Incremental Transmission Loss Allocation Under Pool Dispatch

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***Abstract*—Incremental transmission loss analysis has been used for decades, but recent interest in its application to loss allocation calls for new in-depth results. This paper demonstrates that, for incremental methods to be applied correctly in loss allocation, it is first necessary to specify the load distribution and loss supply strategies. Incremental loss allocation among bus power injections is shown to be arbitrary and, therefore, open to challenge as discriminatory. Loss allocation is possible among incremental loads and/or generators, but the proportion of the total losses assigned to either one is arbitrary. Unique, nonarbitrary incremental loss allocations are however possible among the "equivalent" incremental bilateral exchanges between generators and loads. From these basic components it is possible then to calculate the allocation among generators or loads in any specified proportion. The main results, although developed initially for small increments, are extended to large variations. Finally, a general incremental loss allocation algorithm is developed and tested.**

***Index Terms*—Equivalent bilateral contracts, incremental transmission loss analysis, loss allocation, loss supply, numerical algorithm, pool dispatch.**

I. Introduction

LOSS allocation is a procedure for subdividing the system transmission losses into fractions, the costs of which then become the responsibility of individual users of the power system (gencos, discos, marketers). Loss allocation does not affect generation levels or power flows, however it does modify the distribution of revenues and payments at the network buses among suppliers and consumers. This step is necessary whenever, for reasons of computational simplicity, the base generation dispatch and its clearing price are calculated through a suboptimal merit-order scheme that initially neglects transmission losses [1], [2]. The actual generation of losses is then taken care of by a pre-determined loss supply strategy, usually in the form of a distributed "slack."

The loss allocation schemes developed thus far can be categorized into incremental [3], [4], circuit-based [5], proportional-sharing [6], [7], pro-rata [8], and miscellaneous approaches for bilateral transactions [10]–[12]. In this paper, particular attention is devoted to incremental methods, already in use in Norway [13] or under consideration in the U.K. [14] and Spain [15].

Interest in incremental loss allocation is growing, in part, because it is based on the well-established incremental transmission loss analysis, an approach known to power engineers for at least three decades [16], [17]. Its implementation, moreover, uses algorithms and data consistent with those of the conventional load flow.

The essence of loss allocation by incremental methods is to expand the system losses through a first order sensitivity with respect to the nodal real power generators and loads. Each term in the linear expansion then defines that fraction of the incremental losses allocated to the corresponding incremental generation or load. Although familiar and straightforward, this paper demonstrates that unless correctly defined, such methods can lead to arbitrary loss allocations that can always be challenged as discriminatory. Given the current strong interest in incremental loss allocation schemes, an in-depth investigation of this subject was carried out with the following results.

- The incremental allocation of losses among power *injections* is shown to be *arbitrary* and its use, therefore, is arguably discriminatory.
- The incremental allocation of losses among *equivalent bilateral exchanges* between generators and loads is shown to be *unique* and, therefore, the basis for a valid loss allocation scheme.
- How the unique incremental loss component allocated to a particular generator/load pair is split between the corresponding generator and load is however *not* unique. It could be 50/50, but other arbitrary proportions are also possible (e.g., all incremental losses allocated to the loads, or all to the generators).
- A general purpose incremental loss allocation algorithm is developed and tested for both small and large variations in the operating point.

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II. Distinction Between Loss Supply and Loss Allocation

We begin by pointing out the important difference in the notions of loss supply and loss allocation. Loss supply is the mechanism by which the system losses are generated when these are not accounted for by the original suboptimum dispatch (e.g., merit-order). The loss supply service, whether performed by one generator (the slack) or by a group of generators (distributed slack), directly affects generation levels, power flows and the system losses themselves. Loss allocation, on the other hand, does not directly affect any network variables. The process merely takes the system losses from a power flow solution and divides them in some "fair" fashion among the loads and/or generators for purposes of revenue and payment reconciliation. Thus, a generator that, according to the loss supply scheme, does not generate losses will still be allocated a loss component for which the pool must be compensated. Similarly, a slack generator, even though producing all or part of the losses, will nevertheless also be allocated responsibility for a fraction of the system losses.

An interesting observation is that, in optimally dispatched systems, the loss supply and loss allocation strategies are not required. Each power generation supplies just the right amount of demand and losses to maximize global welfare. Similarly, under optimal power flow, the nodal prices paid by the loads already account for transmission losses and congestion, and no additional loss allocation responsibility need be assigned to the loads.

III. Non-Uniqueness of Loss Allocation Among Incremental Power Injections

The major difficulty in loss allocation is that, for large changes in the operating point, the process is *always* arbitrary. This is so because the system loss is a nonseparable, nonlinear function of the real power generations and loads. One justification for using incremental loss allocation methods is that, for small increments, the linear relation between the incremental system losses and the incremental generation and/or load levels is separable.

