

A CLIPPING-OFF INTERIOR-POINT TECHNIQUE FOR MEDIUM-TERM HYDRO-THERMAL COORDINATION

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ABSTRACT- This paper presents a clipping-off interior-point algorithm for hydro-thermal coordination which allocates hydro and thermal resources in order to minimize the cost. The clipping-off algorithm basically consists on setting the control variables to their upper or lower bounds if certain conditions are met, and eliminating them as variables from the problem formulation thereafter. The number of flops and iterations to obtain the problem solution is considerably reduced as compared to a standard interior-point solution; however, the solution obtained by either method is the same. Based on the hydro-thermal power system of Spain, computational examples are presented; these examples are used to compare the proposed clipping-off predictor-corrector log-barrier algorithm with the standard predictor-corrector and the standard log-barrier algorithms.

KEYWORDS- Hydro-Thermal Coordination, Interior-Point Method, Clip-off.

1. INTRODUCTION

A hydro-thermal coordination program optimizes, in every interval (period) in which the horizon time is divided, the output power and the schedule of every hydro and thermal plant of a power system.

The medium-term horizon-time coordination model is important because it allows to accommodate multi-period energy constraints. The solution of such a coordination program can be used in models that consider shorter horizon times, or as the main block of a yearly planning model.

Several approaches have been used to solve the hydro-thermal coordination problem, including primal decomposition [1], Lagrangian relaxation [2] and tabu search [3].

Interior Point (IP) algorithms seek the optimal solution point progressing through the interior of the problem's feasible region rather than along its vertices as is the case in the linear-programming simplex method. How this search is done, depends on the particular IP algorithm. The first interior-point algorithm was developed by Frisch [4] in 1955. Karmarkar [5] in 1984 developed an interior-point algorithm

for linear programming that was faster, at the time, than any other known algorithm for large LP problems. Karmarkar also proved that his algorithm is a polynomial time algorithm while the simplex is not. Since 1984, IP methods have experienced an extensive development and are being applied to almost every optimization program. Previous use of interior point algorithms in electric power systems are reported in [6, 7] for example.

In 1992, Mehrotra [8] proposed a predictor-corrector method to obtain the search directions of the IP primal-dual method. Mehrotra's predictor-corrector algorithm has proved to be the most efficient of all interior-point methods. Nowadays, almost every implementation of an interior point algorithm is based on his work. Our work is also based on his algorithm.

Christoforodis et al. [9] solved a hydro-thermal coordination problem using the IP algorithm in the commercial package IBM-OSL [10]. The authors have reported experience in solving the hydro-thermal coordination via predictor-corrector algorithms in [11, 12]. In this paper, the predictor-corrector method is enhanced by a clipping-off [13] algorithm that, according to our knowledge, has never been used in conjunction with IP methods. Computer results show that there is some improvement as compared to the standard predictor-corrector and the standard log-barrier IP methods.

The outline of the paper is as follows. The model of the hydro-thermal system is presented in detail in Section 2. The derivation of the proposed clipping-off interior point technique is in Section 3. Finally, in Section 4 some computational results are presented. Section 5 presents the conclusions.

2. PROBLEM FORMULATION

The notation is organized as given data, indices, sets and number of elements in the sets, and variables. Among the variables, there are independent (or control) variables and dependent (or state) variables. Overlining of variables indicates upper bound, and underlining stands for lower bound.

• DATA

- a_j is the fixed operating cost of thermal plant j [\$/h],
- b_j the start-up cost of thermal plant j [\$],
- c_j the shut-down cost of thermal plant j [\$],
- d_{jl} the l -th cost slope of the variable operating cost function of thermal plant j [\$/MWh],
- r_j the maximum up ramp rate of thermal plant j [MW/h],

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- s_j the maximum down ramp rate of thermal plant j [MW/h],
 E_p the minimum energy to be produced by the thermal plants of the production area p during the planning horizon [MWh],
 $\alpha_{\gamma j}$ the amount of pollutant γ emitted by plant j when producing 1 MWh of energy,
 $P_{\gamma e}$ the maximum amount of pollutant γ to be emitted in emission controlled area e during the planning horizon [kg],
 $w_i(k)$ the lateral inflow volume to reservoir of hydro plant i during subperiod k [Hm³],
 x_{i0} the initial storage volume of the reservoir of hydro plant i [Hm³],
 \bar{x}_{iT} the upper bound of the final storage volume of the reservoir of hydro plant i [Hm³],
 \underline{x}_{iT} the lower bound of the final storage volume of the reservoir of hydro plant i [Hm³],
 ρ_{il} the generating characteristic of block l of hydro plant i [MJ/Dm³],
 $d(k)$ the customer power demand in period k [MW],
 m_s the transmission capacity of the interconnections of area s ,
 $R(k)$ the thermal spinning reserve in subperiod k [MW],
 $l(k)$ the duration of subperiod k [h],
 M a large penalty constant and
 $d_s(k)$ the demand in area s during subperiod k .

