A COMPARISON OF INTERIOR-POINT CODES FOR MEDIUM-TERM HYDRO-THERMAL COORDINATION

José Medina (Student M) Victor H. Quintana (Senior M)
Department of Electrical & Computer Engineering
University of Waterloo
Waterloo, Ontario, N2L 3G1, Canada

Antonio J. Conejo (M)
ETSI Industriales
Universidad de Málaga
Málaga, Spain

F. Pérez Thoden
ENDESA
Madrid
Spain

ABSTRACT - This paper studies the performance of newly developed and currently under development interior-point optimization codes as applied to the solution of medium-term hydro-thermal coordination (MTHTC) problems. We compare commercial and research codes, and their main advantages and drawbacks are pointed out. The codes that we study are: CPLEX 3.0-barrier [1] (the latest version of OB1 by Lustig), HOPDM by Gondzio [14], LOQO by Vanderbei [17], PCx by Mehrotra's group [16], LIPSOL by Zhang [22], and IPA1 [25] (a code currently being developed by the authors). All codes have been tested on the Spanish hydro-thermal system.

Keywords: Hydro-Thermal Coordination, Interior-Point Method.

1. INTRODUCTION

A hydro-thermal coordination program deals with the problem of finding the scheduling and the production of every hydro and thermal plant of a system so that the customer demand is supplied at minimum cost with a certain level of security and all constraints related to the thermal and hydro subsystems are satisfied.

There are several reasons for choosing a medium-term time-horizon model. A medium-term time-horizon problem can easily accommodate multiperiod energy constraints that must be included in an accurate model for the generating system of mainland Spain. Medium-term hydro-thermal information can be used in models that consider shorter time horizons. Also, the main block of a yearly planning model is a medium-term model with a time horizon of typically four weeks.

A medium-term hydro-thermal coordination problem results in a very large Non-Linear Mixed-Integer Programming problem. In order to find a solution to such a program, several approaches have been used. Primal decomposition methods are employed in [2, 3]; the solution to the thermal subproblem proposed by Brännlund [3] is based on heuristics and it is not an appropriate approach when the thermal system to be analyzed is complex, which is the case in the Spanish hydro-thermal system. More recently, Nilson et Sjelvgren [8] have described an excellent approach for hydro subsystems, but the thermal subsystem is just described as economic transactions which is not appropriate for the Spanish system. The approach proposed by Dillon [4] to the solution to the thermal subproblem is a computationally expensive mixed-integer linear-programming procedure that requires prohibitively high CPU time to solve systems of realistic size; the thermal subsystem modelling is, however, accurate enough for our purposes; thus a model based on this approach is used for our thermal subsystem.

PE-044-PWRS-16-09-1997 A paper recommended and approved by the IEEE Power System Engineering Committee of the IEEE Power Engineering Society for publication in the IEEE Transactions on Power Systems. Manuscript submitted May 27, 1997; made available for printing September 30, 1997.

Lagrangian relaxation techniques have also been proposed in the literature to solve hydro-thermal coordination problems [5, 6, 7, 9]. A Lagrangian relaxation technique solves the dual problem of the original hydro-thermal coordination problem. However, the dual solution is more often than not primal infeasible, so that perturbation procedures are required to obtain a primal feasible solution. These perturbation procedures may deteriorate the optimality of the solution obtained. Furthermore, solutions to the primal and dual problems are not the same, and the size of the duality gap cannot be foreseen in advance.

Very recently, genetic algorithms are also being applied to solve the hydro-thermal coordination problem. In [19], the hydro schedule is obtained by a genetic algorithm; however, the thermal schedule is obtained by an independent unit commitment program. The thermal solution is then used to evaluate the quality of the hydro schedule. In [20], Bai and Shahidehpour, use a tabu search to improve the solution of a primal decomposition based approach.

Interior-point (IP) methods have experienced an extensive development over the last decade or so, and are being applied to almost every optimization program [26, 27]. However, the only previous work that solves a hydro-thermal coordination problem using IP techniques is the one by Christoforodis et al., [18], which limits itself to solving a proposed model by the commercial package IBM-OSL, given no detailed information on how the model is solved.

An IP algorithm implies that progress towards the optimum is made through the interior of the feasible region rather than along its vertices. How this is done depends on the particular algorithm. The first interior-point algorithm was developed by Frisch [15] in 1955. Karmarkar [12] in 1984 developed an interior-point algorithm for linear programming that was faster than any other known algorithm for large LP problems. Karmarkar also proved that his algorithm is a polynomial time algorithm while the simplex is not. In 1992, Mehrotra [13] developed a predictor-corrector interior-point method to solve the first-order necessary condition of the primal-dual method. The code of this algorithm is called PCx. Lustig et al. [11] applied Mehrotra's predictor-corrector scheme to their algorithm. The name for Lustig's algorithm is OB1. Vanderbei in 1993 developed another implementation of the primal-dual path-following method. Vanderbei's algorithm is called LOQO [17]. In 1995 Zhang and Gondzio developed another two codes based on the predictor-corrector IP algorithm by Mehrotra. Their names are LIPSOL [22] and HOPDM [14], respectively.