Unfortunately, not every linear incremental loss model yields a unique set of separable components. To demonstrate this statement, consider first the expansion of the incremental system losses among the n power injections

$$dP_{\text{loss}} = \sum_{i=1}^{n} \text{ITL}_{i/s} dP_i \tag{1}$$

where $\text{ITL}_{i/s}$ is the sensitivity of the system losses with respect to the power injection P_i holding all power injections constant except at the slack bus, s, that is,

$$\text{ITL}_{i/s} = \left.\frac{\partial P_{\text{loss}}}{\partial P_i}\right|_s. \tag{2}$$

The allocation process then divides the incremental losses in (1) into n components defined by $dL_i = \text{ITL}_{i/s}dP_i; i = 1,\ldots,n$, each component associated with the corresponding incremental power injection. To demonstrate that this separation is arbitrary, consider the identity

$$dP_{\text{loss}} = \sum_{i=1}^{n} dP_i. \tag{3}$$

Let β be any scalar. Now, multiply (1) by 2β and (3) by $2(1-\beta)$, and add the two. Then

$$dP_{\text{loss}} = \sum_{i=1}^{n} \{\beta(\text{ITL}_i) + (1-\beta)\} dP_i. \tag{4}$$

As (4) is a valid linear expansion, the incremental loss component allocated to the power injection at bus i can also be given by

$$dL_i = \{\beta(\text{ITL}_i) + (1-\beta)\} dP_i. \tag{5}$$

By adjusting the free scalar β, the allocated loss components, dL_i, can be modified arbitrarily. Thus, if $\beta = 1$, the slack bus injection at bus s, receives zero loss allocation (since $\text{ITL}_{s/s} = 0$). If, however, $\beta = [1 - \text{ITL}_{r/s}]^{-1}$, then bus r is allocated zero losses.

It is therefore concluded that *the incremental losses cannot be uniquely decomposed among the bus power injections* according to the method of incremental loss analysis. Non-incremental "one-shot" methods have been developed that, attempt to separate the system losses among the injections [5]–[7]. "One shot" methods, however, all contain a degree of arbitrariness and each yields a different allocation.

IV. Distributed Slack Load Flow

Under merit-order dispatch, the pool calculates a set of load distribution factors, $m_i;\ i = 1,\ldots,n$, such that $m_i \geq 0;\ \sum_{i=1}^{n} m_i = 1$. The quantity m_i represents the fraction of the system load supplied by generator i according to the merit-order scheme. Since the m_i coefficients do not account for transmission losses, the pool also specifies a set of loss-supply factors, $\rho_i;\ i = 1,\ldots,n$, assigning to each generator a fraction of the total losses that it must produce to establish a power balance. The loss-supply factors also satisfy the condition, $\rho_i \geq 0;\ \sum_{i=1}^{n} \rho_i = 1$. Note that if there is only one slack generator at bus s, then the losses are supplied only by bus s, and all ρ_i's are set to zero except at bus s where $\rho_s = 1$. In this paper it is assumed that both vectors $\mathbf{m}$ and $\boldsymbol{\rho}$ are known and specified by the pool.

From the above discussion, a slightly more general form of the load flow equations can be formulated. We begin with the basic power flow balance[1],

$$\mathbf{P}_g - \mathbf{P}_d = \mathbf{P}(\boldsymbol{\delta}) \tag{6}$$

however, here the generation vector has the form,

$$\mathbf{P}_g = \mathbf{m}P_d^{\text{sys}} + \boldsymbol{\rho}P_{\text{loss}}. \tag{7}$$

In (6) and (7), $\mathbf{P}_g$ and $\mathbf{P}_d$ are, respectively, the vectors of the n bus generations and demands, $\mathbf{P}(\boldsymbol{\delta})$ is the vector of n real power injections expressed in terms of the vector of the $n-1$ phase angles, $\boldsymbol{\delta}, P_d^{\text{sys}} = \sum_{j=1}^{n} P_{dj}$, while P_{loss} represents the unknown system losses.

The solution of (6) and (7) can be obtained via a simple extension of any standard load flow [18]. The key difference is that a slack generation need not be defined when solving the distributed slack load flow. Instead, all n real power equations are used to find the n unknowns, namely, $\boldsymbol{\delta}$ and P_{loss}. Equations (6) and (7) thus model the network behavior under merit-order dispatch with a specified loss supply scheme.

[1]To simplify the presentation, all bus voltage magnitudes are assumed known. The real power injections therefore depend only on the bus voltage phase angles, δ.

A. Incremental Distributed Slack Load Flow

Assuming now that the load vector varies by a small increment, $d\mathbf{P}_d$, resulting in corresponding changes in all dependent variables, $d\boldsymbol{\delta}$, $d\mathbf{P}_g$, and dP_{loss}, then from (6) and (7) these increments must satisfy the incremental distributed slack load flow

$$\mathbf{m}dP_d^{\text{sys}} + \boldsymbol{\rho}dP_{\text{loss}} - d\mathbf{P}_d = \frac{\partial \mathbf{P}(\boldsymbol{\delta})}{\partial \boldsymbol{\delta}} d\boldsymbol{\delta}. \tag{8}$$

B. Incremental Power Balance Equation

We first recall some fundamental results relating to the incremental behavior of the power flow equations [4]. Note that the rank of the n by $n-1$ Jacobian matrix, $(\partial \mathbf{P}(\boldsymbol{\delta}))/(\partial \boldsymbol{\delta})$, is in general $n-1$, where n is the number of buses. Thus, the solution of (9) yields a single nonzero vector, $\boldsymbol{\alpha}$, of dimension n where

$$\left[\frac{d\mathbf{P}(\boldsymbol{\delta})}{d\boldsymbol{\delta}}\right]^T \boldsymbol{\alpha} = 0. \tag{9}$$