• INDICES, SETS AND NUMBER OF ELEMENTS

- j is the thermal plant index,
 γ the pollutant index,
 e the emission controlled area index,
 p the production area index,
 i the hydro plant index,
 k the subperiod index,
 l the block index for hydro or thermal plants,
 J the set of indices of all thermal plants,
 I the set of indices of all hydro plants
 J_p the set of indices of the thermal plants of the production area p ,
 J_e the set of indices of the thermal plants in emission controlled area e ,
 N_γ the number of pollutants,
 N_e the number of emission controlled areas,
 N_p the number of production areas,
 C_j the number of blocks of the variable operating cost of thermal plant j ,
 U_i the number of blocks of the turbine discharge of hydro plant i ,
 N the number of subperiods of one week,
 J_s the set of indices of the thermal plants which belong to area s ,
 I_s the set of indices of the hydro plants which belong to area s ,
 Ω_i the set of indices of hydro plants upstream the reservoir of plant i ,
 K the set of indices of the subperiods (1, 2, ..., T) of the planning period and
 K_d the set of subperiods for which ramp-rate limits apply.

• CONTROL VARIABLES

- $p_{jl}(k)$ is the l -th power block of the output power above the minimum output power of thermal plant j in subperiod k [MW],
 $v_j(k)$ a 0/1 variable which is equal to 1 if and only if thermal plant j is committed in subperiod k ,
 $y_j(k)$ a 0/1 variable which is equal to 1 if and only if thermal plant j is started-up at the beginning of subperiod k ,

- $s_i(k)$ the spilled outflow volume of hydro plant i during subperiod k [Hm³/s] and
 $u_{il}(k)$ the turbine discharge volume block l of hydro plant i during subperiod k [Hm³/s].

• STATE VARIABLES

- $p_j(k)$ is the output power above the minimum output power of thermal plant j in subperiod k [MW],
 $t_j(k)$ the output power of thermal plant j in subperiod k [MW],
 $z_j(k)$ a 0/1 variable which is equal to 1 if and only if thermal plant j is shut-down at the beginning of subperiod k ,
 $h_i(k)$ the power produced by hydro plant i in subperiod k [MW],
 $x_i(k)$ the storage volume of the reservoir of hydro plant i at the beginning of subperiod k [Hm³],
 $u_i(k)$ the turbine discharge volume of hydro plant i during subperiod k [Hm³/s] and
 $o(k)$ the unserved power in subperiod k [MW].

2.1. Objective Function

The objective function is the total thermal production cost which includes the fixed, start-up, shut-down and variable costs; for all subperiods, the objective function can be written as

$$\sum_{k \in K} \sum_{j \in J} [a_j v_j(k) l(k) + b_j y_j(k) + c_j z_j(k) + \sum_{l=1}^{C_j} d_{jl} p_{jl}(k) l(k)] + M \sum_{k \in K} o(k) \quad (1)$$

We assume that the total-production-cost function is concave. The complete model of the hydro-thermal system is linearized by segmenting the power output of each hydro and thermal plant.

2.2. The Thermal System

The thermal system model comprises the following constraints.

- Each segment (block) of the power output of each thermal plant is constrained by upper and lower bounds, i.e.,
 $0 \leq p_{jl}(k) \leq \bar{p}_{jl} \forall j \in J; \forall k \in K; l = 1, \dots, C_j$ (2)

- The output power of every thermal plant, as the sum of output power blocks, is given by

$$p_j(k) = \sum_{l=1}^{C_j} p_{jl}(k) \quad \forall j \in J, \quad \forall k \in K \quad (3)$$

- Committed thermal plants must operate below their maximum output-power limits, i.e.,

$$p_j(k) \leq \bar{p}_j v_j(k) \quad \forall j \in J, \quad \forall k \in K \quad (4)$$

- The power output of every committed thermal plant is the sum of its minimum power output plus its generation above its lower limit,

$$t_j(k) = \underline{p}_j v_j(k) + p_j(k) \quad \forall j \in J, \quad \forall k \in K \quad (5)$$

- Down and up ramp rates of thermal plants are also bounded, i.e.,

$$t_j(k) - t_j(k+1) \leq s_j l(k) \quad \forall j \in J, \forall k \in K_d \quad (6)$$

$$t_j(k+1) - t_j(k) \leq r_j l(k) \quad \forall j \in J, \forall k \in K_d \quad (7)$$

- For every production area, the generation energy for the whole period of the thermal plants that belong to the area, say p , must be above a prespecified energy limit, i.e.,

$$\sum_{j \in J_p} \sum_{k \in K} t_j(k) l(k) \geq E_p \quad p = 1, \dots, N_p \quad (8)$$

- The amount of every pollutant (γ) emitted in every emission constrained area (e) must be below a prespecified threshold this is,

$$\sum_{j \in J_e} \sum_{k \in K} \alpha_{\gamma j} t_j(k) l(k) \leq P_{\gamma e} \quad e = 1, 2, \dots, N_e; \quad \gamma = 1, 2, \dots, N_g \quad (9)$$

Constraints (8) and (9) are of particular interest for the Spanish system.

