Z-Bus Loss Allocation

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***Abstract*—This paper presents a new procedure for allocating transmission losses to generators and loads in the context of pools operated under a single marginal price derived from a merit-order approach. The procedure is based on the network Z-bus matrix, although all required computations exploit the sparse Y-bus matrix. One innovative feature and advantage of this method is that, unlike other proposed approaches, it exploits the full set of network equations and does not require any simplifying assumptions. The method is based on a solved load flow and is easily understood and implemented. The loss allocation process emphasizes current rather than power injections, an approach that is intuitively reasonable and leads to a natural separation of system losses among the network buses. Results illustrate the consistency of the new allocation process with expected results and with the performance of other methods.**

***Index Terms*—Ancillary service, impedance and admittance matrix, transmission loss allocation.**

I. INTRODUCTION

THE PURPOSE of loss allocation in the context of pool dispatch is to assign to each individual generation and load the responsibility of paying for part of the system transmission losses. This step is necessary whenever, for reasons of computational simplicity, the generation dispatch and its clearing price are calculated through a merit order approach that initially neglects transmission losses [1], [2]. However, as power networks are inevitably lossy, units supplying the required system loss must be compensated for this service, typically at the system marginal price. The loss allocation process then determines how this additional expense should be distributed among all generators and loads in an equitable manner.

As system losses can typically represent from five to ten percent of the total generation, a quantity worth millions of dollars per year, their "fair" allocation among loads and generators has an important impact on their benefits. This is so since the extra cost of the allocated loss component must be subtracted from the revenue of the generators and added to the load payments.

A number of loss allocation methods have been recently proposed in the literature. These fall into the following categories: *pro rata*, incremental transmission loss, proportional sharing, loss formula, and incremental bilateral contract path. A brief description of each category follows.

Pro rata techniques are one of the most common. The loss components allocated are based on the bus generation or load active power levels, but not on their relative location within the network. As a result, remotely located generators or demands benefit at the expense of all others. Examples of electricity markets that use this power-based *pro rata* loss allocation method can be found in mainland Spain [1] and England and Wales [2].

Loss allocation based on incremental transmission loss (ITL) coefficients [3] offer the following characteristics: i) ITLs can be positive or negative, the latter being interpreted as a cross-subsidy. ii) The allocation depends on the choice of the slack generator. iii) Direct application of the coefficients typically results in over-recovery of losses.

The proportional sharing technique [4], [5] provides a computationally efficient procedure for loss allocation. However, only the first Kirchhoff law is enforced and an additional assumption is required, namely the proportional sharing principle, which assumes that the inflows are proportionally shared among the outflows at each network node. Furthermore, the losses allocated to the generating buses relative to the demand buses must be specified arbitrarily (typically 50%).

In the context of bilateral contracts, a number of loss allocation methods have also been put forward. The notion of localized response and second order loss approximations is proposed in [6]. In this case, as indicated by the authors, approximations are poor if the electrical distance between contract participants is small. Quadratic loss approximation formulae are used in the coordinated multilateral trade model [7] and in [8], the latter expressing the loss formula in terms of the transactions rather than injections. The method assumes a single slack bus from which the losses are calculated and distributed among the transactions. In [9], line power flows are first unbundled into a sum of components, each corresponding to a bilateral transaction. The scheme then proposes ways in which the coupling terms among the components appearing in the line losses can be allocated to individual bilateral transactions. Finally, in [10] a process is used whereby individual bilateral transactions are gradually incremented along a given path of variation. Each bilateral transaction may elect to have its losses supplied by a separate slack generator. Once the path of variation and the loss suppliers are specified, the incremental contract loss allocations and their sums are uniquely determined.

The principal difficulty in allocating losses to loads, generators or to bilateral contracts is that, regardless of the approach, the final allocation always contains a degree of arbitrariness. This is due to the fact that the system transmission losses are a nonseparable, nonlinear function of the bus power injections

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which makes it impossible to divide the system losses into the sum of terms, each one uniquely attributable to a generation or load. Thus, the issue of fairness will probably never be fully resolved by any loss allocation method.