From (9), it is clear that α can be determined within a proportionality factor only, however, as will be seen, this arbitrary factor does not affect the loss allocation schemes derived below. In order to determine $\boldsymbol{\alpha}$ for a given operating point, $\boldsymbol{\delta}$, one can solve (9) for the null space of the transpose Jacobian or one can make use of the classical ITL coefficients [4], [16], that is,

$$\alpha_i = 1 - \text{ITL}_{i/s} \tag{10}$$

where $\text{ITL}_{i/s}$ is defined by (2). Just as the proportionality constant in $\boldsymbol{\alpha}$ does not affect the final loss allocation, the arbitrariness in (10) due to the choice of the slack bus, s, also does not influence the final loss allocation [4].

By multiplying both sides of the load flow equations (8) by $\boldsymbol{\alpha}^T$ and using (9), the incremental power balance equation can be obtained

$$(\alpha^T \mathbf{m})dP_d^{\text{sys}} + (\alpha^T \rho)dP_{\text{loss}} - \alpha^T d\mathbf{P}_d = 0. \tag{11}$$

From (11), the incremental losses can now be found in terms of the incremental loads only

$$dP_{\text{loss}} = \frac{\boldsymbol{\alpha}^T d\mathbf{P}_d - (\boldsymbol{\alpha}^T \mathbf{m})dP_d^{\text{sys}}}{\boldsymbol{\alpha}^T \boldsymbol{\rho}}. \tag{12}$$

V. Incremental Loss Allocation

A. Incremental Loss Allocation Among Bus Demands

Combining (12) with $d\mathbf{P}_d = [dP_{d1}, \ldots, dP_{dn}]$ and $dP_d^{\text{sys}} = \sum_{j=1}^n dP_{dj}$, an expression can be found for the incremental system losses consisting of the sum of n terms, each uniquely dependent on a bus load increment, dP_{dj}

$$dP_{\text{loss}} = \sum_{j=1}^n \left[\frac{\alpha_j - \boldsymbol{\alpha}^T \mathbf{m}}{\boldsymbol{\alpha}^T \boldsymbol{\rho}}\right] dP_{dj}. \tag{13}$$

From (13) one can therefore assign to each incremental load the following unique incremental loss component

$$dL_{dj} = \left[\frac{\alpha_j - \boldsymbol{\alpha}^T \mathbf{m}}{\boldsymbol{\alpha}^T \boldsymbol{\rho}}\right] dP_{dj}. \tag{14}$$

The term $(\alpha_j - \boldsymbol{\alpha}^T \mathbf{m})/(\boldsymbol{\alpha}^T \boldsymbol{\rho}) = (\partial P_{\text{loss}})/(\partial P_{dj})$ describes the sensitivity of the system losses with respect to a change in the load at bus j, given that the load increments and corresponding losses are supplied according to the incremental load flow (8). Note that in (14) the arbitrary proportionality constant in the vector α is present in both numerator and denominator and therefore cancels out. A two-bus example is presented in Appendix A to illustrate the derivation process of incremental loss allocation components among loads.

It is clear from (13) and (14) that the incremental losses allocated to the bus loads add up to the total system incremental losses, dP_{loss}. Thus, this particular loss allocation scheme assigns all the losses to the loads and nothing to the generators. This type of allocation is therefore arbitrary in the sense that the loads are given sole responsibility for the losses. What we wish to derive next is an allocation strategy among generators and loads that is unique without requiring that a percentage of the losses be *a priori* assigned to one or the other participant.

B. Unique Allocation Among Equivalent Bilateral Power Exchanges

We begin by recalling that the participation factors add up to one, $\sum_{i=1}^n m_i = 1$, so that the vector of incremental loads can be written as

$$d\mathbf{P}_d = \left[\sum_{i=1}^n m_i\right] d\mathbf{P}_d = \sum_{i=1}^n [m_i d\mathbf{P}_d]. \tag{15}$$

Moreover, $d\mathbf{P}_d$ can be divided into the sum of n vectors

$$d\mathbf{P}_d = \sum_{j=1}^n \mathbf{e}_j dP_{dj} \tag{16}$$

where $\mathbf{e}_j$ is a unit vector containing a one in location j and zeroes elsewhere. Thus, combining (15) and (16)

$$d\mathbf{P}_d = \sum_{i=1}^n m_i \left\{\sum_{j=1}^n \mathbf{e}_j dP_{dj}\right\} = \sum_{i=1}^n \sum_{j=1}^n \mathbf{e}_j [m_i \, dP_{dj}]. \tag{17}$$

