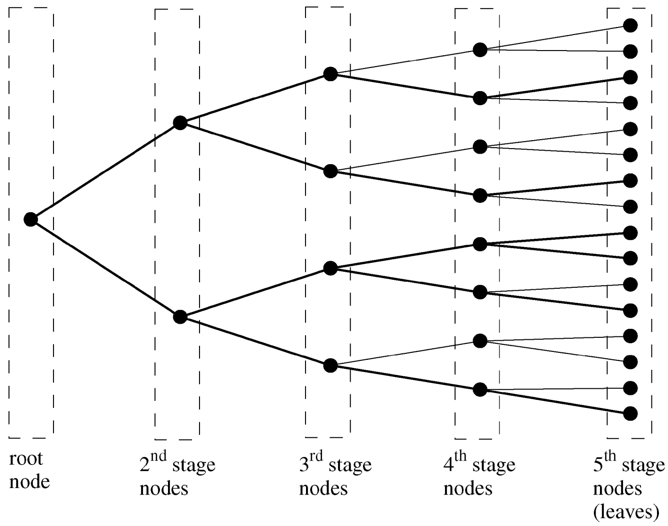

Fig. 2. Scenario tree example.

different realizations of the stochastic variables. In this work, each branch represents a realization of the random variables for one week. Since each week is divided into seven days, each comprising three periods of eight hours, the resulting vector of random variables is made up of 21 components. Fig. 2 shows an example of a 5-stage scenario tree in which 2 branches leave each node, resulting in $2^4 = 16$ scenarios.

The multiple realizations of λ_t^{P} required to build a scenario tree are obtained by assigning different values to the error term ϵ_t. For every period $t \in T$ and every scenario $\omega \in \Omega$ a random value of $\epsilon_t(\omega)$ with a $N(0,\sigma)$ distribution is generated and, through the ARIMA model (1), $\lambda_t^{\mathrm{P}}(\omega)$ is obtained. It should be noted that if two scenarios ω and ω' are coincident in the same branch in period t, the relation $\lambda_t^{\mathrm{P}}(\omega) = \lambda_t^{\mathrm{P}}(\omega')$ must be enforced.

D. Scenario Tree Reduction

The size of the scenario tree obtained by running the scenario generation process is typically very large, resulting in an optimization model that is intractable. To regain tractability, we endeavor to reduce the number of scenarios while still retaining the essential features of the scenario tree. We seek a tractable scenario tree that yields an optimal solution that is close in value to the solution of the original problem.

There has been much attention paid to this problem in the academic literature, and it is still an area of active research [16]. In two-stage stochastic linear programming problems it is possible to reduce a large scenario tree to a simpler tree that is close to the original tree when measured by a so-called *probability distance*. Under mild conditions on the problem data, it can be shown that the optimal value of the simpler problem will be close to the value of the solution to the original problem if the scenario trees are close in the probability metric. Such stability results are no longer valid for stochastic integer programs or for multistage stochastic linear programs. The latter require the introduction of a *filtration distance* [16] that essentially measures how close the branching structures of the two trees are.

The most common probability distance used in stochastic optimization is the *Kantorovich distance*, $D_K(\cdot)$, defined between two probability distributions Q and Q' by equation (2) at the bottom of the page, where $c(\omega, \omega')$ is a nonnegative, continuous, symmetric cost function and the infimum is taken over all joint probability distributions defined on $\Omega \times \Omega$. If

$$c(\omega, \omega') = \|\omega - \omega'\|^r \tag{3}$$

this gives the Wasserstein metric of order r, that can be shown to have some appealing properties when approximating stochastic optimization problems [17].

In the current context, where Q and Q' are finite distributions corresponding to an initial set Ω of scenarios and a reduced set $\Omega' \subset \Omega$, we define

$$c(\omega, \omega') = \sum_{t=1}^{n_T} |\lambda_t^{\mathrm{P}}(\omega) - \lambda_t^{\mathrm{P}}(\omega')|. \tag{4}$$

This can be shown [18] to give the Kantorovich distance

$$D_K(Q, Q') = \sum_{\omega \in \Omega \setminus \Omega'} \pi(\omega) \min_{\omega' \in \Omega'} \left(\sum_{t=1}^{n_T} |\lambda_t^{\mathrm{P}}(\omega) - \lambda_t^{\mathrm{P}}(\omega')| \right) \tag{5}$$

which is attained by assigning the probabilities $\pi(\omega)$ of all scenarios $\omega \in \Omega \setminus \Omega'$ to the "closest" scenario ω' in the remaining set Ω'.

