SHORT-TERM HYDRO-THERMAL COORDINATION BY LAGRANGIAN RELAXATION: SOLUTION OF THE DUAL PROBLEM

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Abstract - This paper addresses the short-term hydro-thermal coordination problem. This problem is large-scale, combinatorial and nonlinear. It is usually solved using a Lagrangian Relaxation approach. In the framework of the Lagrangian Relaxation, this paper provides a novel, non-oscillating and efficient multiplier updating procedure. This procedure advantageously compares with previously reported procedures such us subgradient and bundle methods. A realistic large-scale case study is used to illustrate the behavior of the proposed procedure.

Keywords: Short-term Hydro-thermal Coordination, Large-scale Optimization, Lagrangian Relaxation, Multiplier Updating.

1 Introduction

This paper addresses the short-term hydro-thermal coordination problem. This problem is solved to determine the start-up and shut-down schedule of thermal plants, as well as the power output of thermal and hydro plants during a short-term planning horizon. The objective is to meet customer demand with appropriate levels of spinning reserve so that total operating costs are minimized.

This is a large-scale mixed-integer nonlinear problem. As recognized in the technical literature, the Lagrangian Relaxation (LR) technique is the most promising procedure to solve the short-term hydro-thermal coordination problem [1, 2, 3, 4, 5, 6, 7, 8]. Dynamic programming requires discretization and drastic simplifying assumptions to make the problem computationally tractable [9]. Mixed integer linear programming requires linearization and the solution of large-scale mixed-integer linear problems which is not a trivial task [10, 11, 12].

The LR procedure decomposes the original problem in one subproblem per thermal plant and one subproblem per hydroelectric system. This decomposition allows a precise modeling of the generation system resulting in a very accurate solution technique.

PE-333-PWRS-0-12-1997 A paper recommended and approved by the IEEE Power System Analysis, Computing and Economics Committee of the IEEE Power Engineering Society for publication in the IEEE Transactions on Power Systems. Manuscript submitted July 1, 1997; made available for printing December 12, 1997.

Instead of solving the original problem, the LR technique solves its dual problem. The difference between the objective function optimal values for the primal and dual problems is called the duality gap. In most practical cases the duality gap is below 0.1%. As a byproduct of the solution of the dual problem a solution for the primal problem is obtained. However, more often than not, this primal solution is infeasible. Heuristic procedures are required to get a feasible primal solution.

The LR technique consists of the three phases below:

Phase 1. To solve the dual problem.

Phase 2. To obtain a primal feasible solution.

Phase 3. To exactly dispatch committed generation to meet the demand.

Phase 2 is easily accomplished as stated in most previous LR works [2, 4]. Heuristic techniques are successfully applied to obtain a primal feasible solution, i.e. a primal feasible set of commitment decisios. *Phase 3* is a multiperiod economic dispatch whose solution is well stated in the technical literature [13].

However, *Phase 1* requires the solution of a non-differentiable maximization problem which is not an easy task. The procedures reported in the literature to solve this type of problems are either oscillating or computationally inefficient, or both, oscillating and computationally inefficient.

This paper focuses on *Phase 1.* It provides a solution technique which is non-oscillating, computationally efficient and which does not rely on "messy" heuristics.

The most commonly used technique to address *Phase 1* is the subgradient [2, 4, 5, 6] which is computationally inefficient, and produces oscillating solutions which makes it complicated to define appropriate stopping criteria. The cutting plane technique [14], under mild assumptions, converges to the optimum but makes it very slowly. The recently proposed primal bundle method [15, 16] is a cutting plane method whose objective function has been penalized to control the path to the optimum. This penalty technique is complicated and requires careful tune-up of penalty parameters to be efficient. The required tune-up is problem-dependent and difficult to implement. The dual bundle method [16, 17] exhibits the same type of problems as the primal bundle method. However, it is a less elaborated procedure than the primal one. It is close to the subgradient method, and therefore it is prone to oscillations. The technique proposed in this paper is a cutting plane method which incorporates adaptive control over the feasible region. This is easily accomplished because the feasibility region is simple: just bounds on multipliers. This adaptive control can be performed without involved parameter tune-up, resulting in a non-oscillating, computationally efficient algorithm.

In the framework of de-regulated electric energy markets, the LR technique has a well known and simple yet elegant economic interpretation (see, for instance, [18]) which is described below. Hourly energy price proposals are specified by the market operator for the planning horizon (e.g. the 168 hours of one week). Each generator schedules independently its production along the planning horizon to maximize its benefit. Analogously, each hydroelectric system is scheduled so that its benefit is maximum. After the submission of production proposals by all generators the power balance equation is evaluated in each hour. Excess of committed power will occur in some hours, whereas deficit will occur in some other hours. Hourly prices are updated by the market operator and the previous procedure repeated until the demand is satisfied. The above mechanism constitutes a competitive energy market. Similarly, a spinning reserve market can be established.

The price updating or multiplier updating is in fact a solution procedure for the dual problem. If multipliers are updated proportionally to the power mismatch, the subgradient technique is obtained. If the updating procedure keeps track of the information of all steps to reconstruct total cost as a function of multiplier values, the cutting plane method is obtained. By proper modification of the objective function of the problem to be solved when using the cutting plane technique, the bundle method is derived. And finally, by proper adaptive modification of the feasibility region of the problem to be solved when using the cutting plane technique, the method proposed in this paper is derived.

This paper is organized as follows. Section 2 gives a formulation of the short-term hydro-thermal coordination problem. Section 3 provides a novel approach to update the multipliers in the framework of a LR solution procedure. This updating procedure is compared with previous procedures. Section 4 includes two case studies. The first one is designed to compare the behavior of the different procedures to update multipliers, the second one is a large-scale case study based on the electric energy system of mainland Spain. Section 5 provides conclusions.