As a reaction to all this work in interior-point algorithms, there was also a revival of the simplex method in an effort to improve its performance. With recent improvements, the simplex coding is very competitive to, or even better than, interior-point methods for small to medium size problems. For large size problems, interior-point methods seems to outperform the simplex algorithm.

This paper compares the performance of several interior-point algorithms and simplex algorithms in the solution of medium-term hydro-thermal related problems for the Spanish power system. The purpose of the comparison is to show the performance of several recently-developed optimization codes for solving LP problems and to provide a summary of the main features of these codes.

The paper is organized as follows. In Section 2, the hydro-thermal coordination problem is mathematically formulated. The main features of recently developed interior-point algorithms are discussed in Section 3. Computational results and discussions are presented in Section 4. Section 5 contains the conclusions.

2. PROBLEM FORMULATION

The notation is organized as given data, indices, sets and number of elements in the sets, and variables. Among the variables, there are independent or control variables and dependent or state variables. Overlining indicates upper bound and underlining stands for lower bound.

- DATA

a_j is the fixed operating cost of thermal plant j [\$/h],
b_j the start-up cost of thermal plant j [\$],
c_j the shut-down cost of thermal plant j [\$],
d_{jl} l-th cost slope of the variable operating cost function of thermal plant j [\$/MWh],
r_j the maximum up ramp rate of thermal plant j [MW/h],
s_j the maximum down ramp rate of thermal plant j [MW/h],
E_p the minimum energy to be produced by the thermal plants of the production area p during the planning horizon [MWh],
$\alpha_{\gamma j}$ amount of pollutant γ emitted by plant j when producing 1 MWh of energy,
$P_{\gamma e}$ maximum amount of pollutant γ to be emitted in emission controlled area e during the planning horizon [kg],
$w_i(k)$ the lateral inflow volume to reservoir of hydro plant i during subperiod k [Hm^3],
x_{i0} the initial storage volume of the reservoir of hydro plant i [Hm^3],
$\overline{x}_{iT}$ the upper bound of the final storage volume of the reservoir of hydro plant i [Hm^3],
$\underline{x}_{iT}$ the lower bound of the final storage volume of the r eservoir of hydro plant i [Hm^3],
ρ_{il} the generating characteristic of block l of hydro plant i [MJ/Dm^3],
$d(k)$ is the customer power demand in subperiod k [MW],
$R(k)$ the thermal spinning reserve in subperiod k [MW],
$l(k)$ the duration of subperiod k [h],
M a large penalty constant,
$d_s(k)$ is the demand in area s during subperiod k,
m_s the transmission capacity of the interconnections of area s,

- INDECES, SETS AND NUMBER OF ELEMENTS

j the thermal plant index,
γ the pollutant index,
e the emission controlled area index,
p the production area index,
i the hydro plant index,
k the subperiod index,
l the block index for hydro or thermal plants,
J the set of indices of all thermal plants,
I the set of indices of all hydro plants
J_p the set of indices of the thermal plants of the production area p,
J_e the set of indices of the thermal plants in emission controlled area e,
N_g the number of pollutants,
N_e the number of emission controlled areas,
N_p the number of production areas,
C_j the number of blocks of the variable operating cost of thermal plant j,
U_i the number of blocks of the turbine discharge of hydro plant i,
N the number of subperiods of one week,
J_s the set of indices of the thermal plants which belong to area s,
I_s the set of indices of the hydro plants which belong to area s,
Ω_i the set of indices of hydro plants upstream the reservoir of plant i,
K the set of indices of the subperiods $(1, 2, \cdots, T)$ of the planning period,
K_d the set of subperiods for which ramp-rate limits apply,

- CONTROL VARIABLES

$p_{jl}(k)$ l-th power block of the output power above the minimum output power of thermal plant j in subperiod k [MW],
$v_j(k)$ a 0/1 variable which is equal to 1 if and only if thermal plant j is committed in subperiod k,
$y_j(k)$ a 0/1 variable which is equal to 1 if and only if thermal plant j is started-up at the beginning of subperiod k,
$s_i(k)$ the spilled outflow volume of hydro plant i during subperiod k [Hm^3/s],
$u_{il}(k)$ the turbine discharge volume block l of hydro plant i during subperiod k [Hm^3/s]

- STATE VARIABLES

$p_j(k)$ the output power above the minimum output power of thermal plant j in subperiod k [MW],
$t_j(k)$ the output power of thermal plant j in subperiod k [MW],

$z_j(k)$ a 0/1 variable which is equal to 1 if and only if thermal plant j is shut-down at the beginning of subperiod k,
$h_i(k)$ the power produced by hydro plant i in subperiod k [MW],
$x_i(k)$ is the storage volume of the reservoir of hydro plant i at the beginning of subperiod k [Hm^3],
$u_i(k)$ the turbine discharge volume of hydro plant i during subperiod k [Hm^3/s],
$o(k)$ the unserved power in subperiod k [MW].