As outlined in [19], this formula can be used to derive several heuristics for generating scenario trees that are close to an original tree in the Kantorovich metric. We have chosen to implement the *fast-forward* algorithm as described in [19] and implemented as SCENRED in GAMS [20]. This algorithm is an iterative greedy process starting with an empty tree. In each iteration, from the set of non-selected scenarios, the scenario which minimizes the Kantorovich distance between the reduced and original trees is selected. The algorithm stops if either a specified number of scenarios or a certain Kantorovich distance is reached. Finally, the probabilities of the selected scenarios are updated.

We conclude this section by recalling that the scenario reduction technique we have used is only a heuristic, with no known performance guarantees. The reduced scenario tree generated

$$D_K(Q, Q') = \inf_{\eta} \left\{ \int_{\Omega \times \Omega} c(\omega, \omega') \eta(d\omega, d\omega') : \int_{\Omega} \eta(\cdot, d\omega') = Q, \quad \int_{\Omega} \eta(d\omega, \cdot) = Q' \right\} \tag{2}$$

by the fast-forward algorithm is not guaranteed to be the closest in the Kantorovich metric to the original tree (over all reduced trees of the same cardinality). Moreover, we have no guarantee that the reduced tree will give a good approximation to the optimal value of the original problem. Nevertheless the empirical results reported in the literature (e.g., in [19]) indicate that the reduced trees defined by the fast-forward algorithm perform well in practice.

III. Problem Formulation

The mathematical formulation of the deterministic equivalent of the stochastic problem with recourse faced by the large consumer is stated below:

Minimize equation (6) at the bottom of the page.

Subject to:

1) *CVaR constraints:*

$$\sum_{b\in B}\sum_{t\in T_b}\lambda^{\mathrm{C}}_{b,t}(\omega)P^{\mathrm{C}}_{b,t}(\omega)+\sum_{t\in T}\lambda^{\mathrm{P}}_{t}(\omega)\Big(P^{\mathrm{P}}_{t}(\omega)-P^{\mathrm{S}}_{t}(\omega)\Big)+\sum_{t\in T}C_t(\omega)-\xi-\mu(\omega)\leq 0;\quad \forall\omega\in\Omega' \tag{7}$$

$$\mu(\omega)\geq 0;\quad \forall\omega\in\Omega'. \tag{8}$$

2) *Contract constraints:*

$$0\leq P^{\mathrm{C}}_{b,t}(\omega)\leq P^{\mathrm{C,max}}_{b};\forall b\in B,\forall t\in T_b,\forall\omega\in\Omega' \tag{9}$$

$$P^{\mathrm{C}}_{b,t}(\omega)=0;\quad \forall b\in B,\forall t\in T\setminus T_b,\forall\omega\in\Omega' \tag{10}$$

$$P^{\min}_{b,e}h_b(\omega)\leq \sum_{t\in T_{b,e}}P^{\mathrm{C}}_{b,t}(\omega)\leq P^{\max}_{b,e}h_b(\omega);\quad e=1,\ldots,n_{\mathrm{E}};\forall b\in B,\forall\omega\in\Omega'. \tag{11}$$

3) *Self-production constraints:*

$$0\leq P^{\mathrm{G}}_{t}(\omega)\leq P^{\mathrm{G,max}};\quad \forall t\in T,\forall\omega\in\Omega' \tag{12}$$

$$P^{\mathrm{G}}_{t}(\omega)=\sum_{i=1}^{n_{\mathrm{I}}}P^{\mathrm{G}}_{i,t}(\omega);\quad \forall t\in T,\forall\omega\in\Omega' \tag{13}$$

$$0\leq P^{\mathrm{G}}_{1,t}(\omega)\leq P^{\mathrm{G}}_{1};\quad \forall t\in T,\forall\omega\in\Omega' \tag{14}$$

$$0\leq P^{\mathrm{G}}_{i,t}(\omega)\leq P^{\mathrm{G}}_{i}-P^{\mathrm{G}}_{i-1};\quad i=2,\ldots,n_{\mathrm{I}}-1;\forall t\in T,\forall\omega\in\Omega' \tag{15}$$

$$0\leq P^{\mathrm{G}}_{n_{\mathrm{I}},t}(\omega)\leq P^{\mathrm{G,max}}-P^{\mathrm{G}}_{n_{\mathrm{I}}-1};\ \forall t\in T,\forall\omega\in\Omega' \tag{16}$$

$$C_t(\omega)=\sum_{i=1}^{n_{\mathrm{I}}}F_iP^{\mathrm{G}}_{i,t}(\omega);\quad \forall t\in T,\forall\omega\in\Omega' \tag{17}$$

$$P^{\mathrm{G}}_{t}(\omega)=P^{\mathrm{A}}_{t}(\omega)+P^{\mathrm{S}}_{t}(\omega);\quad \forall t\in T,\forall\omega\in\Omega'. \tag{18}$$