Nevertheless, it is possible to identify a number of characteristics in a loss allocation scheme that are, arguably, reasonable and necessary for the scheme to be equitable or, at least, acknowledged as equitable. These characteristics are also useful in the comparison of the various approaches proposed in the literature. In general, the loss components allocated among the bus generations and loads should:

a) Reflect the magnitude of the power or current injections at each bus.
b) Reflect the relative position of the bus in the network.
c) Reflect both the network topology and the voltage-current relationships.
d) Be simple to understand and implement.
e) Provide effective incentives or disincentives to generators and loads with respect to their relative network locations and magnitudes.
f) Be consistent with a solved load flow.

In this paper, a new method for allocating transmission losses among loads and generators, called *Z-bus loss allocation*, is presented. This method meets all the points suggested above, while introducing three additional characteristics: i) Currents rather than powers are emphasized in the allocation process. This point reflects the fact that currents are the dominant factor in the determination of transmission losses. ii) A natural mathematical separation of the system losses among the various network buses is identified and exploited. iii) Generators or loads distant from the "center of gravity" tend to be allocated proportionately higher losses.

II. Z-BUS LOSS ALLOCATION METHOD

The goal of the *Z-bus loss allocation* method, as in other approaches, is to take a solved power flow and systematically distribute the system transmission losses, P_{loss}, among the n network buses according to,

$$P_{loss} = \sum_{k=1}^{n} L_k. \tag{1}$$

The loss component, L_k, in (1) is the fraction of the system losses allocated to the net real power injection at bus k. This step therefore assigns to each individual bus, k, the responsibility of paying for L_k at the market marginal price, λ. The extra cost due to loss allocation must then be subtracted from the revenue of the generators and added to the load payments so that the pool remains revenue-neutral. In general, if a given bus k has both generation, P_{gk}, and demand, P_{dk}, the allocated loss component, L_k, can be further divided among the two in a *pro rata* manner. For example, if $\gamma_k = (P_{gk}/(P_{gk} - P_{dk}))$, generator k collects a revenue from the pool equal to $\lambda(P_{gk} - \gamma_k L_k)$ while load k pays to the pool an amount equal to, $\lambda(P_{dk} + (1-\gamma_k)L_k)$. If the bus has neither load nor generation then the loss allocation must be zero.

To calculate the terms L_k according to the Z-bus loss allocation method, first consider the network admittance matrix, $Y = G + jB$, typically large, sparse and nonsingular.[1] Suppose that a solved power flow exists, defining, among other quantities, the vector of complex bus current injections, I, and the vector of complex bus voltages, V. The system real losses can be expressed either in terms of Y and V or through Z and I, where $Z = Y^{-1} = R + jX$ is the Z-bus or impedance matrix. Starting with,

$$P_{loss} = \Re\left\{\sum_{k=1}^{n} V_k I_k^*\right\} \tag{2}$$

the system losses can be expressed, either through Y as,

$$P_{loss} = \Re\left\{\sum_{k=1}^{n} V_k \left(\sum_{j=1}^{n} Y_{kj}^* V_j^*\right)\right\} \tag{3}$$

or, through Z as,

$$P_{loss} = \Re\left\{\sum_{k=1}^{n} I_k^* \left(\sum_{j=1}^{n} Z_{kj} I_j\right)\right\}. \tag{4}$$

As it turns out, of the two, the Z matrix formulation is the only one that yields loss allocation numbers that are "reasonable," that is, meeting the six characteristics suggested in the Introduction. This is due to the fact that the losses are directly related to the currents, which are the independent variables in (4).

The basic idea behind the Z-bus approach is to separate the loss formula (4) into two main summations, one due to the resistance matrix, R, and the other to the reactance matrix, X. Then,

$$\begin{aligned} P_{loss} = &\Re\left\{\sum_{k=1}^{n} I_k^* \left(\sum_{j=1}^{n} R_{kj} I_j\right)\right\} \\ &+ \Re\left\{\sum_{k=1}^{n} I_k^* \left(\sum_{j=1}^{n} jX_{kj} I_j\right)\right\}. \end{aligned} \tag{5}$$

In Appendix A, it is shown that the second term in (5) is equal to zero, so that the system losses can be expressed uniquely in terms of the complex currents and the resistance matrix, R. Thus,

$$P_{loss} = \Re\left\{\sum_{k=1}^{n} I_k^* \left(\sum_{j=1}^{n} R_{kj} I_j\right)\right\}. \tag{6}$$

A natural separation of the system losses among the network buses now presents itself. To recast this in matrix form, consider the vector of real power injections expressed as,

$$P = \Re\{\mathrm{diag}\,(I^*)RI\} + \Re\{j\,\mathrm{diag}\,(I^*)XI\}. \tag{7}$$