2.1. Objective Function

The objective function is the total thermal production cost which includes the fixed, start-up, shut-down and variable costs; for all subperiods, the objective function can be written as

$$\sum_{k\in K}\sum_{j\in J}[a_j\, v_j(k)\, l(k)+b_j\, y_j(k)+c_j z_j(k)+ \sum_{l=1}^{C_j} d_{jl}\, p_{jl}(k)\, l(k)]+M\sum_{k\in K} o(k). \quad (1)$$

We assume that the total-production-cost function is concave. The complete model of the hydro-thermal system is linearized by segmenting the power output of each hydro and thermal plant.

2.2. The Thermal System

The thermal system model comprises the following constraints.

- Each segment (block) of the power output of each thermal plant is constrained by upper and lower bounds, i.e.,

$$0\le p_{jl}(k)\le \overline{p}_{jl} \qquad \forall j\in J;\quad \forall k\in K;\quad l=1,\cdots,C_j. \quad (2)$$

- The output power of every thermal plant, as the sum of output power blocks, is given by

$$p_j(k)=\sum_{l=1}^{C_j} p_{jl}(k) \qquad \forall j\in J,\quad \forall k\in K. \quad (3)$$

- Committed thermal plants must operate below their maximum output-power limits, i.e.,

$$p_j(k)\le \overline{p}_j\, v_j(k) \qquad \forall j\in J,\quad \forall k\in K. \quad (4)$$

- The power output of every commited thermal plant is the sum of its minimum power output plus its generation above its lower limit,

$$t_j(k)=\underline{p}_j\, v_j(k)+p_j(k) \qquad \forall j\in J,\ \forall k\in K. \quad (5)$$

- Down and up ramp rates of thermal plants are also bounded, i.e.,

$$t_j(k)-t_j(k+1)\le s_j l(k) \qquad \forall j\in J,\ \forall k\in K_d, \quad (6)$$

$$t_j(k+1)-t_j(k)\le r_j l(k) \qquad \forall j\in J,\ \forall k\in K_d. \quad (7)$$

- For every production area, the generation energy for the whole period of the thermal plants that belong to the area, say p, must be above a prespecified energy limit, i.e.,

$$\sum_{j\in J_p}\sum_{k\in K} t_j(k) l(k)\ge E_p \qquad p=1,\cdots,N_p. \quad (8)$$

- The amount of every pollutant (γ) emitted in every emission constrained area (e) must be below a prespecified threshold this is,

$$\sum_{j\in J_e}\sum_{k\in K}\alpha_{\gamma j}\, t_j(k)\, l(k)\le P_{\gamma e} \qquad e=1,2,\cdots,N_e;\quad \gamma=1,2,\cdots,N_g. \quad (9)$$

  Constraints (8) and (9) are of particular interest for the Spanish system.

- The variables that describe the operation status (running, start-up, shut-down) of the thermal units must satisfy logical constraints in order to guarantee a proper sequence of status changes, i.e.,

$$y_j(k)-z_j(k)=v_j(k)-v_j(k-1) \qquad \forall j\in J,\quad \forall k\in K, \quad (10)$$

$$v_j(k), y_j(k), z_j(k)\in\{0,1\} \qquad \forall j\in J,\quad \forall k\in K. \quad (11)$$

- The number of start-ups and the number of shut-downs of every thermal plant during a week (N subperiods) is limited to 1, i.e.,

$$\sum_{k=n}^{n+N} y_j(k)\le 1;\quad \forall j\in J, \qquad \forall n=1,1+N,1+2N,\cdots,K-N, \quad (12)$$

$$\sum_{k=n}^{n+N} z_j(k)\le 1;\quad \forall j\in J, \qquad \forall n=1,1+N,1+2N,\cdots,K-N. \quad (13)$$

2.3. The Hydro System

The hydro system model comprises the following constraints.

- In every subperiod, the total turbine discharge of every hydro plant is segmented in blocks; thus, it is given by the sum of discharge blocks (segments) of the total plant discharge,

$$u_i(k)=\underline{u}_i+\sum_{l=1}^{U_i} u_{il}(k) \qquad \forall i\in I,\ \forall k\in K. \quad (14)$$

- For every subperiod, the hydro production of every hydro plant is assumed concave; it can be expressed as a combination of the turbine discharge blocks,

$$h_i(k) = \underline{h}_i + \sum_{l=1}^{U_i} \rho_{il}\, u_{il}(k) \;\; \forall i \in I, \forall k \in K. \tag{15}$$

- Each discharge block of every turbine is constrained by upper and lower bounds, i.e.,

$$0 \le u_{il} \le \overline{u}_{il} \qquad \forall i \in I,\; l = 1, 2, \cdots, U_i. \tag{16}$$

- The water conservation equation over all subperiods and for all reservoirs, is given by

$$\begin{aligned} x_i(k+1) &= x_i(k) - u_i(k)l(k) - s_i(k)l(k) \\ &+ \sum_{j \in \Omega_i} [u_j(k) + s_j(k)]l(k) + w_i(k)l(k) \\ &\forall i \in I, \;\; \forall k \in K. \end{aligned} \tag{17}$$

The canals are modeled by adding a variable to Equation (17). This variable represents the flow per period through the canal. The sign of this variable depends on which direction the water is flowing.