4) *Demand constraints:*

$$P^{\mathrm{A}}_{t}(\omega)+P^{\mathrm{P}}_{t}(\omega)+\sum_{b\in B}P^{\mathrm{C}}_{b,t}(\omega)=P^{\mathrm{D}}_{t};\quad \forall t\in T,\forall\omega\in\Omega'. \tag{19}$$

5) *Nonanticipativity constraints:*

$$h_b(\omega)=h_b(\omega+1);\quad \forall b\in B,\omega=1,\ldots,n_{\Omega'}-1;\ \text{if } A(\omega,H(b))=1 \tag{20}$$

$$P^{\mathrm{C}}_{b,t}(\omega)=P^{\mathrm{C}}_{b,t}(\omega+1);\quad \forall b\in B,\omega=1,\ldots,n_{\Omega'}-1;\ \text{if } A(\omega,S(t))=1. \tag{21}$$

6) *Constraints on variables:*

$$h_b(\omega)\in\{0,1\};\quad \forall b\in B,\forall\omega\in\Omega' \tag{22}$$

$$P^{\mathrm{A}}_{t}(\omega),P^{\mathrm{P}}_{t}(\omega),P^{\mathrm{S}}_{t}(\omega)\geq 0;\quad \forall t\in T,\forall\omega\in\Omega'. \tag{23}$$

Note that in the formulation above

$$\lambda^{\mathrm{C}}_{b,t}(\omega)=\frac{\lambda^{\mathrm{C,D}}_{b,t}+\lambda^{\mathrm{P}}_{t}(\omega)}{2};\quad \forall b\in B,\forall t\in T_b,\forall\omega\in\Omega'. \tag{24}$$

Problem (6)–(24) is explained below.

A. Objective Function and CVaR Constraints

The objective function (6) comprises the expected cost of the electrical procurement of the consumer and a penalized risk measure. The expected cost includes (i) the expected cost of buying from bilateral contracts, (ii) the expected net cost of buying from the pool (purchase cost minus revenue from selling), and (iii) the expected cost incurred by the self-production facility. The expected cost is calculated as the sum over all scenarios of the cost in each scenario multiplied by its probability.

The risk measure included in this work is the conditional value-at-risk at α confidence level (α-CVaR) [3]. For a discrete cost distribution, α-CVaR is approximately the expected cost of the $(1-\alpha)100\%$ scenarios with greater cost. In [4] a linear formulation of α-CVaR is provided. Let $\mathrm{DCF}(\omega)$, $\omega\in\Omega'$, be a discrete cost distribution, α-CVaR is the result of the following optimization problem:

$$\text{Minimize}_{\xi,\mu(\omega)}\qquad \xi+\frac{1}{1-\alpha}\sum_{\omega\in\Omega'}\pi(\omega)\mu(\omega). \tag{25}$$

$$\sum_{\omega\in\Omega'}\pi(\omega)\left(\sum_{b\in B}\sum_{t\in T_b}\lambda^{\mathrm{C}}_{b,t}(\omega)P^{\mathrm{C}}_{b,t}(\omega)+\sum_{t\in T}\lambda^{\mathrm{P}}_{t}(\omega)\Big(P^{\mathrm{P}}_{t}(\omega)-P^{\mathrm{S}}_{t}(\omega)\Big)+\sum_{t\in T}C_t(\omega)\right)+\beta\left(\xi+\frac{1}{1-\alpha}\sum_{\omega\in\Omega'}\pi(\omega)\mu(\omega)\right). \tag{6}$$

	Periods																				
	Monday			Tuesday			Wednesday			Thursday			Friday			Saturday			Sunday		
	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
V	×			×			×			×			×								
S		×			×			×			×			×							
P			×			×			×			×			×						
W																×	×	×	×	×	×

V: Valley; S: Shoulder; P: Peak; W: Weekend

Fig. 3. Weekly distribution of subsets of periods.

Subject to:

$$\mathrm{DCF}(\omega) - \xi - \mu(\omega) \leq 0; \quad \forall \omega \in \Omega' \tag{26}$$

$$\mu(\omega) \geq 0; \quad \forall \omega \in \Omega'. \tag{27}$$

The optimal value of ξ, ξ^*, represents the smallest value such that the probability that the cost exceeds or equals ξ^* is less than or equal to $1-\alpha$. Also, ξ^* is known as the value-at-risk (VaR). In addition, $\mu(\omega)$ is the difference between the cost of scenario ω and VaR. Constraints (26) and (27) are equivalent to (7) and (8). The objective function (25) corresponds to the last term of (6).