Thus, the net real power injections are separable into two components, the loss components, L, where,

$$L = \Re\{\mathrm{diag}\,(I^*)RI\} \tag{8}$$

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[1] The requirement of the Z-bus loss allocation method that Y be nonsingular is met in AC networks since transmission lines always have a shunt capacitance to ground.

and the demand components,

$$D = \Re\{j\mathrm{diag}(I^*)XI\}. \tag{9}$$

By comparing (6) and (8), it follows immediately that the sum over all buses of the loss terms, L_k, is equal to the system losses. Moreover, as shown in Appendix A, the sum of the network terms, D_k, is equal to zero. Thus, the system losses are *naturally separated* into the L_k terms defined by (8). Note that although the D_k components add up to zero, in typical networks, their magnitude relative to the loss terms is always dominant at each individual bus, in other words,

$$|D_k| \gg |L_k|. \tag{10}$$

This empirical inequality ensures that, at every bus, the fraction of net power injected allocated to transmission losses is relatively small compared to the power delivered to the loads. From equation (8) it follows that the loss component associated with bus k can be expressed as,

$$L_k = \Re\left\{I_k^*\left(\sum_{j=1}^{n} R_{kj}I_j\right)\right\}. \tag{11}$$

As can be seen from (11), the loss component, L_k, encompasses n terms representing the coupling actions between current injections at all n buses with the current injection at bus k. One characteristic of this natural separation of the system losses is that the loss terms depend primarily on the complex bus current injections[2] This dependence of the loss terms on currents rather than power injections is intuitively reasonable.

It must be emphasized that no special assumptions or approximations have been necessary in deriving the loss allocation terms, L_k. This natural separation of the system losses is based solely on a solved load flow and the exact network equations as characterized by the impedance matrix.

III. USE OF SPARSE ADMITTANCE MATRIX IN LOSS ALLOCATION PROCESS

To avoid explicitly calculating the nonsparse matrix R, all operations in (8) should be based on solutions involving the sparse admittance matrix, Y. Thus, to find the vector of loss components, L, first one solves two sets of algebraic equations, $Y^{-1}(\Re\{I\})$ and $Y^{-1}(\Im\{I\})$. The term RI in (8) is then found by combining these two solutions,

$$RI = \Re\left\{Y^{-1}\left(\Re\{I\}\right)\right\} + j\Re\left\{Y^{-1}\left(\Im\{I\}\right)\right\}. \tag{12}$$

IV. CASE STUDY

The Z-bus loss allocation method has been tested on a set of networks of varying sizes and types, and compared to some of the alternative algorithms described in the literature. In this paper two case studies are summarized, namely the IEEE 14-bus and 118-bus networks, both with slight modifications as defined in Appendix B.

TABLE I
COMPARISON OF RESULTS OF FIVE LOSS ALLOCATION METHODS FOR THE BASIC 14-BUS NETWORK

bus num.	Active power gen. Pg	Active load dem. Pd	Bus current Inject. $\|I_i\|$	Distribution of active power losses Ploss=13.5 MW ; λ=50 $/MWh ; Cost$_{loss}$=677.5 $/h				
				Z-bus	Pro-rata		PS	ITL
					P	I		
	(MW)	(MW)	(A)	($ / h)	($ / h)	($ / h)	($ / h)	($ / h)
1	232.7	0.0	1598	382	323	275	324	307
2	40.0	21.7	188	8	25	32	15	48
3	0.0	94.2	676	139	131	116	144	146
4	0.0	47.8	339	42	66	58	63	63
5	0.0	7.6	55	4	11	9	8	9
6	0.0	11.2	298	24	16	51	12	16
7	0.0	0.0	0	0	0	0	0	0
8	0.0	0.1	190	1	0	33	0	0
9	0.0	29.5	239	26	41	41	39	34
10	0.0	9.0	76	9	12	13	14	10
11	0.0	3.5	27	3	5	5	5	4
12	0.0	6.1	43	5	8	7	8	9
13	0.0	13.5	102	13	19	17	19	16
14	0.0	14.9	112	22	21	19	26	16
sum	272.7	259.1	-	678	678	678	678	678

Comparison of the Z-bus loss allocation algorithm was made against two *pro rata* methods, one based on active power and the other on current magnitude injections. As well, a proportional sharing method [4] and an incremental transmission loss scheme [3], [10] were tested.