- There are upper and lower bounds on reservoir volumes, i.e.,

$$\underline{x}_i \le x_i(k) \le \overline{x}_i \qquad \forall i \in I, \quad \forall k \in K. \tag{18}$$

- Spillage outflows must be positive, i.e.,

$$0 \le s_i(k) \le \overline{x}_i \qquad \forall i \in I, \quad \forall k \in K. \tag{19}$$

- The initial and the final conditions on the reservoir volumes are given by

$$x_i(0) = x_{i0} \qquad \forall i \in I, \tag{20}$$

$$\underline{x}_{iT} \le x_i(T+1) \le \overline{x}_{iT} \qquad \forall i \in I. \tag{21}$$

The power production of a hydro plant (say i), as a function of the water discharge, is dependent upon the head of the reservoir associated with the hydro plant i. Thus, for every reservoir head there is a "power/discharge" curve. The dependency on the head can be taken into account by solving several times the whole hydro-thermal coordination problem and choosing, before every run, the appropriate power/discharge curve. This procedure has proved in practice to be computationally stable.

2.4. Global Constraints

These constraints couple together the thermal and the hydro subsystems and are considered independently for each subsystem when a decomposition algorithm is used.

- Power balance in all subperiods (including unserved power)

$$\sum_{j \in J} t_j(k) + \sum_{i \in I} h_i(k) + o(k) = d(k) \qquad \forall k \in K. \tag{22}$$

- Spinning reserve margin

$$\sum_{j \in J} [\overline{t}_j v_j(k) - t_j(k)] + \sum_{i \in I} [\overline{h}_i - h_i(k)] \ge R(k) \qquad \forall k \in K. \tag{23}$$

- Transmission network constraints

After linearization these constraints are as follows:

$$\Big| \sum_{j \in J_s} t_j(k) + \sum_{i \in I_s} h_i(k) - d_s(k) \Big| \le m_s. \tag{24}$$

2.5. Solution Procedure

The above problem is a large-scale sparse $0/1$ mixed integer LP problem. Our proposed solution relaxes the $0/1$ variables so that they belong to the continuous interval $[0, 1]$. The solution to the resulting LP problem is obtained by using the codes described in the next section; in every subperiod, most relaxed integer variables end up having values 0 or 1, except for a few (typically, one or two) that have values between 0 and 1. This behavior is a consequence of the interaction between the hydro and thermal subproblems; in order to reduce the cost, hydro plants *tend* to substitute *marginal* thermal plants (which are the plants whose relaxed integer variables have values between 0 and 1).

Relaxed integer variables that are between 0 and 1 are treated by a rounding strategy. This strategy assigns a value of 0 or 1 to each of these variables so that the minimum up and down time constraints, the load balance equations and spinning reserve margins are satisfied. After the schedule of thermal plants is known and fixed, we solve the MTHTC problem again.

3. RECENT INTERIOR-POINT CODES

In this section we describe the interior-point codes by Mehrotra, Gondzio, Vanderbei, Zhang, and Lustig; we also outline our proposed code IPA1. All the codes implement Mehrotra's primal-dual path-following predictor-corrector algorithm; this algorithm is basically described in [10, 13].

3.1. LOQO

LOQO [17] is a primal-dual path-following algorithm that solves quadratic (convex) programs as well as linear programs. As implemented, LOQO operates on QP problems directly as they are read from an MPS file. The general form of such programs is

$$minimize \quad f + c^T x + \frac{1}{2} x^T H x$$

subject to

$$b \le Ax \le b + r\,, \quad l \le x \le u$$

where A is an $m \times n$ matrix, and H is an $n \times n$ positive semidefinite matrix, b is the right-hand side, r is the vector of ranges on the constraints and u and l are the upper and lower bounds.

The advantage of solving the program directly in an MPS data structure is that a certain primal-dual symmetry inherent to the MPS format is preserved.

The search directions are obtained by solving the First-Order Necessary Conditions for Optimality (FONCO). The FONCO equations form a large sparse indefinite linear system; however, they are easily turn into a symetric system (providing that the problem is solved directly in the MPS format). LOQO solves the FONCO equations by predetermined reduction on the system (which is done by pivoting without regard to the size of the pivot element). The reason for using predetermined reduction instead of normal pivoting is that it yields faster codes. Reductions are performed until a system of equations in $\triangle x$ and $\triangle y$ is obtained; such system is solved by a modified Cholesky factorization that has been altered to solve symmetric quasy-definite systems. The diagonal pivots are selected based on a global fill-in minimizing heuristic; this global heuristic analyzes the overall structure of the matrix instead of minimizing the fill-in produced in each subsequent stage as myopic heuristics do.