The *scalar* $m_i dP_{dj}$ *can be interpreted as the increment in the component of bus load* j *supplied by generator* i. There exist n^2 such equivalent exchanges between all generators and all loads. In order to express the incremental losses in terms of these equivalent bilateral exchanges, recall the incremental power flow, including the loss and load supply strategies

$$\mathbf{m}dP_d^{\text{sys}} + \boldsymbol{\rho}dP_{\text{loss}} - d\mathbf{P}_d = \frac{\partial \mathbf{P}(\boldsymbol{\delta})}{\partial \boldsymbol{\delta}} d\boldsymbol{\delta}.$$

Since $dP_d^{\text{sys}} = \sum_{j=1}^n dP_{dj}$ and $\mathbf{m} = \sum_{i=1}^n m_i \mathbf{e}_i$, using (17), we can rewrite (8) as

$$\sum_{i,j=1}^n [m_i P_{dj} \mathbf{e}_i] + \rho dP_{\text{loss}} - \sum_{i,j=1}^n [m_i P_{dj} \mathbf{e}_j] = \frac{\partial \mathbf{P}(\delta)}{\partial \delta} d\delta. \tag{18}$$

Then, combining (9) and (18)

$$\sum_{i,j=1}^n [m_i dP_{dj} \alpha_i] + (\boldsymbol{\alpha}^T \boldsymbol{\rho})dP_{\text{loss}} - \sum_{i,j=1}^n [m_i dP_{dj} \alpha_j] = 0 \tag{19}$$

which allows us to expand the incremental losses as a *unique* linear combination of n^2 components, each attributed to an equivalent bilateral exchange $m_i dP_{dj}$

$$dP_{\text{loss}} = \sum_{i,j=1}^{n} \left[\left\{ \frac{\alpha_j - \alpha_i}{\boldsymbol{\alpha}^T \boldsymbol{\rho}} \right\} m_i dP_{dj} \right]. \tag{20}$$

From (20), we can identify the unique incremental loss component associated with the equivalent bilateral exchange $m_i dP_{dj}$, that is,

$$dL_{ij} = \frac{\alpha_j - \alpha_i}{\boldsymbol{\alpha}^T \boldsymbol{\rho}} m_i dP_{dj}. \tag{21}$$

C. Loss Allocation Among Loads or Generators From dL_{ij}

The unique equivalent bilateral allocation terms, dL_{ij}, add up to total incremental losses, that is, $dP_{\text{loss}} = \sum_{i=1}^{n} \sum_{j=1}^{n} dL_{ij}$. This double sum can also be separated among the generators only (index i) or among the loads only (index j). If all the losses are allocated to the generators, then the component assigned to generator i, defined here as dL_{gi}, is found by summing the loss components corresponding to the equivalent exchanges between generator i and all loads $j = 1, \ldots, n$. Thus,

$$dL_{gi} = \sum_{j=1}^{n} dL_{ij} = \sum_{j=1}^{n} \left[\frac{\alpha_j - \alpha_i}{\alpha^T \rho} \right] m_i dP_{dj}. \tag{22}$$

Similarly, if the losses are assigned to the loads only, to determine the incremental loss allocated to load j, defined as dL_{dj}, one adds up all the loss components corresponding to the equivalent exchanges between load j and all generators $i = 1, \ldots, n$. Thus,

$$dL_{dj} = \sum_{i=1}^{n} dL_{ij} = \sum_{i=1}^{n} \left[\frac{\alpha_j - \alpha_i}{\boldsymbol{\alpha}^T \boldsymbol{\rho}} \right] m_i dP_{dj}. \tag{23}$$

It is relatively straightforward to show that (23) and (14) are identical.

It would also be possible to divide the total losses in some a priori proportion among the loads and generators. For example, if the proportion is $50/50$, the allocations in (22) and (23) would each be cut in half.

Thus, *it is not possible to uniquely allocate incremental losses among generators and loads without a priori assigning a proportion of the total losses to each category. It is however possible to assign a unique incremental loss allocation to the equivalent bilateral exchanges between generators and loads*.

VI. Loss Allocation for Large Increments

Assume that it is desired to calculate and allocate losses for a large change in a given load vector, $\mathbf{P}_d$, where the generation is given by the dispatch strategy defined by the vectors $\mathbf{m}$ and $\boldsymbol{\rho}$. If the load vector varies with time in a known fashion, $\mathbf{P}_d(t)$, then the change in loss allocation over a given load trajectory, $\{\mathbf{P}_d(t); t \in [t_0, t_1]\}$, can be calculated by integrating the previously calculated incremental loss allocations over time. For example, from (23), the increment of loss allocated to the load at bus j is

$$\Delta L_{dj} = \int_{t=t_0}^{t_1} \left\{ \sum_{i=1}^{n} \left[\frac{\alpha_j(t) - \alpha_i(t)}{\boldsymbol{\alpha}^T(t) \boldsymbol{\rho}} \right] m_i(t) \frac{dP_{dj}(t)}{dt} \right\} dt. \tag{24}$$

A similar integral applies to the loss components allocated to the generators. Note that the load distribution vector, $\mathbf{m}(t)$, is assumed to be time dependent. This is so, since in general, the merit order dispatch may depend on the system load.

To solve (24) numerically, it is necessary to approximate the integral by the sum of a large number of steps, Δt, at each of which a new load flow and a new vector $\boldsymbol{\alpha}(t)$ must be calculated. This approach yields the "exact" change in loss allocation but is computationally demanding. When a small number of integration steps is taken, the loss allocation components are only approximate and do not sum up to the actual system losses. To correct this imbalance, it is necessary to scale the approximate loss allocations so that their scaled sum matches the exact losses.

