

# Transmission Network Cost Allocation Based on Equivalent Bilateral Exchanges

Francisco D. Galiana, *Fellow, IEEE*, Antonio J. Conejo, *Senior Member, IEEE*, and Hugo A. Gil, *Student Member, IEEE*

**Abstract**—This paper presents a novel methodology for allocating the cost of a transmission network to its users based on the principle of *equivalent bilateral exchanges*, which states that after all physical laws governing the flow of power have been met, each demand is assigned a fraction of each generation and each generator is assigned a fraction of each demand in a uniform manner. Transmission cost allocation based on this principle presents several advantages, namely, independence from the choice of the slack bus, recognition of counter-flows, and transmission use charges that are stable and always positive.

**Index Terms**—Equivalent bilateral exchanges, transmission cost allocation.

## I. INTRODUCTION

THE definition and calculation of transmission cost allocation (TCA) methods can be categorized as either flat rate or flow-based. In the commonly used postage stamp (PS) method [1], loads and generators are charged a flat rate per megawatt hour irrespectively of the extent to which they “use” the various transmission lines of the network. In contrast, flow-based TCA approaches measure the use of individual transmission lines or paths by individual agents and charge each one accordingly [2]. Under this method, each line could charge a different rate for its use.

Flat rates have the advantage that they are simple to calculate and implement; however, it can be argued that users who wheel power across the breadth of the network benefit more from a flat rate than those that transact power within subnetworks with sufficient self-generation and consumption. Flow-based methods, on the other hand, exhibit a locational distinction, assigning a supplementary transmission cost to “heavy” network users and correspondingly less to “light” network users. This additional refinement is believed to promote a more efficient expansion and utilization of generation and transmission resources. How to define network “use” in a fair, systematic, and transparent manner is one of the underlying challenges faced by flow-based TCA methods.

Among the published schemes in the flow-based category, we find the proportional sharing principle (PSP), [3], [4], which

states that any power flow leaving a bus is made up of the flows entering that bus in a proportional manner, thus satisfying Kirchhoff’s current law. The TCA derived from the PSP method also requires a solved load flow together with the directions and magnitudes of the corresponding power flows. Another approach used to characterize network usage is based on the notion of generation shift distribution factors (DF) [6]–[8]. The TCA computed by this method is, however, known to depend on the nonunique and arbitrary choice of the slack bus, a property that may lead to challenges and controversy. In [5], a compromise approach is proposed that applies the proportional sharing principle to a set of geographically separate areas each having its own slack bus.

This paper begins by reviewing the notion of generation shift DF and pointing out the inherent difficulty of assigning transmission use responsibility to individual generators and loads based on such a notion. We then discuss one of the defining characteristics of the PSP method, namely its exclusion of counterflows, in other words, power flow components opposite to the direction of the net flow in a line. Through a simple example, it is demonstrated that the exclusion of counterflows can result in transmission use rates that are highly volatile with respect to the power flow operating point.

The principle of equivalent bilateral exchanges (EBE) for single-area power systems is then enunciated and justified as an alternative flow-based TCA approach that averts some of the disadvantages of other methods. Transmission use rates derived from the EBE principle are characterized by: i) independence from the choice of slack bus; ii) satisfaction of both laws of Kirchhoff; iii) the option to accept or exclude counterflows; iv) location-dependent rates; v) low temporal volatility of transmission use rates and charges; vi) allocation of nonzero charges to all network users. The EBE approach is compared to other existing TCA methods via a number of simulations on the IEEE 24-bus network.

## II. BACKGROUND

For the sake of clarity and conciseness, the presentation of the new TCA approach utilizes a DC load flow model. However, generalizations of these notions to lossy networks, currently under development, are also possible.

Under the DC load flow, let  $\gamma_{ijk}$  denote the generation shift DF [6], [7]. This parameter describes the sensitivity of the power flow through line  $k$ ,  $P_{fk}$ , with respect to  $P_i$ , the power injection at bus  $i$ , under the assumption that the slack bus is located at bus  $j$ . It is then well known that, using the DFs, any power flow can

Manuscript received December 12, 2002. This work was supported in part by the National Sciences and Engineering Research Council (NSERC), Canada, in part by the Fonds Nature et Technologies, QC, Canada, and in part by Ministerio de Ciencia y Tecnología, Project CICYT DPI 2000-0654, Spain.

F. D. Galiana and H. A. Gil are with the Department of Electrical and Computer Engineering, McGill University, Montreal, QC H3A 2A7, Canada (e-mail: galiana@ece.mcgill.ca; hugo.gil@mail.mcgill.ca).