- The variables that describe the operation status (running, start-up, shut-down) of the thermal units must satisfy logical constraints in order to guarantee a proper sequence [14] of status changes, i.e.,

$$y_j(k) - z_j(k) = v_j(k) - v_j(k-1) \quad \forall j \in J, \quad \forall k \in K \quad (10)$$

$$v_j(k), y_j(k), z_j(k) \in \{0, 1\} \quad \forall j \in J, \quad \forall k \in K \quad (11)$$

- The number of start-ups and the number of shut-downs of every thermal plant during a week (N subperiods) is limited to 1, i.e.,

$$\sum_{k=n}^{n+N} y_j(k) \leq 1; \quad \forall j \in J, \quad \forall n = 1, 1+N, 1+2N, \dots, K-N \quad (12)$$

$$\sum_{k=n}^{n+N} z_j(k) \leq 1; \quad \forall j \in J, \quad \forall n = 1, 1+N, 1+2N, \dots, K-N \quad (13)$$

2.3. The Hydro System

The hydro system model comprises the following constraints.

- In every period, the total turbine discharge of every hydro plant is segmented in blocks; thus, it is given by the sum of discharge blocks (segments) of the total plant discharge,

$$u_i(k) = \underline{u}_i + \sum_{l=1}^{U_i} u_{il}(k) \quad \forall i \in I, \forall k \in K \quad (14)$$

- For every period, the hydro production of every hydro plant is assumed concave; it can be expressed as a combination of the turbine discharge blocks,

$$h_i(k) = \underline{h}_i + \sum_{l=1}^{U_i} \rho_{il} u_{il}(k) \quad \forall i \in I, \forall k \in K \quad (15)$$

- Each discharge block of every turbine is constrained by upper and lower bounds, i.e.,

$$0 \leq u_{il} \leq \bar{u}_{il} \quad \forall i \in I, l = 1, 2, \dots, U_i \quad (16)$$

- The water conservation equation, over all periods and for all reservoirs, is given by

$$x_i(k+1) = x_i(k) - u_i(k)l(k) - s_i(k)l(k) + \sum_{j \in \Omega_i} [u_j(k) + s_j(k)]l(k) + w_i(k)l(k) \quad \forall i \in I, \forall k \in K \quad (17)$$

The canals are modeled by adding a variable to Equation (17). This variable represents the flow per period through the canal. The sign of this variable depends on which direction the water is flowing.

- There are upper and lower bounds on reservoir volumes, i.e.,

$$\underline{x}_i \leq x_i(k) \leq \bar{x}_i \quad \forall i \in I, \quad \forall k \in K \quad (18)$$

- Spillage outflows must be positive, i.e.,

$$0 \leq s_i(k) \leq \bar{s}_i \quad \forall i \in I, \quad \forall k \in K \quad (19)$$

- The initial and the final conditions on the reservoir volumes are given by

$$x_i(0) = x_{i0} \quad \forall i \in I \quad (20)$$

$$\underline{x}_{iT} \leq x_i(T+1) \leq \bar{x}_{iT} \quad \forall i \in I \quad (21)$$

The power production of a hydro plant (say, i), as a function of the water discharge, is dependent upon the head of the reservoir associated with the hydro plant i . Thus, for every reservoir head there is a "power/discharge" curve. The dependency on the head can be taken into account by solving several times the whole hydro-thermal coordination problem and choosing, before every run, the appropriate power/discharge curve. This procedure has proved in practice to be computationally stable.

2.4. Global Constraints

These constraints couple together the thermal and the hydro subsystems and are considered independently for each subsystem when a decomposition algorithm is used.

- Power balance in all periods (including unserved power)

$$\sum_{j \in J} t_j(k) + \sum_{i \in I} h_i(k) + o(k) = d(k) \quad \forall k \in K \quad (22)$$

- Spinning reserve margin

$$\sum_{j \in J} [\bar{t}_j v_j(k) - t_j(k)] + \sum_{i \in I} [\bar{h}_i - h_i(k)] \geq R(k) \quad \forall k \in K \quad (23)$$

- Tie line constraints. They limit power flows between areas.

$$\left| \sum_{j \in J_s} t_j(k) + \sum_{i \in I_s} h_i(k) - d_s(k) \right| \leq m_s \quad (24)$$

2.5. Solution Procedure

The above problem is a large-scale sparse 0/1 mixed integer LP problem. Our proposed solution relaxes the 0/1 binary variables so that they belong to the continuous interval $[0, 1]$. The solution to the resulting LP problem is obtained by the Clipping-Off Interior Point Method described in the next section; in every period, most relaxed integer variables end up having values either 0 or 1, except for a few (typically, one or two) that have values between 0 and 1. This behavior is a consequence of the interaction between the hydro and thermal subproblems; hydro plants in order to reduce the cost tend to substitute *marginal* thermal plants (whose binary variables are different from 0 or 1).