The weighting factor β in (6), $\beta \in [0, \infty)$, models the tradeoff between expected procurement cost and risk, and so depends on the preferences of the consumer. A conservative consumer prefers minimizing risk while its demand is satisfied, so it chooses a large value of β to increase the weight of the risk measure in (6). In addition, another consumer might be willing to assume higher risk in the hope of obtaining a lower cost, so its selected value for β would be close to 0. A detailed discussion on how to obtain appropriate values for the weighting factor β is beyond the scope of this paper.

B. Contract Constraints

In this work, volume contracts have been considered. The total energy consumed from a volume contract must satisfy prespecified upper and lower limits. The planning horizon of each contract is usually divided into several subsets of periods depending on the pool prices. For example, if the planning horizon of contract b is divided into n_{E} subsets of periods, $T_{b,e}$, then

$$\bigcup_{e=1,\ldots,n_{\mathrm{E}}} T_{b,e} = T_b; \quad \forall b \in B. \tag{28}$$

In this work, four subsets of periods, i.e., $n_{\mathrm{E}} = 4$, have been defined for each contract, namely *valley* (V), *shoulder* (S), *peak* (P) and *weekend* (W). Fig. 3 shows the distribution of subsets of periods in each week and its relation with the 8-hour periods in which each day is divided into. Taking into account these definitions, constraints (9) set the limits of the energy consumed from every contract in every period. Constraints (10) state that it is not possible to purchase energy outside of the planning horizon of each contract. Constraints (11) set the upper and lower limits for the energy consumed from contracts in each subset of periods.

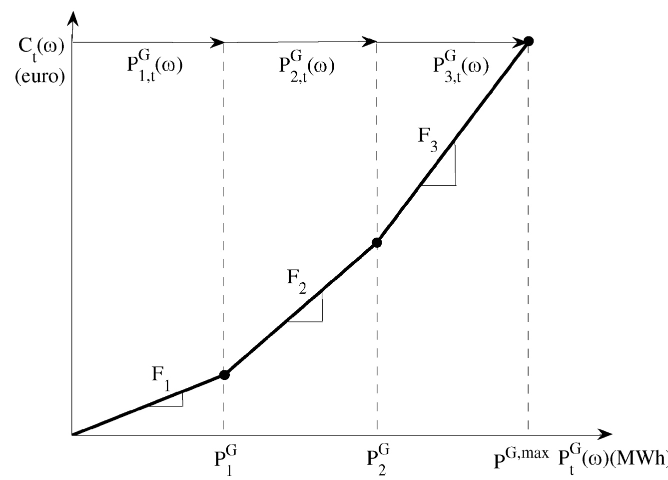

Fig. 4. Piecewise linear production cost.

The above contracting format is motivated by industry practice in Spain. However, note that a different format can be considered and simpler settings are possible, e.g., buying a quantity of energy at constant power and price during a given time period.

C. Self-Production Constraints

Constraints (12) bound the production of the self-production unit by the upper limit of energy that can be produced during one period, which is equal to the capacity of the unit multiplied by the duration of the period. Constraints (13)–(17) are required to model the piecewise linear production cost of the self-production unit [5]. Constraints (18) state that the energy generated by the self-production facility can be used either to satisfy the demand of the consumer or to be sold in the pool. Fig. 4 depicts the piecewise linear production cost of the self-production unit.

D. Demand Constraints

Constraints (19) enforce that the demand must be satisfied in each period of the planning horizon.

E. Nonanticipativity Constraints

Constraints (20) and (21) model the nonanticipativity constraints for $h_b(\omega)$ and $P^{\mathrm{C}}_{b,t}(\omega)$, respectively. Nonanticipativity constraints enforce that if the realizations of the stochastic variables are equal in two scenarios ω and ω' up to stage k, then the value of the *here-and-now* decisions in stage k must be the same. For computational implementation, these variables are made scenario independent.

TABLE II
COMPUTATIONAL SIZE OF PROBLEM (6)–(24)

# of binary variables	$n_{\rm B} n_{\Omega'}$
# of real variables	$n_{\Omega'}\left(n_{\rm T}(n_{\rm B}+n_{\rm I}+5)+1\right)+1$
# of constraints	$n_{\Omega'}(n_{\rm T}(n_{\rm B}+n_{\rm I}+5)+2n_{\rm B}n_{\rm E}+1+$ $\sum_{n\in N}\big(\sum_{b\in B_n}(n_{\Omega',n}-1)+$ $\sum_{b\in\{B_n\bigcup B^{\rm M}\}} n_{\rm W}(n_{\Omega',{\rm n}}-1)\big)$