A. J. Conejo is with Departamento de Ingeniería Eléctrica, Electrónica y Automática, E.T.S.I. Industriales, Universidad de Castilla-La Mancha, Ciudad Real E-13071, Spain (e-mail: Antonio.Conejo@uclm.es).

Digital Object Identifier 10.1109/TPWRS.2003.818689

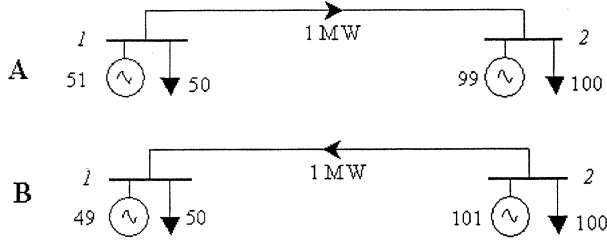


Fig. 1. Case where a small change in generation dispatch reverses the power flow direction.

be expressed as a linear combination of the power injections (see Appendix),

$$P_{fk} = \sum_i \gamma_{ijk} P_i. \quad (1)$$

Note that in (1), the DF corresponding to the slack bus injection  $P_j$  is zero for all lines  $k$ , and for any choice of slack bus  $j$ , that is

$$\gamma_{ijk} = 0; \quad \forall j, k. \quad (2)$$

Although mathematically correct, this result suggests, illogically and inequitably, that the arbitrarily chosen injection  $P_j$  has no influence on any of the network line flows, and therefore, that this injection should be allocated zero line use. Furthermore, as the Appendix shows, by changing the slack bus from  $j$  to a different arbitrary bus  $s$  the DFs are modified according to

$$\gamma_{isk} = \gamma_{ijk} - \gamma_{sjk}. \quad (3)$$

Thus, by arbitrarily changing the slack bus position, not only does the DF of the new slack bus become zero but also all other DFs change radically by the subtraction of the amount  $\gamma_{sjk}$ . The latter conclusion implies that, for a given load flow, the influence of an arbitrary injection  $P_i$  on the flow through line  $k$ , that is  $\gamma_{ijk} P_i$ , also changes significantly with the arbitrary slack bus location. The inescapable conclusion is that since transmission use and transmission charges derived directly from the DF model (1) are nonunique and arbitrary, their suitability in TCA is highly questionable.

The above and other arguments have motivated the development of methods where the TCAs do not depend on the arbitrary choice of the slack bus; in particular, the proportional sharing principle (PSP) [3]. This method, however, can display highly volatile TCA charges with respect to temporal load variations and to generator dispatch strategies. This drawback is illustrated by the simple example of Fig. 1.

This example illustrates the volatility of the TCA charges under PSP to the generation strategies; however, a similar argument could be used to show the same volatile behavior with respect to the demands.

Let the line cost in Fig. 1 be U.S.\$ 1000/h. Under PSP, at operating point A, since the power flow (1 MW) is from bus 1 to bus 2, generator 1 and load 2 share the entire line cost (U.S.\$ 500/h each), while generator 2 and load 1 pay nothing. In con-

trast, at operating point B, a small change in the dispatch that reverses the direction of the flow from 1 MW to  $-1$  MW results in a radical change in the transmission use charges. Now, generator 2 and load 1 share the entire line cost, while generator 1 and load 2 pay nothing. This volatile behavior is particularly troubling for the loads whose transmission payments are at the mercy of how the generators participate in the market.

The principal reason for the above-illustrated volatile behavior exhibited by the PSP method is that the use of a line is absolutely tied to the direction of the net power flow. More specifically, this type of volatility is related to the exclusion of counterflows in the TCA scheme.

We now introduce a new principle for flow-based network cost allocation with two main characteristics. The first is that a slack bus need not be defined, and the second is that counterflows are permitted, a property that will be shown to limit drastically the volatility of the TCAs to changes in operating point.

### III. PRINCIPLE OF EQUIVALENT BILATERAL EXCHANGES

We begin with a solved lossless dc optimal power flow that satisfies all generation and line flow constraints. From this solution, we obtain the vector of generations  $P_g$ , plus the vector of power flows  $P_f$ .

We now propose what we term the principle of equivalent bilateral exchanges (EBE). This principle states that since a solved optimal power flow meets the laws of Kirchhoff without violating any line flow or generation limit, each generation injection flows without impediment toward all of the demands, while each demand is fed by all injected generations. As such, each demand is supplied by a fraction of each generator uniformly divided among all generators. Analogously, each generator supplies a fraction of each demand uniformly divided among all demands.

As with the proportional sharing principle, the principle of EBE cannot be proved or disproved. All that can be said is that it is a reasonable assumption for the allocation of network use. However, as the results show, the EBE approach exhibits a number of advantages over other existing methods.