Note also that the integral in (24) provides only a variation, and not the absolute loss allocation. To obtain an absolute result, it is necessary to start the integration at a point where the allocation of losses is already known. In the absence of such initial information, one can assume the existence of a linear and uniform load trajectory starting at zero loads. Clearly, when the loads are zero, so are the losses and the allocations. Thus, if a vector of loads, $\mathbf{P}_d$, is known but not its time trajectory, to obtain the allocation of losses using the above integration procedure, the trajectory assumed is, $\{\mathbf{P}_d(t) = t\mathbf{P}_d;\ 0 \leq t \leq 1\}$, at the end of which the load vector is equal to the specified level.

A. Successful Approximate Large Step Loss Allocation

Experimental results have demonstrated that accurate and fast loss allocations can be obtained by approximating the loss allocation integrals by as few as one single integration step over the whole trajectory. The following steps are required.
Let $\hat{L}_{dj}$ be the approximate loss allocated to load j after one or several large integration steps, Δt. Let P_{loss} be the *exact losses calculated via a load flow* at the final specified load levels. Then, as the sum of the $\hat{L}_{dj}$ does not equal P_{loss}, one can define a new scaled loss allocation component for load j

$$L_{dj}^{\%} = \frac{\hat{L}_{dj}}{\sum_{k=1}^{n} \hat{L}_{dk}} P_{\text{loss}}. \tag{25}$$

Experimental results indicate that, for constant values of vectors $\mathbf{m}$ and $\boldsymbol{\rho}$ along the whole integration path, the percent allocation coefficients $L_{dj}^{\%} = 100(\hat{L}_{dj})/(\sum_{k=1}^{n} \hat{L}_{dk})$ are practically independent of the number of integration steps. This means that with as few as a single integration step, the scaled loss allocation terms, $L_{dj}^{\%}$, are almost indistinguishable from the exact values L_{dj} defined by (24). As shown in the results, if the vector m varies with system demand, then the error in the approximate allocations worsens, but remains within acceptable bounds.

B. Uncertainty in the Vectors $\mathbf{m}$ and $\boldsymbol{\rho}$

An allocation problem where only a load flow solution is provided, with no knowledge of either how the losses are supplied

TABLE I
COMPARISON OF PERCENTAGE VALUES OF LOSS ALLOCATION COEFFICIENTS FOR VARIOUS NUMBER OF INTEGRATION STEPS WITH CONSTANT m AND ρ

					Number of integration steps					
					1		10		100	
		Losses (pu)			0.068		0.068		0.068	
		Estimated Losses (pu)			0.140		0.075		0.069	
Bus Num.	Power gen. (MW)	Power demand (MW)	m	ρ	$L_g^{\%}$ (%)	$L_d^{\%}$ (%)	$L_g^{\%}$ (%)	$L_d^{\%}$ (%)	$L_g^{\%}$ (%)	$L_d^{\%}$ (%)
1	1.26	0.00	0.46	1	67.0	0.0	67.0	0.0	67.0	0.0
2	0.4	0.22	0.15	0	12.4	1.7	12.4	1.7	12.4	1.7
3	0	0.94	0.00	0	0.0	57.3	0.0	57.1	0.0	57.1
4	0	0.48	0.00	0	0.0	11.5	0.0	11.6	0.0	11.6
5	0	0.08	0.00	0	0.0	1.6	0.0	1.6	0.0	1.6
6	0	0.11	0.00	0	0.0	3.6	0.0	3.6	0.0	3.6
7	0	0.00	0.00	0	0.0	0.0	0.0	0.0	0.0	0.0
8	1	0.00	0.39	0	20.7	0.0	20.6	0.0	20.6	0.0
9	0	0.30	0.00	0	0.0	4.4	0.0	4.4	0.0	4.4
10	0	0.09	0.00	0	0.0	2.0	0.0	2.0	0.0	2.0
11	0	0.04	0.00	0	0.0	1.0	0.0	1.0	0.0	1.0
12	0	0.06	0.00	0	0.0	2.9	0.0	2.9	0.0	2.9
13	0	0.14	0.00	0	0.0	6.6	0.0	6.6	0.0	6.6
14	0	0.15	0.00	0	0.0	7.4	0.0	7.4	0.0	7.4
Sum	2.66	2.59	1.00	1	100.0	100.0	100.0	100.0	100.0	100.0