The evaluation of each method can be based on the values of the allocated components measured in MW, in per unit, as a percentage of the system losses, or in dollars per hour. The dollar per hour evaluation was selected in this paper, as it most emphatically describes the monetary impact of loss allocation and the significant differences among the various buses and methods.

Tables I–III compare the loss allocation components per bus for different methods. The results assume a system marginal price of $50/MWh. Columns 5 through 9 represent the cost of the allocated bus losses for the five different allocation methods tested. These are respectively: Z-bus, *pro rata* based on active power injections (P), *pro rata* based on current magnitude injections (I), proportional sharing (PS), and incremental transmission loss method (ITL).

In Table I, generator 1 with about 85% of the total generation, always gets allocated the highest cost according to all methods. Similarly, all five methods allocate the next highest cost to the load at bus 3 comprising 36% of the system load. The Z-bus method allocates $139/h or 21% of the losses to load 3. The remaining buses receive varying allocations depending on the method, with some differences being relatively significant.

The Z-bus method in general places a stronger emphasis on current injections than other methods. Thus, the allocation to generator 1 is the highest for the Z-bus method ($382/h or 56% of the total cost of losses). Generator 2 receives a relatively low

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[2] The loss allocation terms are invariant with respect to the current reference angle since (11) is defined by terms with products of I and I^*.

TABLE II
COMPARISON OF RESULTS OF FIVE LOSS ALLOCATION METHODS FOR THE MODIFIED 14-BUS NETWORK

bus num.	Active power gen. Pg	Active load dem. Pd	Bus current inject. $\|I_i\|$	Distribution of active power losses Ploss=6.2 MW ; λ=50 $/MWh ; Cost_{loss}=308 $/h				
				Z-bus	Pro-rata		PS	ITL
					P	I		
	(MW)	(MW)	(A)	($ / h)	($ / h)	($ / h)	($ / h)	($ / h)
1	125.3	0.0	857	116	80	72	111	90
2	40.0	21.7	136	4	12	11	11	26
3	0.0	94.2	677	124	60	57	92	79
4	0.0	47.8	335	13	31	28	17	25
5	0.0	7.6	54	1	5	5	4	4
6	0.0	11.2	293	23	7	25	8	10
7	0.0	0.0	0	0	0	0	0	0
8	100.0	0.1	703	-9	64	59	32	44
9	0.0	29.5	237	3	19	20	0	7
10	0.0	9.0	75	3	6	6	1	3
11	0.0	3.5	27	1	2	2	2	2
12	0.0	6.1	43	5	4	4	6	5
13	0.0	13.5	102	11	9	9	16	8
14	0.0	14.9	111	15	10	9	8	5
sum	265.3	259.1	-	308	308	308	308	308

TABLE III
COMPARISON OF RESULTS OF FIVE LOSS ALLOCATION METHODS FOR THE 118-BUS NETWORK; ONLY BUSES WITH AN ABSOLUTE LOSS ALLOCATION PAYMENT ABOVE $80/h, AS CALCULATED BY THE Z-BUS METHOD, ARE SHOWN.

bus num	Active power gen. Pg	Active load dem. Pd	Bus current inject. $\|I_i\|$	Distribution of active power losses Ploss=132.7 MW ; λ=50 $/MWh ; Cost_{loss}=6635 $/h				
				Z-bus	Pro-rata		PS	ITL
					P	I		
	(MW)	(MW)	(A)	($ / h)	($ / h)	($ / h)	($ / h)	($ / h)
1	0	51	447	84.3	45.8	49.8	56.4	65.0
10	450	0	3126	211.0	401.4	349.0	406.1	294.6
15	0	90	679	134.0	80.3	75.6	77.6	112.8
18	0	60	447	84.9	53.7	49.8	45.1	72.3
25	220	0	1903	205.0	196.4	213.0	284.0	164.5
26	314	0	2428	144.6	280.0	271.4	284.6	209.7
27	0	71	536	90.9	63.7	59.7	82.9	74.3
32	0	59	533	82.3	52.4	59.7	83.6	63.7
40	0	66	497	164.5	59.1	55.7	82.3	120.1
41	0	37	287	89.6	33.2	31.8	63.0	70.3
42	0	96	719	223.6	85.6	80.3	165.9	171.2
45	0	53	427	80.3	47.1	47.8	80.9	66.4
54	48	113	537	107.5	57.7	59.7	77.0	124.1
55	0	63	497	106.8	56.4	55.7	61.0	80.9
56	0	84	656	138.7	75.0	73.6	98.9	106.8
65	391	0	3386	291.3	349.0	378.2	229.6	252.1
66	392	39	2729	356.3	315.2	305.2	435.3	281.3
69	514	0	3647	631.7	458.5	407.4	478.4	316.5
76	0	68	573	96.2	60.4	64.4	92.9	61.7
80	477	130	2787	380.8	309.9	311.2	170.5	364.9
88	0	48	360	-118.8	43.1	40.5	16.6	-37.2
89	607	0	4377	1987.2	542.1	489.0	587.9	1184.3
90	0	163	1206	-324.5	145.3	134.7	114.8	-70.3
90	0	163	1206	-324.5	145.3	134.7	114.8	-70.3
92	0	65	502	-126.1	57.7	56.4	34.5	-35.8
100	252	37	1636	179.8	191.8	183.1	222.9	184.5
107	0	50	381	88.9	44.5	42.5	82.3	67.7
112	0	68	548	172.5	60.4	61.0	106.2	128.7