#### A. Equivalent Bilateral Exchanges

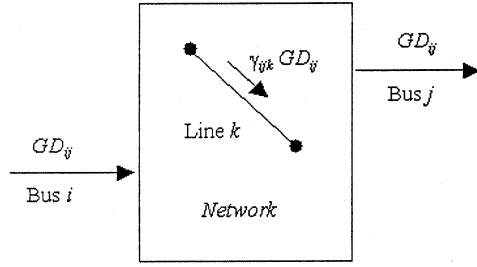
Consider a single area power system<sup>1</sup>. Following the EBE principle enunciated above, define an equivalent bilateral power exchange between the generation at bus  $i$  and the demand at bus  $j$  by,

$$GD_{ij} = \frac{P_{gi} P_{dj}}{P_d^{sys}} \quad (4)$$

where, in a lossless network, the system demand  $P_d^{sys} = \sum_j P_{dj} = \sum_i P_{gi}$ .

The equivalent exchange  $GD_{ij}$  can be viewed as the fraction of the generation  $P_{gi}$  that supplies the bus demand  $P_{dj}$ , or equivalently, the fraction of the demand  $P_{dj}$  supplied by the generation  $P_{gi}$ . It is now straightforward to decompose each indi-

<sup>1</sup>The multiarea EBE, currently under development, imposes additional constraints on cross-area equivalent bilateral exchanges.


 Fig. 2. Effect of equivalent bilateral exchange  $GD_{ij}$  on line flow  $k$ .

vidual generation and demand level into linear combinations of the equivalent bilateral exchanges

$$P_{gi} = \sum_i GD_{ij} \quad (5)$$

$$P_{dj} = \sum_i GD_{ij}. \quad (6)$$

With this decomposition, the effect of the equivalent bilateral exchange  $GD_{ij}$ , on the power flow in line  $k$ ,  $P_{fk}$ , is determined uniquely by  $\gamma_{ijk} GD_{ij}$ , an operation that does *not* require the definition of a slack bus and that satisfies both laws of Kirchhoff. As illustrated in Fig. 2, this relation can also be found by solving a load flow consisting of a single power injection of  $GD_{ij}$  at bus  $i$  and a single power extraction of the same amount at bus  $j$ , the latter bus becoming the slack bus for this particular power exchange. Thus, in effect, each EBE has a built-in slack bus.

The net flow in an arbitrary line  $k$  can therefore be expressed uniquely in terms of the equivalent bilateral exchanges as

$$P_{fk} = \sum_{i,j} \gamma_{ijk} GD_{ij}. \quad (7)$$

### B. Transmission-Use Components and Charges Based on Equivalent Bilateral Exchanges

Under the EBE principle, each flow component  $\gamma_{ijk} GD_{ij}$ , is deemed to “use” line  $k$  irrespectively of its sign with respect to the net flow in line  $k$ ,  $P_{fk}$ , a property that, in essence, substantiates the existence of counterflows. The “use” of transmission line  $k$  by the bilateral exchange  $GD_{ij}$ , is then defined by

$$U_{ijk} = |\gamma_{ijk}| GD_{ij}. \quad (8)$$

Some general properties of network “use” under the EBE principle derived from (8) are

- i) Bilateral exchanges between generators and loads at the same bus  $i$  do not make use of the network, that is

$$U_{ijk} = 0. \quad (9)$$

- ii) With some exceptions involving radial subnetworks, a nonzero exchange  $GD_{ij}$  with  $i$  different from  $j$  affects all line flows  $k$ , that is

$$U_{ijk} > 0; \quad \forall k. \quad (10)$$

This characteristic is important as it implies that every generator and load contribute a positive amount to the combined network cost.

- iii) Even if  $P_{fk} = 0$ , from (10) it follows that the use of any line  $k$  is always nonzero. This property ensures that the rate of use of such lines remains stable with respect to the system operating point.