3.2. PCx

The PCx code has been developed by Mehrotra's group [16], and is a variant of Mehrotra's algorithm. It reads the data in MPS format and converts it to the standard LP format. The primal and the dual of a problem in standard form are, respectively,

$$minimize \;\; c^T x$$

subject to

$$Ax = b\,, \quad x + s = u\,, \quad x \geq 0\,,\; s \geq 0$$

and

$$maximize \;\; b^T y - u^T w$$

subject to

$$A^T y + z - w = c\,, \quad z \geq 0\,,\; w \geq 0$$

PCx finds the search directions by performing a predetermined reduction of the FONCO equations and leads to the Normal-Equation (NE). The NE system is symmetric positive-definite and is solved by an efficient and robust procedure that includes Liu's multiple minimum degree ordering strategy [24], and Ng and Peyton code [21] for sparse Cholesky factorization; the code is slightly modified to handle the small pivots that usually arise in the later iterations of interior-point methods.

The NE matrix can be much denser than the restriction matrix A if A contains dense columns. The authors of PCx are currently working on a new version of the algorithm using Schur complements to avoid fill-ins in the normal-equation.

3.3. HOPDM

The algorithm implemented in HOPDM [14] is a variant of Mehrotra's algorithm that uses Multiple Corrector Steps of Centrality (MCSC) in order to try to reduce the number of iterations.

The FONCO equations are solved by a predetermined reduction. Computing the solution of the resulting reduced system is done in two steps: factorization of the matrix into some easily invertible form and a solution procedure that exploits this factorization. In general, factorization usually takes up to 90 % of the total CPU time needed to solve an optimization program. HOPDM tries to reduce the number of factorizations at the expense of some extra time per iteration. The idea behind MCSC is that although the theory requires that iterates remain in the neighborhood of the central path, in practical computing experience, they may stay quite far away from it without adverse consequences on the ability to take long steps. What really prevents a primal dual algorithm from taking long steps is a large discrepancy between complementary products $x_j z_j$ and $s_j w_j$. HOPDM looks for a corrector step $\triangle c$ that allows for longer step sizes in the composite direction $\triangle t = \triangle p + \triangle c$, where $\triangle p$ is the predictor step. The corrector step direction is chosen as the one that minimizes the diferences among the complementary products. The number of times that the corrector step is repeatedly applied depends on the effort needed in each factorization.

3.4. LIPSOL

LIPSOL has been developed by Zhang [22], and implements another version of Mehrotra's algorithm. LIPSOL is a Matlab-based software package for solving linear programs [23]. It uses Matlab's sparse-matrix data-structure and MEX external interface facility. At the same time, LIPSOL takes advantage of existing efficient Fortran codes for solving large, sparse, symmetric positive-definite linear systems: a sparse Cholesky factorization package by Esmond Ng and Barry Peyton [21] and a multiple minimum-degree ordering package by Joseph Liu [24]. Matlab's high level programming enviroment makes the code more simple and versatile than codes in Fortran or C.

3.5. IPA1

IPA1 [25] is yet another Mehrotra's predictor-corrector algorithm that is programmed in Matlab by the authors. It handles variables with and without upper bounds. IPA1 solves the normal-equation system by a symmetric minimum-degree reordering and a Cholesky factorization; it uses Matlab's sparse-matrix data-structure.

3.6. CPLEX-Logbarrier

CPLEX-Logbarrier [1] is a solver option of the commercial package CPLEX. It is the latest version of Lustig's code OB1. It is also based on Mehrotra's algorithm. CPLEX-Logbarrier solves linear and quadratic problems.

Depending on the normal-equation structure, two possible reorderings can be chosen by the user to reduce the number of non-zeros in the Cholesky triangular matrix. The resulting system is then solved by Cholesky factorization.

Table 1: PRO1 Solution Times

CODE	PRO1 n=1296, m=819, nonzeros=3120	
	CPU s	Iterations
PCx	4.653	9
LOQO	6.719	12
HOPDM	3.647	9
LIPSOL	20.342	11
IPA1	> 25	11
CPLEX-BAROPT	4.23	9
CPLEX-OPT	4.82	1041
CPLEX-TRANOPT	2.92	435
CPLEX-NETOPT	4.18	599

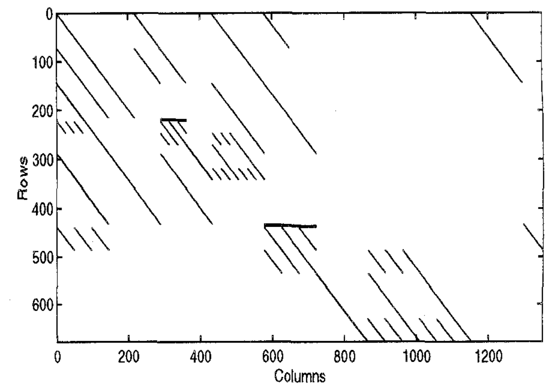

Figure 1: Structure of the restriction matrix of PRO1.

4. RESULTS

In this section we compare the performance of the codes described above when solving two relaxed hydro-thermal coordination problems. One of these is a small-medium size problem called PRO1; the other is a realistic size hydro-thermal coordination problem based on the electric energy system of mainland Spain, named PRO2. The solution times reported refers to a SUN SparcStation 2 with 64 MB of RAM. The optimality criterion tolerance is set to 10^{-8} for all the codes and for both problems.

LIPSOL and IPA1 are programmed in Matlab. This means that the codes are simpler, more versatile and easier to understand but at the same time slower than the rest of the codes. For this reason, their execution CPU-times haven't been considered when comparing the codes.