TABLE II
PERCENT LOSS ALLOCATION COEFFICIENTS FOR VARIOUS LOSS SUPPLY STRATEGIES

			Number of integration steps =1								
	Losses (pu)		0.0659			0.0651			0.0665		
Bus Num	Power demand (MW)	m	ρ	P_g (pu)	$L_g^{\%}$ (%)	ρ	P_g (pu)	$L_g^{\%}$ (%)	ρ	P_g (pu)	$L_g^{\%}$ (%)
1	0.00	0.46	0	1.19	66.4	0	1.19	65.6	0.46	1.22	66.3
2	0.22	0.15	1	0.47	12.7	0	0.40	12.2	0.15	0.41	12.4
3	0.94	0.00	0	0.00	0.0	0	0.00	0.0	0.00	0.00	0.0
4	0.48	0.00	0	0.00	0.0	0	0.00	0.0	0.00	0.00	0.0
5	0.08	0.00	0	0.00	0.0	0	0.00	0.0	0.00	0.00	0.0
6	0.11	0.00	0	0.00	0.0	0	0.00	0.0	0.00	0.00	0.0
7	0.00	0.00	0	0.00	0.0	0	0.00	0.0	0.00	0.00	0.0
8	0.00	0.39	0	1.00	20.9	1	1.07	22.2	0.39	1.03	21.3
9	0.30	0.00	0	0.00	0.0	0	0.00	0.0	0.00	0.00	0.0
10	0.09	0.00	0	0.00	0.0	0	0.00	0.0	0.00	0.00	0.0
11	0.04	0.00	0	0.00	0.0	0	0.00	0.0	0.00	0.00	0.0
12	0.06	0.00	0	0.00	0.0	0	0.00	0.0	0.00	0.00	0.0
13	0.14	0.00	0	0.00	0.0	0	0.00	0.0	0.00	0.00	0.0
14	0.15	0.00	0	0.00	0.0	0	0.00	0.0	0.00	0.00	0.0
sum	2.59	1.00	1.0	2.66	100.0	1.0	2.66	100.0	1.00	2.66	100.0

or how the load is distributed among the generators, cannot be solved by incremental methods. In such cases, "one-shot" allocation schemes are the only alternative. To use incremental methods in loss allocation, as a minimum, one should know the load flow solution together with either the loss supply strategy, $\boldsymbol{\rho}$, or the load distribution vector, $\mathbf{m}$. Knowing one of the two vectors then allows us to estimate the other. This estimation is as follows: Suppose that the load flow solution is known including the vector of generations, $\mathbf{P}_g$, the system losses, P_{loss}, and the total demand, P_d^{sys}. If, in addition, the loss supply vector, $\boldsymbol{\rho}$, is known, then since $\mathbf{P}_g = \mathbf{m}P_d^{\text{sys}} + \boldsymbol{\rho}P_{\text{loss}}$, one can estimate the load distribution vector as, $\mathbf{m} = (\mathbf{P}_g - \boldsymbol{\rho}P_{\text{loss}})/P_d^{\text{sys}}$. Alternatively, if only $\mathbf{m}$ is known, the loss supply vector can be estimated from, $\boldsymbol{\rho} = (\mathbf{P}_g - \mathbf{m}P_d^{\text{sys}})/P_{\text{loss}}$. Both vector estimates have to be normalized in order to add up to one.

C. Dependence of $\mathbf{m}$ Vector on System Load

During the process of integration (24) from zero to the final load values, it is possible for the merit order load distribution vector, $\mathbf{m}$, to vary with the level of system load as new generators are dispatched and others reach their limit. It is not reasonable then to assume that the final value of $\mathbf{m}$ is valid over the entire load trajectory. In such cases, the integration process must be broken up into segments inside of which the vector $\mathbf{m}$ is constant. These segments are defined by the merit order dispatch following well-known methods [16].

TABLE III
COMPARISON OF PERCENT VALUES OF LOSS ALLOCATION COEFFICIENTS FOR VARIOUS NUMBER OF INTEGRATION STEPS UNDER A "SEGMENTED" INTEGRATION PATH

							Integration steps per segment		
							1	10	100
			Losses (pu)				0.068	0.068	0.068
Bus Num.	P_g (pu)	P_d (pu)	m P_d<1	m 1<P_d<1.3	m P_d>1.3	ρ	$L_d^\%$ (%)	$L_d^\%$ (%)	$L_d^\%$ (%)
1	1.26	0	1	0.55	0.02	1	0.0	0.0	0.0
2	0.40	0.22	0	0.45	0.21	0	1.6	0.1	-0.2
3	0	0.94	0	0	0	0	53.2	54.2	54.4
4	0	0.48	0	0	0	0	13.5	13.8	13.8
5	0	0.08	0	0	0	0	1.7	1.5	1.5
6	0	0.11	0	0	0	0	3.3	2.9	2.9
7	0	0.00	0	0	0	0	0.0	0.0	0.0
8	1	0.00	0	0	0.77	0	0.0	0.0	0.0
9	0	0.30	0	0	0	0	6.6	7.0	7.1
10	0	0.09	0	0	0	0	2.5	2.6	2.6
11	0	0.04	0	0	0	0	1.1	1.1	1.1
12	0	0.06	0	0	0	0	2.6	2.5	2.5
13	0	0.14	0	0	0	0	6.2	6.2	6.2
14	0	0.15	0	0	0	0	7.5	8.0	8.1
sum	2.66	2.59				1	100.0	100.0	100.0

VII. CASE STUDIES

The incremental loss allocation method was tested on a number of networks. We present here the results for a slightly modified IEEE 14 bus network, with an additional generator at bus 8. The numerical data and one-line diagram of the study network are given in Appendix B.