Next, we define the use of line  $k$  by generation  $P_{gi}$  as the sum of the use of line  $k$  by all equivalent exchanges involving generator  $i$ , that is

$$UG_{ik} = \sum_j |\gamma_{ijk}| GD_{ij}. \quad (11)$$

Analogously, the use of line  $k$  by demand  $P_{dj}$  is the sum of the use of line  $k$  by all equivalent exchanges involving demand  $j$ , that is

$$UD_{jk} = \sum_i |\gamma_{ijk}| GD_{ij}. \quad (12)$$

The total use of line  $k$  due to all equivalent bilateral exchanges is

$$UL_k = \sum_{i,j} |\gamma_{ijk}| GD_{ij}. \quad (13)$$

If a line  $k$  is entitled to charge a known rate  $r_k$  in U.S./MWh for its use, then the *TUC* allocated to generation  $P_{gi}$  for the use of all lines, according to (11) becomes<sup>2</sup>,

$$TUCG_j = \frac{\sum_k r_k UG_{jk}}{2}. \quad (14)$$

Similarly, the *TUC* allocated to demand  $P_{dj}$  for the use of all lines, according to (12) becomes

$$TUCD_j = \frac{\sum_k r_k UD_{jk}}{2}. \quad (15)$$

If the line cost  $C_k$  in U.S./h rather than the line use rate  $r_k$  is specified, then the latter would be defined by the specified line cost divided by the total line use over all equivalent exchanges, that is

$$r_k = \frac{C_k}{UL_k}. \quad (16)$$

Two other relevant rates are also defined. One, denoted by  $rg_i$  denotes the rate in U.S./MWh at which generator  $i$  is charged for the use of the network. The second, denoted by  $rd_j$  denotes the rate in U.S./MWh at which demand  $j$  is charged for the use of the network

$$\begin{aligned} rg_i &= \frac{TUCG_i}{P_{gi}}; \quad \forall i \text{ with } P_{gi} > 0 \\ rd_j &= \frac{TUCD_j}{P_{dj}}; \quad \forall j \text{ with } P_{dj} > 0. \end{aligned} \quad (17)$$

## IV. APPLICATIONS AND RESULTS

### A. Numerical Example

To explain the use of the EBE methodology, we first consider the simple lossless test system shown in Fig. 3, including loads,

<sup>2</sup>Assuming a 50/50 split in transmission costs between generation and demand.

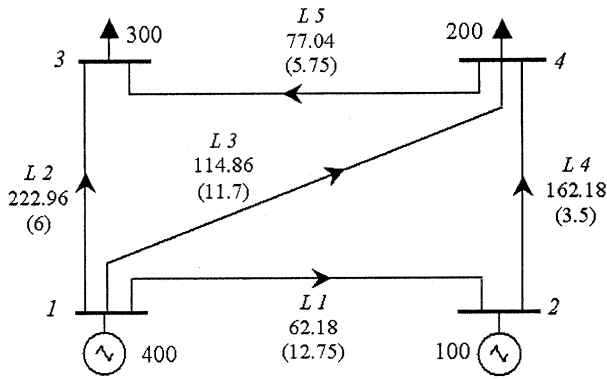


Fig. 3. Four-bus test system.

TABLE I  
LINE USE RATES UNDER EBE

| Line          | 1     | 2     | 3     | 4     | 5     |
|---------------|-------|-------|-------|-------|-------|
| Rate (\$/MWh) | 0.114 | 0.027 | 0.094 | 0.022 | 0.032 |

generations, and line flows, all in megawatt [3]. The line costs in U.S.\$/h are shown in brackets adjacent to the power flows.

In this example, according to (4), the only nonzero equivalent bilateral exchanges in MW are

$$GD_{13} = 240, \quad GD_{14} = 160, \quad GD_{23} = 60, \quad GD_{24} = 40.$$

According to (13), the use of line 1 is

$$\begin{aligned} UL_1 &= |\gamma_{131}|GD_{13} + |\gamma_{141}|GD_{14} + \gamma_{231}|GD_{23} \\ &\quad + |\gamma_{241}|GD_{24} \\ &= 0.1635 * 240 + 0.3 * 160 + 0.3052 * 60 \\ &\quad + 0.1687 * 40 \\ &= 112.30 \text{ MW.} \end{aligned}$$

From (16), the corresponding line use rate is

$$r_1 = \frac{C_1}{UL_1} = \frac{12.75}{112.30} = 0.114 \frac{\$}{\text{MWh.}}$$

Table I shows the line use rates for all lines in the example.

Now, using (14), the transmission charge applied to generator 1 is

$$TUCG_1 = \frac{[r_1UG_{11} + r_2UG_{12} + r_3UG_{13} + r_4UG_{14} + r_5UG_{15}]}{2}$$

where  $UG_{11} = |\gamma_{131}|GD_{13} + |\gamma_{141}|GD_{14} = 0.1635 * 240 + 0.3 * 160 = 87.24$  MW, and so on. Then, the use of the entire

TABLE II  
CHARGES FOR USE OF THE NETWORK

|             | Bus   |      |       |      | Total |
|-------------|-------|------|-------|------|-------|
|             | 1     | 2    | 3     | 4    |       |
| TUCG (\$/h) | 16.21 | 3.64 | -     | -    | 19.85 |
| TUCD (\$/h) | -     | -    | 11.19 | 8.66 | 19.85 |