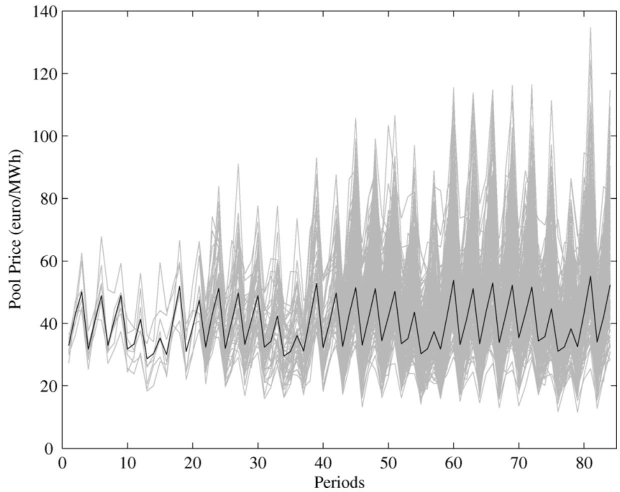

Fig. 5. Pool prices.

F. Constraints on Variables and Others

Constraints (22) and (23) constitute variable declarations. Finally, the expressions (24) define the final price of energy purchased from bilateral contracts. In this study we consider that the final price is equal to the average of the pool prices and a deterministic price which represents the expected pool prices. This kind of contracts is known as contract for differences. It should be noted that a different manner to state the price for the energy transacted using bilateral contracts can be used.

Problem (6)–(24) is a mixed-integer linear optimization problem that can be solved by commercially available branch-and-cut software [21]–[23].

Table II provides the size of problem (6)–(24) expressed as the number of binary variables, real variables and constraints.

IV. Case Study

The performance of the proposed decision-making approach is illustrated through a case study based on the electricity market of mainland Spain [1]. A time series of 11 months has been used to estimate pool prices for December 2004. In order to represent the probability distribution of pool prices 2401 equiprobable scenarios have been considered corresponding to a scenario tree where 7 branches leave each node ($7^4 = 2401$). Fig. 5 shows the 2401 scenarios of pool prices for the 84 periods considered. The bold line in Fig. 5 represents the expected pool prices obtained using the ARIMA model (1). The forecast demand of the consumer is plotted in Fig. 6.

The consumer has the possibility of signing two monthly bilateral contracts as well as four weekly contracts, one per week. The energy consumption limits of each contract are reported in Table III. For the sake of conciseness, both monthly contracts

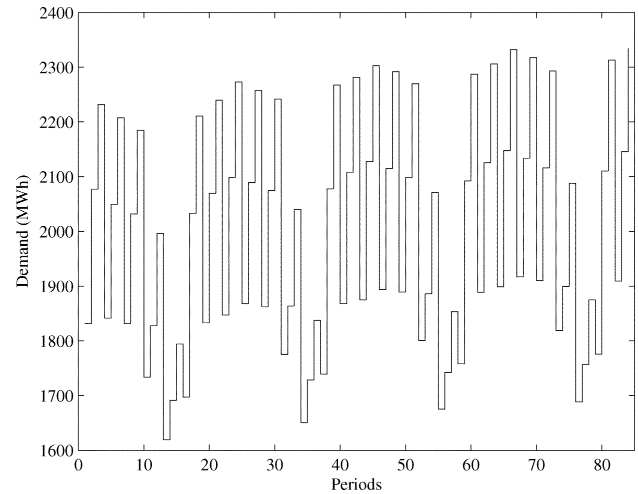

Fig. 6. Consumer demand.

TABLE III
ENERGY CONSUMPTION LIMITS OF THE BILATERAL CONTRACTS (MWh)

	Monthly contracts		Weekly contracts	
	$P_{b,e}^{\rm min}$	$P_{b,e}^{\rm max}$	$P_{b,e}^{\rm min}$	$P_{b,e}^{\rm max}$
V	3540	7870	550	1250
S	6125	12360	1000	2310
P	7525	16850	1425	3365
W	2125	4880	150	550

TABLE IV
PRICES OF THE MONTHLY CONTRACTS (Euro/MWh)

	Week # (monthly contract 1)				Week # (monthly contract 2)			
	1	2	3	4	1	2	3	4
V	42.0	43.0	44.0	45.0	42.0	43.0	44.0	45.0
S	52.8	53.8	54.8	55.8	55.2	56.2	57.2	58.2
P	62.4	63.4	64.4	65.4	64.8	65.8	66.8	67.8
W	44.4	45.4	46.4	47.4	43.2	44.2	45.2	46.2

TABLE V
PRICES OF THE WEEKLY CONTRACTS (Euro/MWh)

	Week 1	Week 2	Week 3	Week 4
V	44.0	46.2	48.0	49.2
S	57.4	59.7	60.0	61.2
P	66.5	67.2	69.0	69.8
W	45.0	48.6	50.4	51.6

are identical in terms of energy limits. Analogously, the four weekly contracts share the same consumption limits. Also, the upper limit of energy purchased in one period, $P_b^{\rm C,max}$, is fixed to 800 MWh in all contracts. Price data of the monthly and weekly contracts are provided in Tables IV and V, respectively.