Relaxed integer variables that are between 0 and 1 are treated by a rounding strategy; this strategy assigns a value of 0 or 1 to each of these variables so that the minimum up and down time constraints, the load balance equations and spinning reserve margins are satisfied. This results in a thermal schedule. With this thermal schedule fixed, the complete hydro-thermal program is solved again. Previous use of this method for handling binary variables have been reported by the authors in [15, 12].

3. THE PROPOSED ALGORITHM

3.1. Introduction

The Medium-Term Hydro-Thermal Coordination (MTHTC) problem as modelled in Section 2 results in a linear problem with bounded variables. By referring to control variables as u (see notation), and to the rest of variables as x , the MTHTC problem can be mathematically expressed as

$$\begin{aligned} \min \quad & \{C_1^T x + C_2^T u\} \\ \text{subject to} \quad & \end{aligned} \quad (25)$$

$$A_1 x + A_2 u = b \quad (26)$$

$$\underline{x} \leq x \leq \bar{x} \quad (27)$$

$$\underline{u} \leq u \leq \bar{u} \quad (28)$$

where A_1 is a $m \times n_s$ matrix, A_2 is a $m \times n_c$ matrix, b is the right-hand side vector of dimension $n = n_s + n_c$, $\text{col}[C_1, C_2]$ is the objective function gradient, x is the state variable vector of dimension n_s and u is the control variable vector of dimension n_c . As is well known, control or independent variables govern the behavior of the system, i.e., once they are known, the state or dependent variables can be obtained.

The clipping-off technique [13] basically consists on fixing the control variables u to their upper or lower bounds if certain conditions are met. Once these variables are fixed, they are considered as parameters for the rest of the numerical solution. To apply the clipping-off technique, we relax the bounds of the control variables in the MTHTC problem, and execute Mehrotra's predictor-corrector algorithm for such a relaxed problem. After a couple of iterations, as a result of this relaxation, most control variables have values beyond their actual limits. For each control variable that is under this condition, we check its search direction. If the search direction points towards increasing its infeasibility gap, we clip the

variable value off, i.e., we give this variable the value of its violated bound, keeping it constant during the optimization subsequent iterations, unless the direction of search changes in sign at any of these iterations. In such a case, the variable ceases to be fixed, and is treated as a state variable for the rest of the algorithm. For instance, if at iteration k we find that the control variable is $u_i^k > \bar{u}_i$, with its search direction pointing towards increasing infeasibility (which will make u_i^{k+1} even larger in the next iteration), then we set $u_i^{k+1} = \bar{u}_i$.

Every time a control variable ceases to be on one of its bounds, the constraint matrix $[A_1 A_2]$, the state variable vector x and the control variable vector u are updated to account for the reduction (or increase) in the number of elements.

3.2. Derivation of the Proposed Algorithm

The first step to derive the clipping-off IP algorithm is to introduce slack variables in the Program (25-28), that is,

$$\min \quad \{C_1^T x + C_2^T u\} \quad (29)$$

subject to

$$A_1 x + A_2 u = b \quad (30)$$

$$x + s = \bar{x} \quad (31)$$

$$x - v = \underline{x} \quad (32)$$

$$u + t = \bar{u} \quad (33)$$

$$u - g = \underline{u} \quad (34)$$

$$s, v, t, g \geq 0 \quad (35)$$

For the first iterations of the clipping-off IP algorithm, we relax the bounds of the control variables u , i.e., we make $\bar{u} = M$ and $\underline{u} = -M$ in Equations (33, 34), where M is a large positive number. If we add and subtract these last two constraints we obtain $2u - g + t = 0$ and $g + t = 2M$. Since g and t are forced to be greater than 0 and M is an arbitrary positive large number, this last equation is not strictly necessary. We keep the former of these equations, dropping the 2 and scaling the slacks, and replace Equations (33, 34) with $u - g + t = 0$. In order to derive a log-barrier algorithm, we append the non-negativity conditions on s, v, t and g to the objective function as a parameterized logarithmic penalty term.

The primal of our log-barrier MTHTC program can now be written as

$$\min \quad \{C_1^T x + C_2^T u - \mu \left[\sum_{i=1}^{n_s} (\ln s_i + \ln v_i) \right. \right.$$

$$\left. \left. + \sum_{i=1}^{n_c} (\ln t_i + \ln g_i) \right] \right\}$$

subject to

$$A_1 x + A_2 u = b$$

$$x + s = \bar{x} \quad (36)$$

$$x - v = \underline{x}$$

$$u + t - g = 0$$

The parameter μ is a decreasing sequence of positive barrier parameters such that $\lim_{k \rightarrow \infty} \mu_k = 0$. If the feasible re-

gion of the original (before adding the log-barrier terms to the objective function) problem is bounded, the optimum of the original problem is the same as the solution of the Program (36) [16] when this is solved as a sequence of subproblems with $\mu_0 > \mu_1 > \dots > \mu_\infty = 0$.