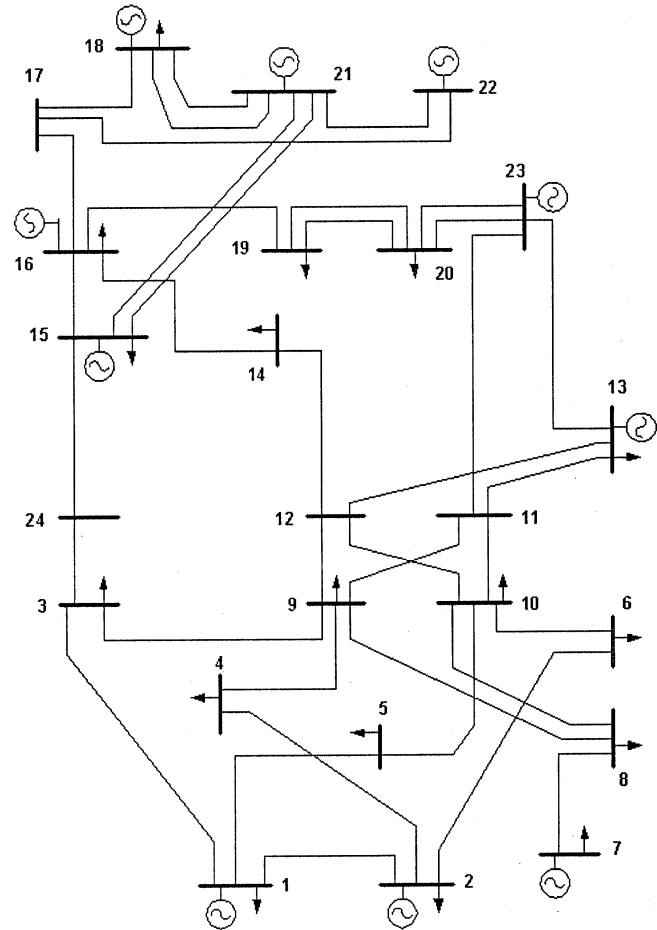


Fig. 4. RTS96 one-area system.

network by generator 1 is (see equation at the bottom of the page).

Charges to all generators and demands can be calculated by applying the same procedure. The results are shown in Table II.

Adding up the total payments from Table II results in U.S.\$ 39.7/h, which is the total network cost.

### B. Comparison With Other TCA Methodologies

The EBE methodology was compared with two alternative TCA methods on the RTS96 system (Fig. 4), [9].

The two methodologies selected for comparison were

$$TUCG_1 = \frac{[0.114 * 87.24 + 0.027 * 187.75 + 0.094 * 116.14 + 0.022 * 87.24 + 0.032 * 139.60]}{2} = 16.21 \frac{\$}{h}$$

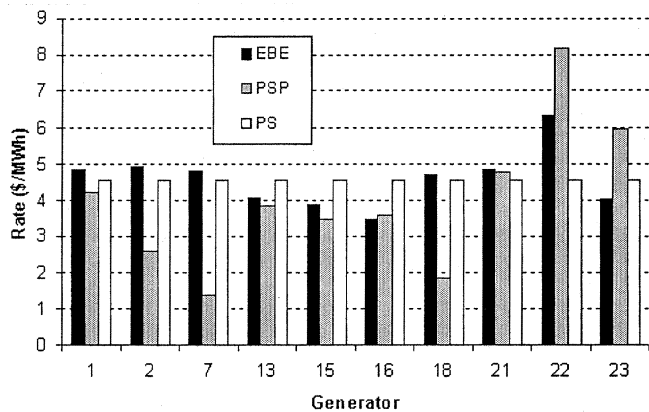


Fig. 5. Network-use generator rates.

TABLE III  
GENERATOR NETWORK USE RATE STATISTICS

|                   | EBE  | PSP  | PS   |
|-------------------|------|------|------|
| Minimum (\$/MWh)  | 3.49 | 1.41 | 4.52 |
| Maximum (\$/MWh)  | 6.34 | 8.15 | 4.52 |
| Average (\$/MWh)  | 4.60 | 3.99 | 4.52 |
| $\sigma$ (\$/MWh) | 0.80 | 1.98 | 0    |
| Volatility (%)    | 17.3 | 49.7 | 0    |

- *Postage stamp (PS)*: This methodology allocates the costs of transmission to generators and demands in proportion to their production and consumption, irrespectively of their relative network location.
- *Power sharing principle (PSP)* [3], [4]: This flow-based methodology states that any power flow leaving a bus is made up of the flows entering that bus in a proportional manner. Unlike the EBE method, PSP does not permit counterflows. Moreover, whereas under EBE the use of a line depends on the line flow distribution factors  $\{\gamma_{ijk}\}$ , which are derived from both laws of Kirchhoff, under PSP, line use is defined accounting only for Kirchhoff's current law.