The consumer owns a 100-MW self-production unit. Since each time period comprises eight hours, the maximum energy that can be produced in each period, $P^{\rm G,max}$, is equal to 800 MWh. Table VI lists the data of the 3-piece linear cost function considered.

After applying the scenario reduction technique mentioned in Section II-D, the resulting tree contains just 200 scenarios. The decision on the number of scenarios of the reduced tree is based on two sound metrics: Kantorovich distance and expected cost. The relative distance between the original tree and reduced trees is depicted in Fig. 7. The relative distance is defined as the

TABLE VI
PRODUCTION COST DATA OF THE COGENERATION UNIT

P_1^{G} (MWh)	P_2^{G} (MWh)	F_1 (€/MWh)	F_2 (€/MWh)	F_3 (€/MWh)
160	480	33	36	39

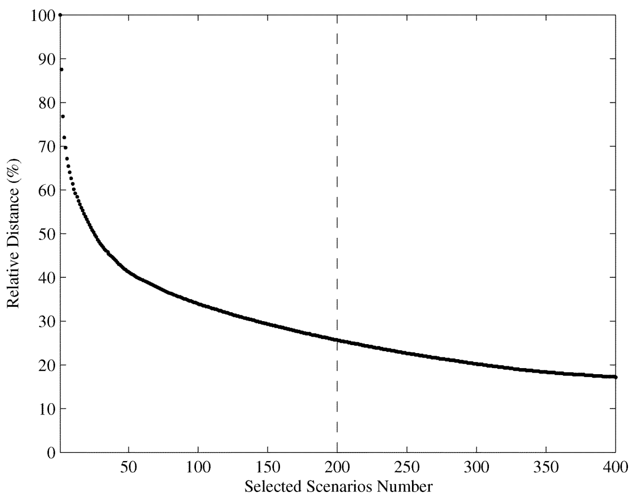

Fig. 7. Relative distance between the original tree and reduced trees.

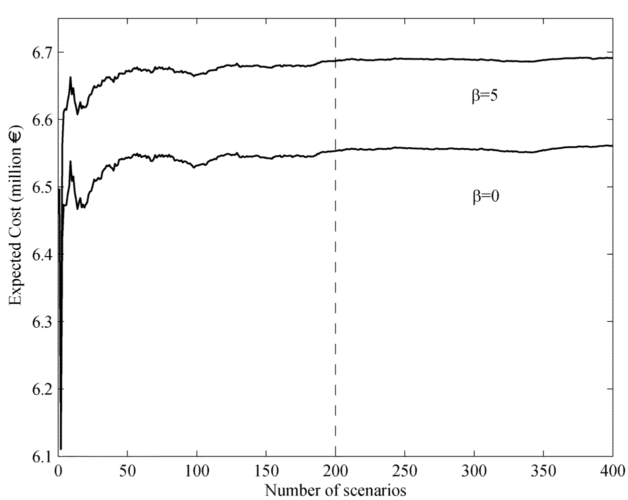

Fig. 8. Expected cost as a function of the number of scenarios considered.

Kantorovich distance between the original and reduced probability distributions divided by the Kantorovich distance between a 1-scenario tree and the original distribution, where the distance between two trees is calculated with (5). As can be observed, using a 200-scenario tree instead of a 400-scenario tree only incurs a 5% deterioration in the relative distance (Fig. 7). This result is corroborated by Fig. 8, which shows the evolution of the optimal expected cost with the number of scenarios for $\beta = 0$ and $\beta = 5$. Fig. 8 shows that the expected cost difference between the 200- and 400-scenario trees is just 0.12%. Based on both results, a 200-scenario tree is an appropriate choice.

Similar curves to those plotted in Fig. 8 have been obtained for other values of the risk parameter β. For all the simulations, a confidence level of $\alpha = 0.95$ has been used in the calculation of CVaR.