The Lagrangian of Program (36) can be defined as

$$\begin{aligned} \mathcal{L} \equiv & C_1^T x + C_2^T u - \mu_k \left(\sum_{i=1}^{ns} \ln s_i + \sum_{i=1}^{ns} \ln v_i + \sum_{i=1}^{nc} \ln t_i \right) \\ & + \sum_{i=1}^{nc} \ln g_i - y^T (A_1 x + A_2 u - b) + w^T (x + s - \bar{x}) \\ & - z^T (x - v - \underline{x}) - q^T (u + t - g) \end{aligned} \quad (37)$$

The First-Order Necessary Conditions of Optimality (FONCO) of \mathcal{L} are

$$\frac{\partial \mathcal{L}}{\partial y} \equiv A_1 x + A_2 u - b = 0 \quad (38)$$

$$\frac{\partial \mathcal{L}}{\partial x} \equiv A_1^T y + z - w - C_1 = 0 \quad (39)$$

$$\frac{\partial \mathcal{L}}{\partial u} \equiv A_2^T y + q - C_2 = 0 \quad (40)$$

$$\frac{\partial \mathcal{L}}{\partial s} \equiv W S e - \mu e = 0 \quad (41)$$

$$\frac{\partial \mathcal{L}}{\partial v} \equiv Z V e - \mu e = 0 \quad (42)$$

$$\frac{\partial \mathcal{L}}{\partial g} \equiv Q G e - \mu e = 0 \quad (43)$$

$$\frac{\partial \mathcal{L}}{\partial t} \equiv -Q T e - \mu e = 0 \quad (44)$$

$$\frac{\partial \mathcal{L}}{\partial w} \equiv x - v - \underline{x} = 0 \quad (45)$$

$$\frac{\partial \mathcal{L}}{\partial z} \equiv x + s - \bar{x} = 0 \quad (46)$$

$$\frac{\partial \mathcal{L}}{\partial q} \equiv u + t - g = 0 \quad (47)$$

where uppercase letters stand for diagonal matrices whose entries are the elements of the corresponding lowercase-letter vectors.

To derive the clipping-off technique, we substitute $-Q$ in Equation (44) by R and add the equation $q - r = 0$ to the FONCO equations. The primal-dual search directions are derived from the FONCO equations above. They are obtained, at each iteration, by taking one step of either the Newton's method or the predictor-corrector method, for a given value of μ . In this paper we use the predictor-corrector method, originally proposed by Mehrotra [8]. The advantage of using a predictor-corrector algorithm is that, although it needs some extra computational work per iteration, it includes second order information and allows to dynamically estimate the value of the barrier parameter μ at every iteration. The use of a predictor-corrector algorithm leads to less number of iterations and less CPU time as compared to the normal Newton's algorithm.

Defining a vector p as $\text{col}[x, u, y, s, w, v, t, g, z, q, r]$, the predictor-corrector's search directions are obtained by sub-

stituting the new point $p^{k+1} = p^k + \Delta p$ directly into the FONCO equations, yielding

$$\begin{bmatrix} A_1 & A_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 & 0 & -I & 0 & I & 0 \\ 0 & 0 & A_1^T & 0 & -I & 0 & I & 0 & 0 & 0 & 0 \\ I & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & W & S & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Z & V & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q & G & 0 & 0 \\ 0 & 0 & A_2^T & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & I \\ I & 0 & 0 & 0 & 0 & -I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R & T \end{bmatrix}$$

$$\times \begin{bmatrix} \Delta x \\ \Delta u \\ \Delta y \\ \Delta s \\ \Delta w \\ \Delta v \\ \Delta z \\ \Delta g \\ \Delta q \\ \Delta t \\ \Delta r \end{bmatrix} = \begin{bmatrix} r_b \\ r_u \\ r_{c1} \\ r_{\bar{x}} \\ r_{SW} \\ r_{ZV} \\ r_{GQ} \\ r_{c2} \\ r_q \\ r_{\underline{x}} \\ r_{TR} \end{bmatrix} \quad (48)$$

where the right-hand side includes nonlinear terms and the parameter μ , which has to be determined at every iteration. The way this is handled in the predictor-corrector scheme is by splitting the right-hand side vector into two. The predictor step assumes μ and the non-linear terms equal zero; the corresponding right-hand side vector is given by

$$r_b = b - A_1 x - A_2 u \quad (49)$$

$$r_u = g - u - t \quad (50)$$

$$r_{c1} = c_1 - A_1^T y - z + w \quad (51)$$

$$r_{\bar{x}} = \bar{x} - x - s \quad (52)$$

$$r_{SW} = -S W e \quad (53)$$

$$r_{ZV} = -Z V e \quad (54)$$

$$r_{GQ} = -G Q e \quad (55)$$

$$r_{c2} = c_2 - A_2^T y - q \quad (56)$$

$$r_q = -q - r \quad (57)$$

$$r_{\underline{x}} = \underline{x} - x + v \quad (58)$$

$$r_{TR} = -R T e \quad (59)$$

Using this predictor right-hand vector the System 48 is solved for the search directions $\Delta \hat{p}$. Next, the primal and dual variables are updated and the barrier parameter for the current iteration is calculated; the way to compute μ is explained in Subsection 3.5..