Fig. 5 compares the generator network usage rates  $rg$  under the three TCA methodologies, while Table III summarizes a number of statistics from this figure. Only nonzero charges are shown. In all three methods, the line costs are allocated evenly between generators and loads.

We see from these results that the generator network usage rates under PSP show the highest variability with respect to the network location. This “spatial volatility” can be viewed as a locational signal indicating to potential generation investors the preferred network locations from the point of view of transmission use charges.

From Table III, we note that the average rate with the EBE methodology is U.S.\$ 4.60/MWh compared to U.S.\$ 3.99/MWh under PSP. As the PS is a flat-rate, all generators are charged at U.S.\$ 4.52/MWh. The standard deviation with the PSP method is however much higher, reflecting the higher rate volatility (defined as the ratio of the standard deviation and the average). Among the three methods, the EBE approach lies in an intermediate position in terms of locational volatility.

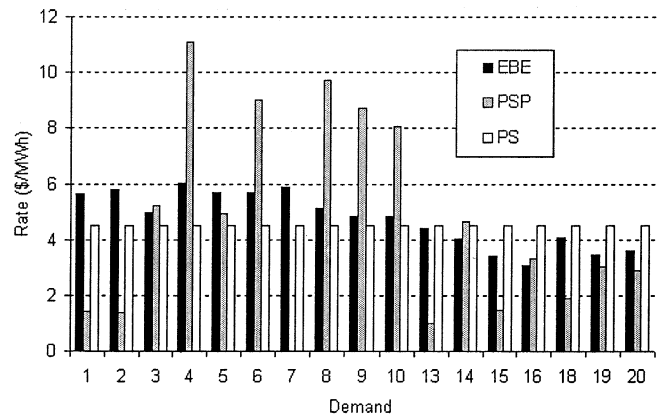


Fig. 6. Network-use demand rates.

TABLE IV  
DEMAND NETWORK USE RATE STATISTICS

|                   | EBE  | PSP   | PS   |
|-------------------|------|-------|------|
| Minimum (\$/MWh)  | 3.06 | 0     | 4.52 |
| Maximum (\$/MWh)  | 6.01 | 11.07 | 4.52 |
| Average (\$/MWh)  | 4.75 | 4.90  | 4.52 |
| $\sigma$ (\$/MWh) | 0.99 | 3.61  | 0    |
| Volatility (%)    | 20.8 | 73.7  | 0    |

With respect to the demands, Fig. 6 and Table IV show that the usage rates  $rd$  display an even higher spatial volatility, particularly under PSP.

Referring to Fig. 6, both EBE and PSP charge higher rates to the demands located in the “southern” region of the system and lower rates to those demands located in the north, as power tends to flow from north to south. Since the loads at buses 1, 2, and 7 at the far south of the network are fed by lines with large counterflows, under the PSP method these are charged relatively low rates. In fact, the load at bus 7 is allocated a zero rate under PSP. In contrast, under EBE, the same loads pay a higher rate than under both PSP and PS, accounting for the fact that the south is a predominantly net importer of power, and therefore, makes heavy use of the network.

Because only net flows are considered under the PSP methodology, generators and demands are generally charged for the use of fewer lines but at a higher line-use rate. On the other hand, the EBE methodology generally charges for the use of all lines but at a lower line rate.

As indicated before, a strong locational signal (i.e., high spatial volatility in the network-use rate) may be desirable as it leads new agents (correctly or incorrectly) to consider their future location in the network. On the other hand, for *ex-post* methodologies like the EBE or PSP, high spatial volatility can also lead to temporal volatility under which network-use rates fluctuate significantly over time. This behavior raises concern because agents (generators and demands) prefer *stable* transmission charges in order to better financially and economically plan their projects.

To study temporal volatility under EBE and PSP, network-use rates were calculated for every hour of the year according to

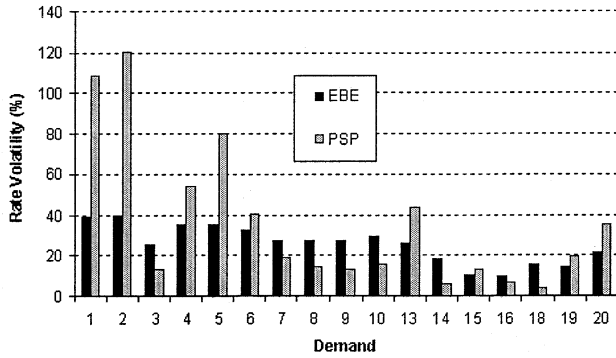


Fig. 7. Demand network use rate temporal volatility.