4.1. PRO1

The small-medium size hydro-thermal coordination problem PRO1 has the structure shown in Figure 1 where a blank means a zero element and a dot stands for a non-zero element. The problem objective is to optimize the operation of three thermal plants and three coupled hydro units over 48 time periods. The system is modelled according to Section 2, with the simplification that only one block is considered in the total-production-cost function of the thermal plants, and only one block is considered in the hydro-power/discharge function of the hydro plants. This formulation leads to a minimization problem with 819 constraints and 1296 variables; there are 3120 nonzero elements in the constraint matrix. The vector of variables X is $col[v, y, z, p, t, x, u, s, s_1, s_2]$, where s_1 are the slack-variable vector for the minimum output-power constraints, i.e., Constraints (4), and s_2 is the slack-variable vector for the spinning reserve margin constraints and energy constraints imposed on the thermal plants, i.e., Constraints (23) and (8-9); the meaning of the remaining of the variable vectors can be found in Section 2.

The performance of the various codes is shown in Table 1. According to the execution times shown in this table, all codes perform quite evenly for this small-medium system; the same performance is observed with other small-medium size problems. The fastest algorithm for PRO1 is the TRANOPT option of CPLEX (56 % faster than the slowest code which is LOQO); the option TRANOPT runs a simplex algorithm on the dual of the original problem.

4.2. PRO2

The problem objective of PRO2 is to optimize the operation of the Spanish hydro-thermal generating system over 42 time periods. The thermal subsystem consists of 30 thermal plants; the hydro subsystem is composed of 29 coupled hydro plants of the Sil river; the rest of the Spanish hydro subsystem is aggregated into another hydro plant. Once the binary variables are relaxed, the hydro-thermal coordination problem results in an LP problem formulation with 7830 constraints and 13500 variables, and 37538 nonzero elements. The structure of PRO2 is shown in Figure 2.

The demand (-), and the optimal thermal (-o) and hydro production (-*) are shown in Figure 3. The computer results show that the optimal hydro production flattens the demand curve, absorving all the changes in demand. This hydro generation pattern prevents the thermal plants from regulating the load, which is expensive and may cause damage to the boilers.

The restriction matrix of PRO2 does not apparently have dense columns since *(number of non-zeros / number of rows)* > 0.05 . The structure of PRO2 favours interior-point codes that reduce the FONCO equations to the normal-equation system as only few fill-ins may occur. In fact, our results show that LOQO is the slowest of the interior-point codes tested since LOQO does not use the normal-equation. The fastest code is CPLEX-logbarrier which is 45 % faster than LOQO; the second fastest is HOPDM with a speedup of 42 % with respect to LOQO. All interior-point codes are faster than the simplex-based methods for PRO2. Results are shown in Table 2.

We also solve the thermal and the hydro subproblems of PRO2 independently. The interior-point solution time for each subsystem is about half the time needed for the complete solution. However, the simplex based algorithms spend approximately one tenth of the complete solution time for

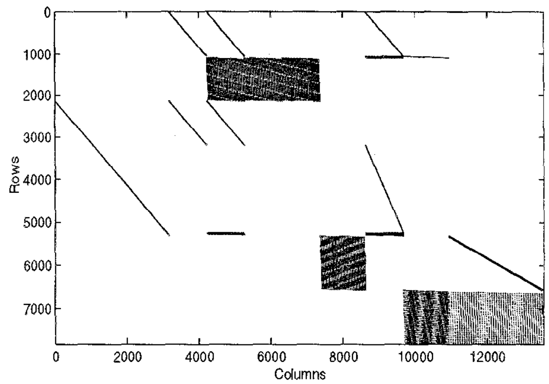

Figure 2: Structure of the restriction matrix of PRO2.

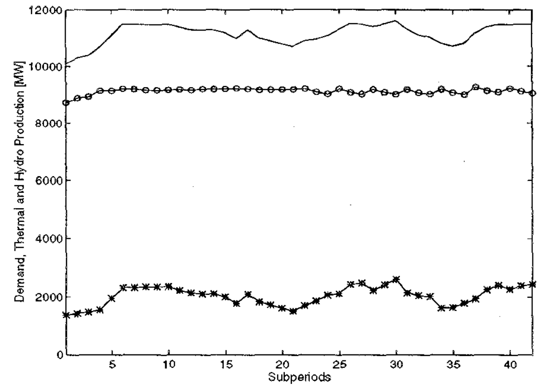

Figure 3: Load, Thermal and Hydro Production.

Table 2: PRO2 Solution Times

	PRO2 n=13500, m=7830, nonzeros=37538	
CODE	CPU s	Iterations
PCx	233.89	39
LOQO	312.73	35
HOPDM	180.38	36
LIPSOL	653.77	34
IPA1	> 1000	> 30
CPLEX-BAROPT	169.55	33
CPLEX-OPT	424.63	1197
CPLEX-TRANOPT	535.86	5553
CPLEX-NETOPT	564.43	7131

each subsystem; this performance confirms, once more, the well known fact that simplex solution times grow faster with problem size as compared to interior-point techniques.