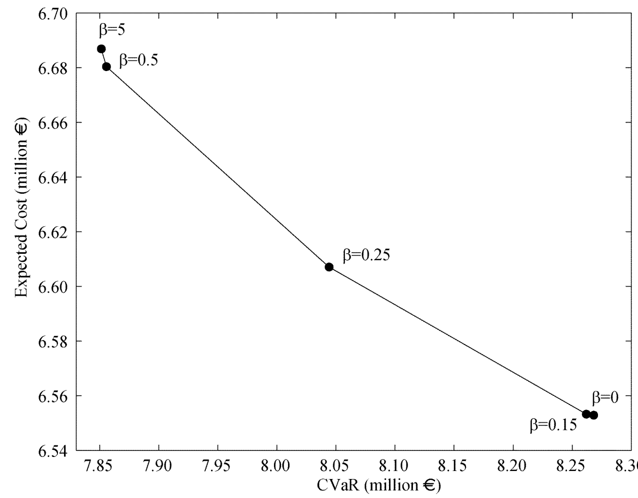

Fig. 9. Expected cost versus CVaR.

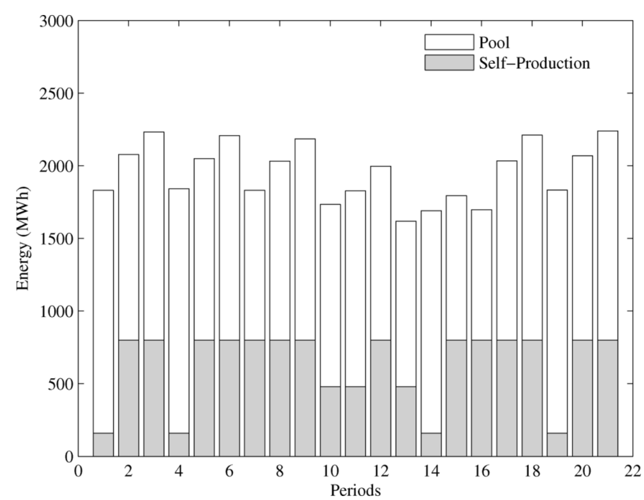

Fig. 10. Electricity procurement in the first week for a single scenario ($\beta = 0$).

The resulting problem, characterized by 326288 constraints, 236602 real variables, and 1200 binary variables, has been solved for different values of the weighting factor β using CPLEX 9.0 under GAMS [20]. Fig. 9 provides a plot of expected cost versus CVaR for different values of β. The expected cost ranges from 6.55 million Euro if risk is ignored ($\beta = 0$), to 6.68 million Euro if risk is accounted for ($\beta = 5$). In other words, a 5% reduction in CVaR from 8.27 to 7.85 million Euro results in a 2.4% increase in expected cost. This result is significant since it provides relevant and useful information for the decision maker on the energy procurement problem. The CPU time required to solve problem (6)–(24) for different values of β with a Dell PowerEdge 6600 with two processors at 1.60 GHz and 2 GB of RAM memory was less than 130 s.

Figs. 10 and 11 show the energy procurement of a single scenario in the first week for two levels of risk. For a risk neutral consumer with $\beta = 0$ no contract is signed (Fig. 10) and the self-production unit is operating at its capacity in many periods because the scenario considered comprises pool prices higher than the production cost.

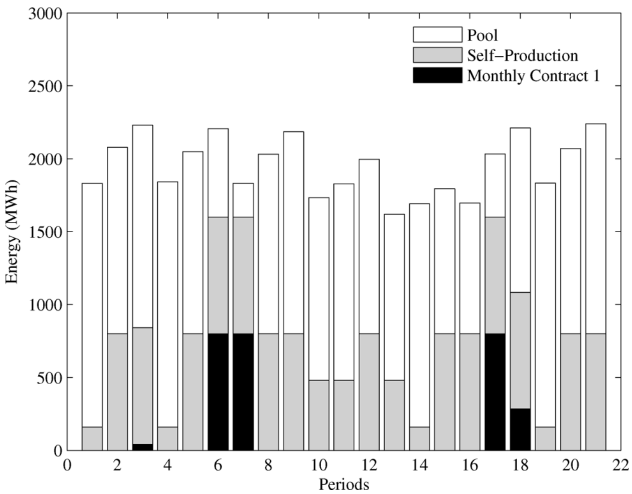

Fig. 11. Electricity procurement in the first week for a single scenario ($\beta = 0.25$).