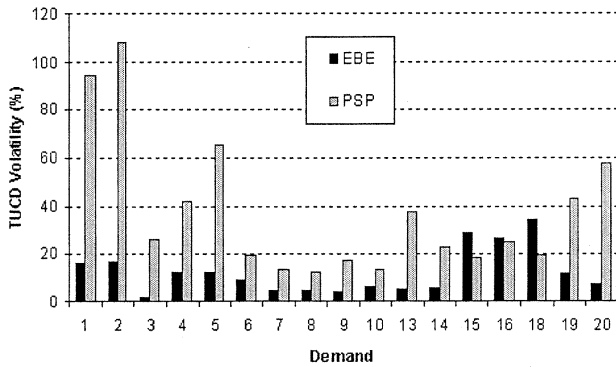


Fig. 8. Demand network use charge temporal volatility.

the seasonal load profile presented in [9]. From the perspective of the generators, the temporal volatility is very close to the volatility of the actual generation dispatch, and the three methodologies presented similar behavior.

From the point of view of the demands, the comparative EBE and PSP temporal volatilities are shown in Figs. 7 and 8, the former with respect to the demand use rates,  $rd$ , and the latter with respect to the transmission charges,  $TUCD$ .

Fig. 7 shows that under the PSP methodology, certain demands face considerably higher temporal rate volatility due to their southern location in an area with strong counterflows. Fig. 8 shows that, with some exceptions, the volatility of the charges under EBE is comparatively even lower than under PSP.

The relevant fact from these results is that typically the EBE methodology provides demands with substantially more stable charges as the load conditions change over time.

## V. CONCLUSIONS

This paper has presented a novel methodology for allocating transmission use costs to generators and demands. It is based on the principle of EBE under which every demand is assigned a fraction of each generation uniformly divided among all generators. In a similar way, each generator is assigned a fraction of each demand uniformly divided among all generators.

This methodology has the advantage of accounting for counterflows while all payments from generators and demands are always positive. Also, this methodology has the important advantage of being independent from the choice of a slack bus, an always-controversial issue.

The EBE methodology exhibits temporal and operating point stability, a most important request from network users who desire fair and stable charges in order to better plan their undertakings. In addition, the EBE method displays a good degree of variability in the usage rates and charges across the network, an important price signal needed to stimulate the efficient expansion of the transmission network.

A current project is investigating the application of the EBE principle to large multiarea interconnected networks that account for control areas and tie-lines. The idea here is to limit the definition of equivalent exchanges to within control areas, thus ensuring that no user is charged for the use of external interconnected networks.

## APPENDIX

Let the DC load flow equations be denoted by

$$B\delta = P_g - P_d. \quad (18)$$

In (18), the DC load flow matrix  $B$  is of dimension  $n$  by  $n-1$ ,  $\delta \in \mathbb{R}^{n-1}$  is the vector of nodal phase angles,  $P \in \mathbb{R}^n$  is the vector of net real power bus injections,  $P_g \in \mathbb{R}^n$  is the vector of real power bus generations, and  $P_d \in \mathbb{R}^n$  is the vector of specified real power bus demands. If  $P_{fk}$  is the real power flow across an arbitrary line  $k$  then, according to the DC load flow approximation, this power flow can be expressed as the product of the line susceptance and the angle difference across the line. Mathematically this relation can be expressed by

$$P_{fk} = h_k^T \delta. \quad (19)$$

As the matrix  $B$  is singular with rank  $n-1$ , the vector of injections  $P$  cannot be arbitrarily specified in (18). However, certain injection vectors, namely those belonging to the range space of  $B$ , do satisfy (18) uniquely. One such example occurs when 1 MW of power is injected into bus  $i$  and extracted at bus  $j$  (the slack generator). Denote this vector of injections  $e_{ij}$  and let the corresponding vector of phase angles be  $\delta_{ij}$ . Then

$$B\delta_{ij} = e_{ij}. \quad (20)$$

The corresponding power flow is then the line flow distribution factor (DF),  $\gamma_{ijk}$ , that is

$$\gamma_{ijk} = P_{fk} = h_k^T \delta_{ij}. \quad (21)$$

Clearly, if  $i = j$  the vector  $e_{ij} = 0$  so that  $\gamma_{ijk} = 0$ , that is, injecting and extracting one megawatt at the same bus has zero effect on the line flows.