Table I shows a comparison of the percent loss allocation coefficients calculated for one, ten, and 100 integration steps assuming constant $\mathbf{m}$ and $\boldsymbol{\rho}$ vectors. Two percent loss allocation coefficients are presented, $L_g^\%$, with the losses entirely allocated to the generators, and $L_d^\%$, with the losses entirely allocated to the loads. Also, in Table I, the loss supply vector, $\boldsymbol{\rho}$, is such that only the generator at bus 1 supplies system losses. Finally, the estimated (calculated through the integration process) and "exact" values (from load flow solution) of the system losses are shown. As Table I indicates, the differences between the exact and estimated values of system losses can be quite significant and highly depend on the number of integration steps. However, the percentage values of the loss allocation coefficients are *nearly independent of the number of steps*. This powerful result reveals that the proposed incremental method can be applied to large variations in operating point without significant loss of accuracy with only a single integration step and knowledge of the exact P_{loss}.

Table II presents the percent values of the loss allocation coefficients for three different strategies of loss supply

1) when bus 2 supplies the losses;
2) when bus 8 supplies the losses;
3) when the losses are distributed among all generators in proportion to their outputs, that is, $\rho_i = (P_{gi})/(\sum_i P_{gi})$.

For lack of space, only the loss allocation among the generators is presented in this table. In all cases the value of $\mathbf{m}$ is the same as in Table I. The slight change in the percent values of the loss coefficients among the given cases is due to different operating points created by the varying loss supply vector. This, therefore, leads to slightly different load flows and system losses. It is interesting to note also that although the generation levels may change quite significantly when ρ changes, the percent and absolute loss allocations remain relatively unchanged.

Results for the case when the integration path is split into segments, each segment having a different vector $\mathbf{m}$, are shown in Table III. In this example, three segments are defined:

1) for $P_d^{\text{sys}} \leq 1$ pu, in which segment only the generator at bus 1 is dispatched;
2) for $1 \leq P_d^{\text{sys}} \leq 1.3$ pu, in which generators 1 and 2 share the load in the proportions of 55% for G1 and 45% for G2;
3) for $P_d^{\text{sys}} \geq 1.3$ pu, in which the third generator (at bus 8) is also dispatched.

The final dispatch proportions defining the vector $\mathbf{m}$ are as given in Table III. Although the final load flow solution for the examples of Tables I and III are identical, in the case of a segmented m, more integration steps are needed to accurately estimate the percent loss allocation coefficients $L_g^\%$ and $L_d^\%$. Still, even in this segmented m case, the difference between one and 100 integration steps is relatively small, with the exception of a few buses where the loss allocation is not large, as in bus 2.

VIII. CONCLUSION

Although incremental transmission loss analysis has been in use for several decades, recent interest in its application to loss allocation calls for new rigorous results dictating the conditions under which the allocated losses are unique and nondiscriminatory. Thus, it is shown that incremental loss allocation among bus power injections is arbitrary and therefore cannot be used to allocate losses in a nondiscriminatory manner. Mathematical formulae are developed showing that unique incremental loss allocations are possible for equivalent incremental power exchanges between generators and loads. Unique allocations among individual bus loads or bus generators are also possible, but it is necessary to specify a priori in which proportion the losses are to be divided among the two. It is also shown that incremental loss allocation requires the specification of two vector quantities, the loss supply and the load distribution parameters. One of these can be left unspecified and estimated, provided that a load flow solution is also known. The allocation algorithms developed and tested show that incremental methods can be applied to large changes in operating point with little loss of accuracy if use is made of loss allocation parameters expressed as a percentage of the exact system losses.

APPENDIX A

To clarify the derivation of the incremental loss allocation components, an illustrative two-bus example is presented here. The line and load data are shown in Fig. 1. The vector of load distribution factors is, $\mathbf{m} = [0.8\ 0.2]^T$ while the vector of loss-supply factors is $\boldsymbol{\rho} = [0.6\ 0.4]^T$. The load flow equations are

$$\begin{bmatrix} P_{g1} \\ P_{g2} \end{bmatrix} = \underbrace{\begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix}}_{\mathbf{m}} P_d^{\text{sys}} + \underbrace{\begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix}}_{\tilde{n}} P_{\text{loss}} \tag{A.1}$$

$$= \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix} P_d^{\text{sys}} + \mathbf{P}(\boldsymbol{\delta}) \tag{A.2}$$

Thus, (8) becomes

$$\begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix} dP_d^{\text{sys}} + \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix} dP_{\text{loss}} - \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix} dP_d^{\text{sys}} = \begin{bmatrix} \frac{\partial P_1}{\partial \delta} \\ \frac{\partial P_2}{\partial \delta} \end{bmatrix} d\delta. \tag{A.3}$$

Equation (9) takes the form

$$\frac{\partial P_1}{\partial \delta}\alpha_1 + \frac{\partial P_2}{\partial \delta}\alpha_2 = 0. \tag{A.4}$$