allocation with the Z-bus method since the net injected current is low. However, the current magnitude injections at the bus cannot by themselves fully explain the allocated losses with the Z-bus method. As equation (8) indicates, the loss allocated also depends on coupling terms due to injections at all other buses and the corresponding transfer resistances.

Table II describes the results obtained when a new generator is added to bus 8 of the 14-bus network with an output of 100 MW. The loads and network parameters for this case remain the same as in Table I. The goal here is to examine a case where generation and load are more evenly distributed throughout the network. As Table II shows, the most notable change is that the system losses have decreased by more than 50%. The loss allocation percentages have also changed significantly. For example, generator 1 that is now producing 47% of the total generation is allocated $116/h or 38% of the total cost of losses. On the other hand, load 3 which still constitutes 36% of the total load is now allocated $124/h or 40% of the system losses. One rough explanation for this behavior is that load 3 is now not as close to the "center of gravity" of the generation as it was in the case of Table I.

Table III shows some selected results for the 118-bus network. Where the Z-bus method differs significantly from other methods (for example, buses 10, 69 and 89), a reasonable explanation is that our method is more sensitive to network location, particularly at those buses whose current injections have a strong impact on network losses. Full results for the entire 118-bus network are given in [12].

One characteristic of the Z-bus method, also present in the incremental transmission loss algorithm, is the possibility of negative loss allocations, as seen in Tables II and III. For example, comparing bus 8 in Tables I and II, we see that the loss allocation changes from $1 to 9/h. This change from positive to negative loss allocation is due to two reasons. One is that the system losses have sharply dropped from 13.5 MW to 6.2 MW, a result directly attributed to the relocation of part of the generation from bus 1 (the slack) to bus 8. The second reason is that the relocation of generation produces a more even distribution of generation and load. Negative allocation provides monetary incentives to those generators "well" positioned in the network. Alternatively, generators or loads "poorly" positioned receive proportionately higher loss allocations.

V. CONCLUSION

A new transmission loss allocation method for use in the context of merit-order pool dispatch is proposed and tested. It is based on a solved power flow and has the following principal characteristics:

- It is based on the exact network equations as defined by the complex impedance matrix and the complex nodal injections. All calculations are based on the sparse admittance matrix.
- The system losses are shown to be separable among the individual buses in a natural manner.
- No special approximations are required.
- In allocating losses, the method emphasizes the interactions among complex current injections as opposed to power flows.
- It is extremely simple to formulate and implement numerically.
- In a similar way to incremental loss allocation methods, the Z-bus algorithm can yield negative allocations to reward generators or loads that are strategically well positioned in the network.

Simulation studies show that the Z-bus method yields allocation levels generally consistent with expectations and with alternative loss allocation methods. The method proposed in this paper does however yield results that at some buses may differ significantly from other methods. We, however, re-emphasize that no loss allocation scheme can be deemed exact, including the Z-bus method. Any scheme can only be judged on the basis of "reasonable" criteria such as the ones suggested in this paper. In the final analysis, the acceptance of a loss allocation approach will depend on its perceived fairness by all pool participants.