As can be seen in Fig. 11, monthly contract 1 is used in the first week for $\beta = 0.25$. It should be noted that in periods 6, 7, and 17, the monthly contract 1 reaches its limit $P_b^{\mathrm{C,max}} = 800$ MWh. Fig. 12 illustrates the energy procurement of the consumer and the use of bilateral contracts during the first week for different values of β. The energy represented in each sector of the pie charts is the weighted average over all of the scenarios. As expected, the share of bilateral contracts considerably increases with β in order to hedge against risk exposure to pool prices volatility. With $\beta = 0$ the consumer mostly relies on the pool and no contracts are signed. For $\beta = 0.25$, 6.6% of the energy consumed in the first week is procured exclusively through monthly contract 1. Finally, for $\beta = 5$ the volume of energy purchased from contracts for the first week rises up to 15.6%, being equally split between monthly contracts 1 and 2, because both are used at their maximum level in the same number of periods. Note that the weekly contract is not signed due to its higher price (Tables IV and V). It is also remarkable that the volume of energy self-produced by the consumer keeps at a stable share around 20% and experiences a slight drop with β due to the relatively high minimum consumption limits of bilateral contracts.

V. Conclusions

This paper provides a methodology for the electricity procurement of a large consumer based on risk-constrained stochastic programming. The use of an ARIMA model to characterize the pool price stochastic nature renders appropriate results. It provides sufficient accuracy for moderate computational effort and conceptual simplicity. The scenario reduction technique based on the Kantorovich distance proves adequate to make the resulting optimization problem tractable while retaining sufficient accuracy in the representation of the probabilistic distribution of the price. The procurement framework including bilateral contracts, the pool and self-production is general enough to represent many real world situations. The technique proposed provides the expected cost as a function of cost volatility, which is required to carry out informed decision

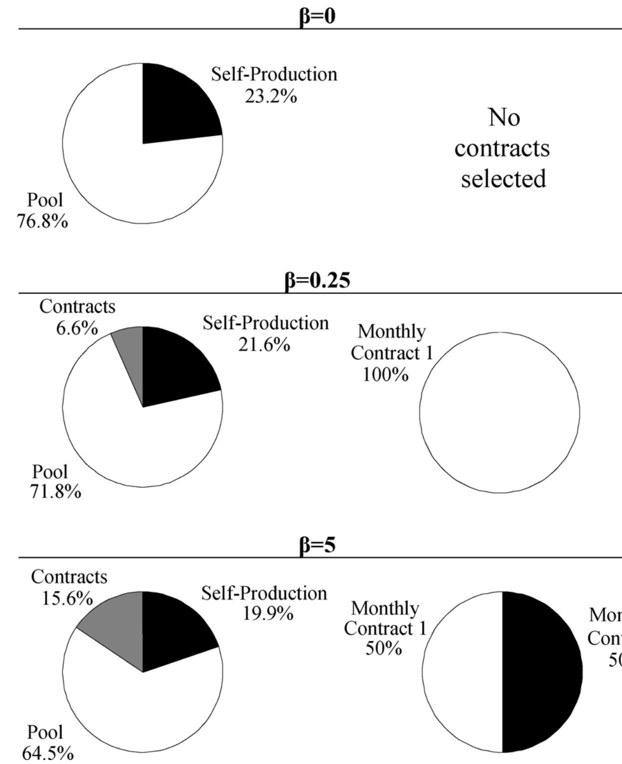

Fig. 12. Electricity procurement and use of contracts for the first week.

making. The validity and practical interest of the methodology proposed are shown through a realistic case study.

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Miguel Carrión (S'04) received the Ing. Ind. degree from the Universidad de Castilla-La Mancha, Ciudad Real, Spain, in 2003. He is currently working toward the Ph.D. degree at the Universidad de Castilla-La Mancha. His research interests are in the fields of power systems economics, stochastic programming, and electricity markets.

Andy B. Philpott (M'04) is a Professor in the Department of Engineering Science, The University of Auckland, Auckland, New Zealand, and leads the Electric Power Optimization Center. His research interests encompass optimization under uncertainty, with particular reference to optimization modeling in electricity markets.

Antonio J. Conejo (F'04) received the M.S. degree from MIT, Cambridge, MA, in 1987, and the Ph.D. degree from the Royal Institute of Technology, Stockholm, Sweden, in 1990.

He is currently a full Professor at the Universidad de Castilla-La Mancha, Ciudad Real, Spain. His research interests include control, operations, planning and economics of electric energy systems, as well as statistics and optimization theory and its applications.

José M. Arroyo (S'96–M'01–SM'06) received the Ing. Ind. degree from the Universidad de Málaga, Málaga, Spain, in 1995 and the Ph.D. degree in power systems operations planning from the Universidad de Castilla-La Mancha, Ciudad Real, Spain, in 2000.

From June 2003 through July 2004, he held a Richard H. Tomlinson Postdoctoral Fellowship at the Department of Electrical and Computer Engineering, McGill University, Montreal, QC, Canada. He is currently an Associate Professor of Electrical Engineering at the Universidad de Castilla-La Mancha. His research interests include operations, planning and economics of power systems, as well as optimization and parallel computation.