Taking into account that a simplex decomposition-based solution procedure requires, on the average, at least three iterations on both subsystems, the total CPU-times required by simplex decomposition-based codes is larger than the time needed by one-step interior-point techniques. Also, dealing with global constraints and related convergence problems is avoided by using one-step interior-point methods. Interior-point methods can solve larger problems avoiding decomposition.

CPLEX-logbarrier is the only code that includes a post-processor that obtains all the basis information. The solution times on the tables do not show the time needed by CPLEX-logbarrier postprocessor in order to make a fair comparison of the codes. CPLEX-BARRIER postsolve needed 13 % of the whole CPU-time required to obtain a basic solution for PRO1, and 16.07 % for PRO2.

CPLEX-logbarrier needs less iterations and less time to solve PRO1 when the presolve option is off. However, for PRO2, the solution time is faster (8.76 %) with the presolve option on; the number of iterations is the same.

The proposed heuristic for obtaining the thermal schedule, deteriorates the value of the LP objective function by less than 0.5 %. We have compared our heuristic technique for handling integer variables to the CPLEX Mixed Integer Solver. The minimum up time and minimum down time constraints of the MTHTC problem are not included in the comparison because they lead to non-linear constraints that cannot be handled by the CPLEX. The CPLEX solution time is about 27.5 % slower than the time required to obtain the complete solution by our method.

It should also be considered that all the codes but the CPLEX are written for research purposes and are organized to be simple to understand by other researchers; they avoid programming techniques that make commercial codes faster but difficult to understand. Vanderbei [17] claims that up to 50 % of speedup can be obtained by standard programming techniques.

Based on the results presented above and on our experience, we cannot conclude that one code is better than the others. The structure of the problem proved to be a very important factor when choosing an algorithm. The available budget, and whether obtaining a basic solution is important or not, among other things, are also important issues to take into account when choosing an algorithm.

5. CONCLUSIONS

The hydro-thermal coordination problem is presented in detail in this paper. The main difficulties encountered by other means of solution are pointed out. The main features of four very recently developed research algorithms and CPLEX-logbarrier code are discussed. Their performance in finding a solution to the hydro-thermal coordination problem is compared to the performance of simplex-based methods. The solution to a realistic medium-term hydro-thermal coordination problem that is based on the mainland Spain power system is briefly presented. The results show that interior-point methods can solve large hydro-thermal coordination problems faster than simplex decomposition-based methods.

6. ACKNOWLEDGMENTS

The authors want thank the NSERC of Canada and the Spanish Ministry of Education and Science for the financial support.

REFERENCES

[1] *Using the Cplex Callable Library.* (Version 3.0) 1989-1994 CPLEX Optimization, Inc.

[2] J. Medina, A.Conejo, N. Jimenez, F. Pérez Thoden, J. González del Santo *An LP Based Decomposition Approach to Solve the Medium-Term Hydro-Thermal Coordination Problem.* 12th Power System Computation Conference (PSCC), Vol.1, pp.397-405, Dresden 1996.

[3] H. Brännlund, D. Sjelvgren and J. Bubenko. *Short Term Generation Scheduling with Security Constraints.* IEEE Transactions on Power Systems. Vol.1, No.3, pp.310-316. October 1988.

[4] T.S. Dillon, K.W. Edwin, H.D. Kochs and R.J. Tand. *Integer Programming Approach to the Problem of Optimal Unit Commitment with Probabilistic Reserve Determination.* IEEE Transactions on Power Apparatus and Systems. Vol.PAS-97, No.6, pp.2154-2166. November/December 1978.

[5] J. Batut and A. Renaud. *Daily Generation Scheduling Optimization with Transmission Constraints: An New Class of Algorithms.* IEEE Transactions on Power Systems. Vol. 7, No. 3, pp. 982-989. August 1992.

[6] H. Yan, P.B. Luh X. Guan and P.M. Rogan. *Scheduling of Hydrothermal Power Systems.* IEEE Transactions on Power Systems. Vol.8, No.3, pp.1135-1365. August 1993.

[7] V.M. Mendes, L.A.F.M. Ferreira, P. Roldão, R. Pestana. *Optimal Short-Term Scheduling in Large Hydrothermal Power Systems.* Proceeding of the Eleventh Power Systems Computation Conference (PSCC). Vol.II, pp.1297-1303. Avignon, France, August-September 1993.

[8] O. Nilson, D. Sjelvgren. *Mixed-Integer Programming Applied to Short-Term Planning of a Hydro-Thermal System.* IEEE Transactions on Power Systems. Vol.11, No.1, pp.281-286. February 1996.

[9] S. Ružić. *Lagrangian Relaxation Technique in Power Systems Operation Planning: Multipliers updating Problem.* Proceedings of the IEEE/KTH Stockholm Power Tech Conference, pp.61-66. Stockholm, Sweden, June 1995.

[10] I.J. Lustig, R.E. Marsten and D.F. Shanno. *On Implementing Mehrotra's Predictor Corrector Interior Point Method for linear programming.* SIAM Journal on Optimization, Vol.2, No.3, pp.435-449. August 1992.