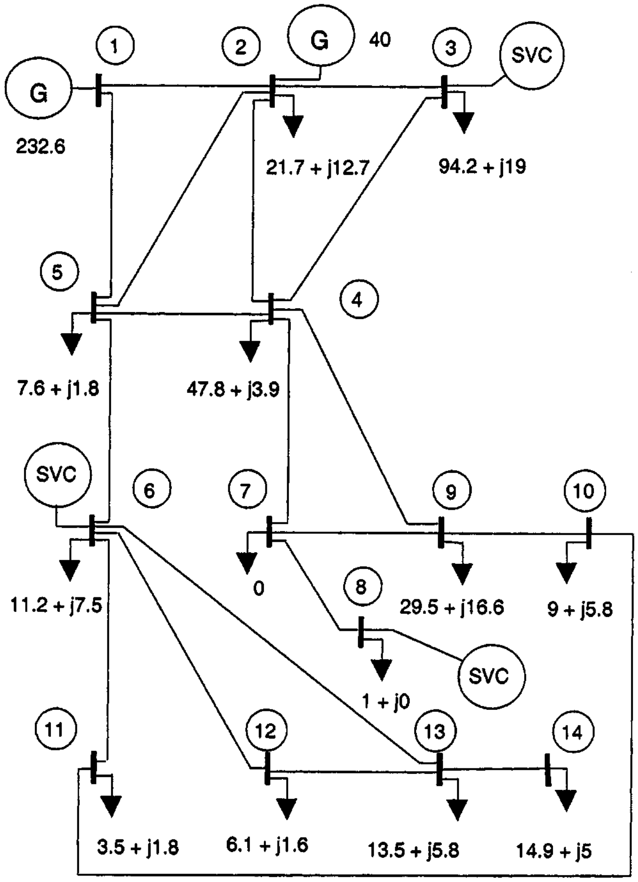

Fig. 1. One-line of the 14-bus network. Power demand and generation at every bus are given in MW.

TABLE IV
LINE DATA FOR THE 14-BUS NETWORK USED IN THE CASE STUDIES

line number	from bus	to bus	r (pu)	x (pu)	b (pu)
1	1	2	0.0194	0.0592	0.0528
2	1	5	0.054	0.223	0.0528
3	2	3	0.047	0.198	0.0438
4	2	4	0.0581	0.1763	0.0374
5	2	5	0.057	0.1739	0.034
6	3	4	0.067	0.171	0.0346
7	5	4	0.0134	0.0421	0.0128
8	4	7	0.0001	0.2091	0
9	4	9	0.0001	0.5562	0
10	5	6	0.0001	0.252	0
11	6	11	0.095	0.1989	0
12	6	12	0.1229	0.2558	0
13	6	13	0.0662	0.1303	0
14	7	8	0.0001	0.1762	0
15	7	9	0.0001	0.11	0
16	9	10	0.0318	0.0845	0
17	9	14	0.1271	0.2704	0
18	10	11	0.082	0.1921	0
19	12	13	0.2209	0.1999	0
20	13	14	0.1709	0.348	0

APPENDIX A

Proof that,

$$\Re\left\{(I^*)^T(jX)I\right\}=0. \tag{A1}$$

Proof: Since,

$$\Re\left\{(I^*)^T ZI\right\}=\Re\left\{(I)^T Z^* I^*\right\}=\Re\left\{(I^*)^T (Z^*)^T I\right\}. \tag{A2}$$

Then,

$$\Re\left\{(I^*)^T(R+jX)I\right\}=\Re\left\{(I^*)^T(R-jX)^T I\right\}. \tag{A3}$$

But since Z is generally symmetric,

$$\Re\left\{(I^*)^T(jX)I\right\}=-\Re\left\{(I^*)^T(jX)I\right\}. \tag{A4}$$

That is,

$$\Re\left\{(I^*)^T(jX)I\right\}=0. \tag{A5}$$

Q.E.D.

APPENDIX B

Fig. 1 provides the one-line of the 14-bus system [11] used in the simulations. Active and reactive load demands are specified (MW, MVAr) in the Fig. 1 underneath the load symbols, while the generators active outputs (MW) are specified next to the generator symbols. Line data for the 14-bus system are given in Table IV. The following bus voltage magnitudes are specified (in p.u.): $V_1 = 1.06$, $V_2 = 1.045$, $V_3 = 1.01$, $V_6 = 1.07$ and $V_8 = 1.09$. The base quantities are 138 kV and 100 MVA.

For the 118-bus IEEE network, standard line, voltage magnitudes of controlled buses and power injection data are used. Negative power generations are treated as loads.

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