Since the DC load flow is linear, for any feasible set of injections<sup>3</sup>, the corresponding line flows are given by

$$P_{fk} = \sum_i \gamma_{ijk} P_i. \quad (22)$$

Now, suppose that for a solved load flow case, the slack bus is changed from  $j$  to  $s$ .

<sup>3</sup>In the DC load flow, a set of injections is feasible if and only if  $\sum_{i=1}^n P_i = 0$

Although the power flows remain unchanged, we show next that the distribution factors must vary. To see this, first use the power balance relation to express the new slack generation  $P_s$  in terms of the other injections

$$P_s = - \sum_{i \neq s} P_i. \quad (23)$$

Substituting into (22) gives

$$\begin{aligned} P_{fk} &= \sum_{i \neq s} \gamma_{ijk} P_i + \gamma_{sjk} \left( - \sum_{i \neq s} P_i \right) \\ &= \sum_i (\gamma_{ijk} - \gamma_{sjk}) P_i. \end{aligned} \quad (24)$$

Since (24) is true for any set of injections, then

$$\gamma_{isk} = \gamma_{ijk} - \gamma_{sjk}. \quad (25)$$

Using (25), it is straightforward to calculate the DFs with respect to any slack bus from those DFs found with respect to one specific slack bus.

#### REFERENCES

- [1] H. H. Happ, "Cost of wheeling methodologies," *IEEE Trans. Power Syst.*, vol. 9, pp. 147–156, Feb. 1994.
- [2] J. W. M. Lima, "Allocation of transmission fixed charges: An overview," *IEEE Trans. Power Syst.*, vol. 11, pp. 1409–1418, Aug. 1996.
- [3] J. Bialek, "Topological generation and load distribution factors for supplement charge allocation in transmission open access," *IEEE Trans. Power Syst.*, vol. 12, pp. 1185–1193, Aug. 1997.
- [4] D. Kirschen, R. Allan, and G. Strbac, "Contributions of individual generators to loads and flows," *IEEE Trans. Power Syst.*, vol. 12, pp. 52–60, Feb. 1997.
- [5] C. Vásquez, L. Olmos, and I. J. Pérez-Arriaga, "On the selection of the slack bus in mechanisms for transmission network cost allocation that are based on network utilization," in *Proc. 14th Power Syst. Comput. Conf.*, Sevilla, Spain, June 24–28, 2002.

- [6] H. D. Limmer, "Techniques and applications of security calculation applied to dispatching computers," in *Proc. 3rd Power Syst. Comput. Conf.*, Rome, Italy, 1969.
- [7] W. Y. Ng, "Generalized generation distribution factors for power system security evaluations," *IEEE Trans. Power App. Syst.*, vol. PAS-100, pp. 1001–1005, Mar. 1981.
- [8] ANEEL—Agencia Nacional de Energia Eletrica, Brazil. [Online]. Available: <http://www.aneel.gov.br>
- [9] "The IEEE reliability test system—1996," *IEEE Trans. Power Syst.*, vol. 14, pp. 1010–1020, Aug. 1999.

**Francisco D. Galiana** (F'92) received the B.Eng. (Hons.) degree from McGill University, Montreal, QC, Canada, in 1966, and the S.M. and Ph.D. degrees from the Massachusetts Institute of Technology, Cambridge, in 1968 and 1971, respectively.

Currently, he is Professor of electrical engineering at McGill University. He spent several years at the Brown Boveri Research Center, Baden, Switzerland, and the University of Michigan, Ann Arbor. His current research interests are in the analysis of power systems under competition.

**Antonio J. Conejo** (S'86–M'91–SM'98) received the B.S. degree in electrical engineering from the Universidad P. Comillas, Madrid, Spain, in 1983, the M.S. degree in electrical engineering from the Massachusetts Institute of Technology, Cambridge, in 1987, and the Ph.D. degree in electrical engineering from the Royal Institute of Technology, Stockholm, Sweden, in 1990.

Currently, he is Professor of Electrical Engineering at the Universidad de Castilla-La Mancha, Ciudad Real, Spain. His research interests include control, operations, planning and economics of electric energy systems, as well as optimization theory and its applications.

**Hugo A. Gil** (S'00) received the Eng. degree in electrical engineering from the Universidad Nacional de Colombia in 1995, and the Dr. Eng. degree in electrical engineering from the Universidade Federal de Santa Catarina, Brazil, in 2001.

Currently, he is a Post-Doctoral Fellow at McGill University, Montreal, QC, Canada. His research interests are power systems expansion planning, transmission cost allocation, as well as optimization and microeconomics and their applications to power systems.