These predictor search directions and updated parameter μ are substituted into the original right-hand side vector to obtain relevant components of the corrector right-hand side vector. The components of this vector that do not contain μ and non-linear terms are set equal to zero. The non-zero components of the corrector right hand side vector are

$$r_{SW} = \mu e - \Delta \hat{W} \Delta \hat{S} e \quad (60)$$

$$r_{ZV} = \mu e - \Delta \hat{Z} \Delta \hat{V} e \quad (61)$$

$$r_{GQ} = \mu e - \Delta \hat{G} \Delta \hat{Q} e \quad (62)$$

$$r_{TR} = \mu e - \Delta \hat{T} \Delta \hat{R} e \quad (63)$$

The symbol $\hat{\cdot}$ over some variables is used to express that they are no longer unknown as they now take the values of the predictor solution.

Once the corrector search directions are computed by solving the System 48, the actual search directions of the current iteration, Δp , are obtained by adding the predictor and corrector search directions. The new point is obtained as $p^{k+1} = p^k + \alpha \Delta p$, except for the control variables that are updated as $u^{k+1} = \underline{u}$ or as $u^{k+1} = \bar{u}$, depending on which restriction limit they are not meeting. α is the step size (Subsection 3.4).

For solving purposes, the System 48 is put into an equivalent system by making a number of substitutions, leading to the following normal equation for Δy ,

$$\begin{aligned} (A_1 D_1 A_1^T + A_2 D_2 A_2^T) \Delta y &= r b + A_1 D_1 [S^{-1} (r_{SW} \\ - W r_{\bar{x}}) - V^{-1} (r_{ZV}) + Z r_{\underline{x}} + r_{c1}] &+ A_2 [D_2 r_{c2} \\ - Q^{-1} r_{GQ} - r_u + R^{-1} (r_{TR} + T r_q)] \end{aligned} \quad (64)$$

where $D_1 = (V^{-1} Z + S^{-1} W)^{-1}$ and $D_2 = G Q^{-1} + R^{-1} T$. The above normal equation has a symmetric positive definite matrix that allows for a robust efficient solution. In this paper, the normal equation is solved by direct Cholesky factorization in conjunction with a minimum degree reordering (to reduce the number of fill-ins in the Cholesky factor). Since the predictor and corrector systems differ only on the right-hand side vector, both systems use the same matrix factorization.

By back-substitution, we can obtain the remaining search directions, i.e.,

$$\Delta x = D_1 [A_1^T \Delta y + V^{-1} (Z r_{\underline{x}} + r_{ZV}) + S^{-1} (W r_x - r_{SW}) - r_{c1}] \quad (65)$$

$$\Delta u = D_2 (A_2^T \Delta y - r_{c2}) + r_u + Q^{-1} r_{GQ} + R^{-1} (T r_q - r_{TR}) \quad (66)$$

$$\Delta v = \Delta x - r_{\underline{x}} \quad (67)$$

$$\Delta s = -\Delta x + r_{\bar{x}} \quad (68)$$

$$\Delta w = s^{-1} (-w \Delta s + r_{SW}) \quad (69)$$

$$\Delta z = V^{-1} (-Z \Delta v + r_{ZV}) \quad (70)$$

$$\Delta q = (I + G^{-1} Q R^{-1} T)^{-1} G^{-1} \{r_{GQ} - Q[\Delta u + r_u + R^{-1} (T r_q - r_{TR})]\} \quad (71)$$

$$\Delta r = r_q - \Delta q \quad (72)$$

$$\Delta t = R^{-1} (r_{TR} - T \Delta r) \quad (73)$$

$$\Delta g = \Delta u + \Delta t - r_u \quad (74)$$

To simplify the notation on the following subsections, we define $x = \text{col}[x, u]$, $z = \text{col}[z, q]$, $v = \text{col}[v, g]$, $s = \text{col}[s, t]$, $w = \text{col}[w, r]$, $c = \text{col}[c_1, c_2]$ and $A_1 = [A_1 A_2]$.

3.3. The Initial Point

One of the main advantages of primal-dual IP algorithms is that the initial point need not be feasible. It only has to meet the strict positiveness conditions on the slack variables. The choice of the initial point is far more important when using

the predictor-corrector algorithm for the search directions. The initial values cannot be too small because this can cause instability [17].