[11] I.J. Lustig, R.E. Marsten and D.F. Shanno. *Computational Experience with a Globally Convergent Primal-Dual Predictor-Corrector Algorithm for Linear Programming.* Mathematical Programming Vol.66, pp.123-135, 1994.

[12] N. Karmarkar. *A new polynomial-time Algorithm for Linear programming.* Combinatorica Vol.4, pp.373-395, 1984

[13] S. Mehrotra. *On the implementation of a (primal-dual) Interior Point Method.* SIAM Journal on Optimization Vol.2, No.4, pp.575-601, 1992.

[14] J. Gondzio and M. Makowski. *HOPDM: Modular Solver for LP Problems. User's Guide to version 2.12.* WP-95-50 IIASA, June 1995

[15] K.R. Frisch. *The Logarithmic Potential Method for Convex Programming.* Unpublished manuscript, Institute of Economics, University of Oslo, Norway, May 1955.

[16] J. Czyzyk, S. Mehrotra and S.J. Wright. *PCx User Guide.* Optimization Technology Center. Technical Report OTC 96/01, May 1996.

[17] R.J. Vanderbei. *LOQO: An Interior Point Code for Quadratic Programming.* Technical Report, Princeton University, 1996.

[18] M.Christoforodis, M.Aganagic, B.Awobamise, S.Tong, and A.F. Rahimi. *Long-Term/Mid-Term Resource Optimization of a Hydro-Dominant Power System Using Interior Point Method* IEEE Transactions on Power Systems, Vol.11, No.1, pp. 287-294, February 1996.

[19] Po-Hung Chen, and Hon-Chang. *Genetic Aided Scheduling of Hydraulically Coupled Plants in Hydro- Thermal Coordination.* IEEE Transactions on Power Systems Vol.11, No.2, pp.975-981, May 1996.

[20] X.Bai, and S.M.Shahidehpour. *Hydro-Thermal Scheduling by Tabu Search and Decomposition Method.* IEEE Transactions on Power Systems Vol.11, No.2, pp.968-974, May 1996.

[21] E.Ng, and B.W.Peyton. *Block Sparse Cholesky Algorithms on Advanced Uniprocessor Computers.* SIAM Journal on Scientific Computing, Vol.14, pp.1034-1056, 1993.

[22] Y. Zhang *User's Guide to LIPSOL Linear Programming Interior Point Solvers v0.3* Department of Mathematics and Statistics. University of Maryland Baltimore County. Baltimore, Maryland 21228-5398, U.S.A. July 1995.

[23] *Matlab User's Guide.* The MathWorks, Inc.

[24] J.W. Liu, E.G. Ng, and B.W. Peyton. *On finding Supernodes for Sparse Matrix Computations.* SIAM Journal on Matrix Analysis and Applications, Vol.1, pp.242-252, 1993.

[25] J. Medina, V.H. Quintana. *An Interior Point Based Method for Hydro-Thermal Coordination.* Department of Electrical & Computer Engineering. University of Waterloo. Waterloo, Ontario. Technical Report UW E&CE 96-11. August 1996.

[26] K. Ponnambalam, V.H. Quintana, A. Vannelli. *A Fast Algorithm for Power System Optimization Problems Using an Interior Point Method.* IEEE Transactions on Power Systems, Vol.7, No.2, May 1992, pp.892-899.

[27] X. Yan, V.H. Quintana. *An Efficient Predictor-Corrector Interior Point Algorithm for Security Constrained Economic Dispatch.* 96 SM 506-6 PWRS. Presented at the 1996 IEEE/PES Summer Meeting, Denver 1996.

BIOGRAPHIES

J. Medina (IEEE Student M) is persuing his Ph.D. degree in Electrical Engineering from the Universidad Pontificia Comillas, Madrid, Spain. Currently he is a Visiting Scholar at the University of Waterloo, Waterloo, Canada.

V.H. Quintana (IEEE SM'73) recieved the Dipl. Ing. degree from the State Technical University of Chile in 1959, and the M.Sc. and Ph.D. degrees in Electrical Engineering from the University of Wiscosin, Madison in 1965, and the University of Toronto, Ontario in 1970, respectively. Since 1973 he has been with the University of Waterloo, Dpt. of Electrical & Computer Engineering, where he is a full professor. Dr. Quintana is an Associate Editor of the International Journal of Energy System, a member of the association of Professional Engineers of the Province of Ontario and a member of the NSERC of Canada.

A. J. Conejo (IEEE M) received the Ingeniero Industrial Eléctrico degree from the Universidad Pontificia Comillas, Madrid, Spain in 1983; the M.Sc. and Ph.D. degrees in Electrical Engineering from the MIT, Massachusetts, in 1987, and the Royal Institute of Technology, Stockholm, Sweden in 1990, respectively. He is currently an associate professor at the Universidad de Málaga, Málaga, Spain.

F. Pérez Thoden received the Ingeniero Industrial Eléctrico from the Universidad Pontificia Comillas, Madrid, Spain in 1983. He is currently working for ENDESA, Madrid, Spain, on the regulation of electric power systems.