Clearly, the vector α is not unique, but can be determined within a proportionality factor. In order to solve (A.4), we write the power injections

$$\begin{aligned} P_1 &= g(1 - \cos\delta) - b\sin\delta \\ P_2 &= g(1 - \cos\delta) + b\sin\delta \end{aligned} \tag{A.5}$$

from which

$$\begin{aligned} \frac{\partial P_1}{\partial \delta} &= g\sin\delta - b\cos\delta \\ \frac{\partial P_2}{\partial \delta} &= g\sin\delta + b\cos\delta. \end{aligned} \tag{A.6}$$

Now, (13) becomes

$$dP_{\text{loss}} = \frac{0.2(\alpha_1 - \alpha_2)}{0.6\alpha_1 + 0.4\alpha_2} dP_{d1} + \frac{0.8(\alpha_2 - \alpha_1)}{0.6\alpha_1 + 0.4\alpha_2} dP_{d2}. \tag{A.7}$$

Substituting (A.6) into (A.4) gives the unique proportionality factor between the elements of the vector $\boldsymbol{\alpha}$

$$\alpha_1 = -\frac{\frac{\partial P_2}{\partial \delta}}{\frac{\partial P_1}{\partial \delta}}\alpha_2 = -\frac{g\sin\delta + b\cos\delta}{g\sin\delta - b\cos\delta}\alpha_2 \tag{A.8}$$

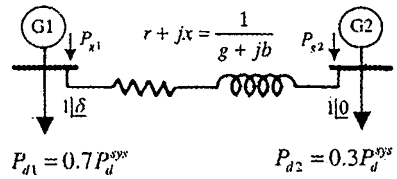

Fig. 1. Two-bus example system.

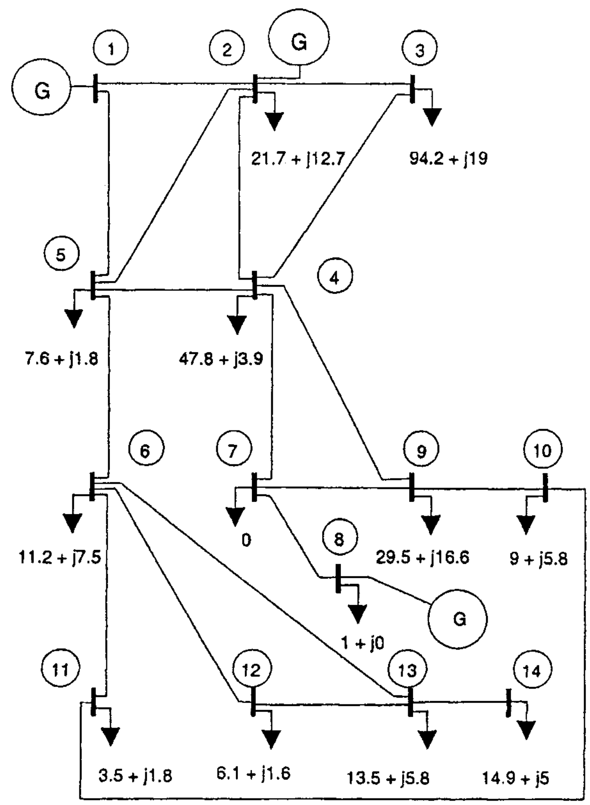

Fig. 2. One-line diagram for the 14-bus network. Active and reactive power demands and generations at every bus are given in MW and MVAr, respectively.

which, when further substituted in (A.7) gives the unique incremental loss allocation among the bus demands [as shown in (13) and (14)], that is,

$$dP_{\text{loss}} = \underbrace{\frac{2g\sin\delta}{g\sin\delta + b\cos\delta}dP_{d1}}_{dL_{d1}} + \underbrace{\frac{-8g\sin\delta}{g\sin\delta + b\cos\delta}dP_{d2}}_{dL_{d2}}. \tag{A.9}$$

A similar analysis can be conducted for loss allocation among equivalent power exchanges.

APPENDIX B

Fig. 2 provides the one-line diagram of the 14-bus system used for the simulations. The active and reactive load demands are specified (MW, MVAr) in the figure underneath the load symbols. Line data for the 14-bus system are given in Table IV. All bus voltage magnitudes are set to one per unit.

TABLE IV
LINE DATA FOR THE 14-BUS NETWORK USED IN THE CASE STUDIES

line number	from bus	to bus	r (pu)	x (pu)	b (pu)
1	1	2	0.0194	0.0592	0.0528
2	1	5	0.0540	0.2230	0.0528
3	2	3	0.0470	0.1980	0.0438
4	2	4	0.0581	0.1763	0.0374
5	2	5	0.0570	0.1739	0.0340
6	3	4	0.0670	0.1710	0.0346
7	5	4	0.0134	0.0421	0.0128
8	4	7	0.0001	0.2091	0
9	4	9	0.0001	0.5562	0
10	5	6	0.0001	0.2520	0
11	6	11	0.0950	0.1989	0
12	6	12	0.1229	0.2558	0
13	6	13	0.0662	0.1303	0
14	7	8	0.0001	0.1762	0
15	7	9	0.0001	0.1100	0
16	9	10	0.0318	0.0845	0
17	9	14	0.1271	0.2704	0
18	10	11	0.0820	0.1921	0
19	12	13	0.2209	0.1999	0
20	13	14	0.1709	0.3480	0

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