The predictor-corrector algorithm can start from an infeasible positive point that meets the positiveness condition of the slacks. As the solution progresses, the point is moved towards the feasible region. In this paper, we follow Lustig et al. [17] to obtain an initial point. First, an estimate of the primal variable x is obtained as

$$\bar{x} = A_1^T (A_1 A_1^T)^{-1} b$$

Next, the variables are initialized as follows:

$$\begin{aligned} x_j^0 &= \max\{\bar{x}_j, \epsilon_1\} \\ s_j^0 &= \max\{\epsilon_1, \bar{x}_j - x_j^0\} \\ v_j^0 &= \max\{\epsilon_1, x_j^0 - \underline{x}_j\} \\ y^0 &= 0 \\ z_j &= c_j + \epsilon_2 \quad w_j = \epsilon_2 \quad \text{if} \quad c_j \geq \epsilon_2 \\ z_j &= -c_j \quad w_j = -2c_j \quad \text{if} \quad c_j < -\epsilon_2 \\ z_j &= c_j + \epsilon_2 \quad w_j = \epsilon_2 \quad \text{if} \quad 0 \leq c_j \leq \epsilon_2 \\ z_j &= \epsilon_2 \quad w_j = -c_j + \epsilon_2 \quad \text{if} \quad -\epsilon_2 \leq c_j \leq 0 \end{aligned}$$

where ϵ_1 and ϵ_2 are defined as

$$\begin{aligned} \epsilon_1 &= \max\{-\min_{1 \leq j \leq n} \bar{x}_j, \epsilon_3, \frac{\|b\|}{100}\} \\ \epsilon_2 &= 1 + \epsilon_4 \|c\| \end{aligned}$$

Lustig et al. in [17] uses fixed values for ϵ_3 and ϵ_4 , equal to 100 and 1, respectively. Parameters ϵ_3 and ϵ_4 has been introduced by Yan and Quintana in [7]. The best results are obtained with $\epsilon_3 = 100$ and $\epsilon_4 = 10$.

3.4. Step Length

The step length is determined based on the non-negativity conditions on the slack variables s, v, z and w ,

$$\begin{aligned} \hat{\alpha}_p &= \min \left\{ 1, \min_j \left[\frac{s_j}{-\Delta s_j} / \Delta s_j < 0, \frac{v_j}{-\Delta v_j} / \Delta v_j < 0 \right] \right\} \\ \hat{\alpha}_d &= \min \left\{ 1, \min_j \left[\frac{z_j}{-\Delta z_j} / \Delta z_j < 0, \frac{w_j}{-\Delta w_j} / \Delta w_j < 0 \right] \right\} \end{aligned}$$

In order to stay away from the boundary, these values of α are multiplied by a number smaller than one to obtain the actual primal and dual step sizes, i.e., $\alpha_p = 0.99995 \hat{\alpha}_p$ and $\alpha_d = 0.99995 \hat{\alpha}_d$.

3.5. Estimating μ

Once the predictor search directions are found, the complementary gap is evaluated as

$$\hat{g} = (v + \delta_p \Delta \hat{v})^T (z + \delta_d \Delta \hat{z}) + (s + \delta_p \Delta \hat{s})^T (w + \delta_d \Delta \hat{w})$$

where the δ 's are the predictor step sizes, obtained as the α 's in Subsection 3.4.

Our estimate for μ is based on Mehrotra's work [8], and is given by

$$\mu = \left(\frac{\hat{g}}{x^T z + s^T w} \right)^2 \left(\frac{\hat{g}}{n} \right) \quad (75)$$

when $x^T z + s^T w \geq 1$. However, if $x^T z + s^T w < 1$, then μ is updated by

$$\mu = \frac{x^T z + s^T w}{\phi(n)} \quad (76)$$

where n is the number of variables (all of them) in Program (25-28), $\phi(n)$ is n^2 if $n \leq 5000$, and $\phi(n)$ is $n^{3/2}$ if $n > 5000$. This estimate chooses a small μ when good progress can be made in the predictor direction, and a large μ when the predictor direction produces little improvement. This makes sense, as little progress in the predictor direction generally means the need for more centering, that is, a larger value of μ is needed [17].

3.6. Stopping Criteria

The algorithm stops when optimality, and primal and dual feasibility are obtained within a given tolerance. The optimality is measured by the duality gap

$$\frac{|c'x - b'y - v'z + u'w|}{1 + |b'y + v'z - u'w|}$$

The primal infeasibility is measured by

$$\frac{|Ax - b| + |x + s - \bar{x}| + |x - v - \underline{x}|}{1 + |b|}$$

And the dual infeasibility is measured by

$$\frac{|C_1 + w - z - A_1^T y| + |C_2 - q - A_2^T y|}{1 + |c|}$$

When these three criteria are below a certain tolerance, the algorithm stops.

4. RESULTS

In this section we test and compare the standard predictor-corrector and the standard log-barrier IP algorithms with the proposed clipping-off algorithm.

The system used in the tests is based on the electric power system of Spain. This system contains 25 thermal units and 12 cascaded hydro plants. The objective function to be minimized is the production cost over a considered planning time horizon. Four scheduling problems are solved, which are named PRO1, PRO2, PRO3 and PRO4. The number of periods ranges from 6 in the first scheduling problem to one week in hourly intervals in PRO4. The second and third problem consider 24 and 96 periods respectively. The characteristics of each of these problems are given in Table 1, where nnz is the number of non-zeros in the constraint matrix $[A_1 A_2]$.

The structure of the restriction matrix $[A_1 A_2]$ for PRO1 is shown in Figure 1. This structure is shared by all the problems.

The algorithm is implemented in Matlab 4.2c [18], and is executed in a Sun Sparc 10 with 96 Megs of RAM under SunOS 5.5 operating system.

The accuracy of the solution is set to eight digits. Some computer results of the tests are shown in Table 2, where N

Table 1: Characteristics of the Problems.

	Rows	Columns	nnz
PRO1	584	1100	2551
PRO2	2186	4538	11005
PRO3	8594	18002	43981
PRO4	15002	31466	76957

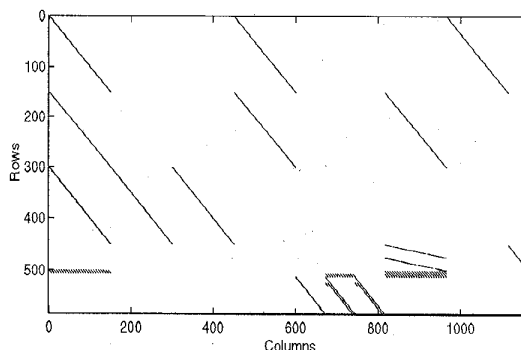


Figure 1: Structure of the Restriction Matrix of PRO1.

is the number of control variables that are clipped-off and "It" is the number of iterations; the number of FLOPS is given in millions, and nc means that convergence is not reached. Best results are obtained when the clip-off technique is applied to only the discharge variables, i.e., vector u .

As seen in Table 2, the clip-off algorithm requires less number of FLOPS than the standard predictor-corrector log-barrier algorithm, and less number of FLOPS than the standard log-barrier interior point algorithm; the clip-off algorithm requires at least 11 % less FLOPS than the standard predictor-corrector log-barrier, and almost 50 times less FLOPS than the standard log-barrier algorithm. When solving PRO4, the number of FLOPS using the clipping-off technique is reduced by 68 % as compared to the predictor-corrector algorithm. Also, it needs less number of iterations to convergence: almost a third of iterations less in the case of PRO4. The standard log-barrier algorithm could not reach the optimal solution of PRO2, PRO3 and PRO4 in a reasonable time. The RAM memory required by the clipping-off technique is also smaller than the memory needed by the predictor-corrector and the standard log-barrier algorithm. Solution times are in the order of minutes for the first two problems and in the order of hours for PRO3 and PRO4. These solution times can be reduced by using a compilable programming language as Fortran or C.

All the algorithms used in the comparison incorporate a minimum degree reordering scheme. This reordering reduces the number of FLOPS enormously. For instance, when solving a problem half the size of PRO1 with the standard log-barrier algorithm without reordering, 136.27 times more FLOPS are needed as compared to the same algorithm with reordering. This reduction in the number of FLOPS increases with the size of the problem.

Table 2: Performance of the Clipping-Off Technique

	CLIP-OFF			PRE-CO		Log-bar	
	FLOPS	It	N	FLOPS	It	FLOPS	It
PRO1	4.18	13	72	5.56	18	193.60	29
PRO2	23.96	18	288	28.64	20	nc	nc
PRO3	220.52	20	1152	245.00	23	nc	nc
PRO4	317.065	20	2028	999.15	29	nc	nc

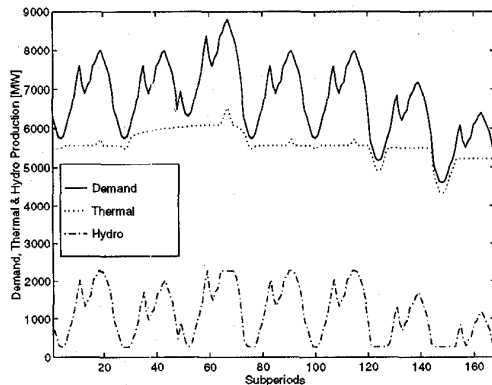


Figure 2: Load, Thermal and Hydro Production.

Experience has shown that once a control variable hits a boundary, it does not leave it during subsequent iterations.

In Figure 2, the demand, and the optimal thermal and hydro production of PRO4 are shown. It can be seen that the optimal hydro production flattens the demand curve, absorbing all the changes in demand. This hydro generation pattern prevents the thermal plants from regulating the load, which is expensive and may cause damage to the boilers.

5. CONCLUSIONS

This paper presents a log-barrier primal-dual algorithm that uses both a predictor-corrector scheme and a clipping-off technique for control variables. The algorithm has been tested on three medium-range hydro-thermal coordination problems that are based on the Spanish electric power system. Test results show that the clip-off technique leads to savings in the number of FLOPS and iterations as compared to the standard predictor-corrector and the standard log-barrier algorithms.

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