2 Problem formulation

The short-term hydro-thermal coordination problem can be formulated as follows:

Minimize$_{(x,y)}$

$$f(x) = \sum_i f_i(x_i) \tag{1}$$

subject to

$$s_i(x_i) \le 0 \qquad \forall i \tag{2}$$

$$s_j(y_j) \le 0 \qquad \forall j \tag{3}$$

$$g(x,y) = G - \sum_i g_i(x_i) - \sum_j g_j(y_j) \le 0 \tag{4}$$

$$h(x,y) = H - \sum_i h_i(x_i) - \sum_j h_j(y_j) = 0 \tag{5}$$

where $(x,y) = (x_i, y_j;\ \forall i, \forall j)$, variables x_i are the variables related to thermal plant i, and y_j are the variables related to the hydroelectric system j. G, $g_i(x_i)$, $g_j(y_j)$, H, $h_i(x_i)$ and $h_i(y_j)$ are vectors of dimension equal to the number of subperiods in the planning horizon.

Equation (1) is the production cost to be minimized, equations (2) are the constraints related to every thermal unit, equations (3) are the constraints related to every hydroelectric system, equation (4) enforces spinning reserve constraints, and equation (5) enforces demand constraints. It should be noted that time is embedded in the above formulation.

Equations (4) and (5) are global constraints which couple together thermal-related and hydro-related variables.

Problem (1)-(5) is denominated the primal problem (PP).

Global constraints are incorporated in the objective function of the primal problem through Lagrangian multipliers to obtain the Lagrangian function:

$$\begin{aligned}\mathcal{L}(x,y,\lambda,\mu) = \\ \sum_i f_i(x_i) + \lambda^{\mathsf{T}}\left[H - \sum_i h_i(x_i) - \sum_j h_j(y_j)\right] \\ +\mu^{\mathsf{T}}\left[G - \sum_i g_i(x_i) - \sum_j g_j(y_j)\right].\end{aligned} \tag{6}$$

where λ and μ are vectors of dimension equal to the number of subperiods in the planning horizon. The dual problem (DP) of the original primal problem (1)-(5) has the form

$$\text{Maximize}_{(\lambda,\mu)} \quad \phi(\lambda,\mu) \tag{7}$$

$$\text{subject to} \quad \mu \ge 0 \tag{8}$$

where

$$\phi(\theta) = \phi(\lambda,\mu) = \lambda^{\mathsf{T}} H + \mu^{\mathsf{T}} G + d(\lambda,\mu), \tag{9}$$

where $\theta = (\lambda, \mu)$, and $d(\lambda,\mu)$ is the solution of the decomposed primal problem (DPP) stated below

Minimize$_{(x,y)}$

$$\begin{aligned}\sum_i \left[f_i(x_i) - \lambda^{\mathsf{T}} h_i(x_i) - \mu^{\mathsf{T}} g_i(x_i)\right] \\ - \sum_j \left[\lambda^{\mathsf{T}} h_j(y_j) + \mu^{\mathsf{T}} g_j(y_j)\right]\end{aligned} \tag{10}$$

subject to

$$s_i(x_i) \le 0 \qquad \forall i \tag{11}$$

$$s_j(y_j) \le 0 \qquad \forall j. \tag{12}$$

The above problem decomposes in one subproblem per thermal unit and one subproblem per hydroelectric system.

The subproblem associated to thermal unit i is

$$\begin{aligned} \text{Minimize}_{x_i} \quad & f_i(x_i) - \lambda^{\mathsf{T}} h_i(x_i) - \mu^{\mathsf{T}} g_i(x_i) && (13) \\ \text{subject to} \quad & s_i(x_i) \leq 0, && (14) \end{aligned}$$

and the subproblem associated to hydroelectric system j is

$$\begin{aligned} \text{Maximize}_{y_j} \quad & \lambda^{\mathsf{T}} h_j(y_j) + \mu^{\mathsf{T}} g_j(y_j) && (15) \\ \text{subject to} \quad & s_j(y_j) \leq 0. && (16) \end{aligned}$$

The Lagrangian Relaxation procedure to solve the dual problem works as follows:

1. Initialize multiplier vector $\theta = (\lambda, \mu)$.
2. Solve the decomposed primal problem by solving 1 subproblem per thermal plant (13)-(14) and 1 subproblem per hydroelectric system (15)-(16).
3. Update the multiplier vector using any of the procedures stated below (Section 3).
4. If the convergence criterion is satisfied, stop. Otherwise go to 2. Convergence criteria are stated below (Section 3).

3 Multiplier updating

The column vector of constraint mismatches at iteration ν constitutes a subgradient as stated in [14], i.e.

$$s^{(\nu)} = \text{column}[h(x^{(\nu)}, y^{(\nu)}), g(x^{(\nu)}, y^{(\nu)})] \tag{17}$$

is a subgradient vector which is used below.

3.1 Subgradient method (SG)

The multiplier vector is updated as stated below [19].

$$\theta^{(\nu+1)} = \theta^{(\nu)} + k^{(\nu)} \frac{s^{(\nu)}}{|s^{(\nu)}|} \tag{18}$$

where

$$\lim_{\nu \to \infty} k^{(\nu)} \to 0 \quad \text{and} \quad \sum_{\nu=1}^{\infty} k^{(\nu)} \to \infty \tag{19}$$

The SG method is simple to implement and its computational burden is small. However, it progresses slowly to the optimum in an oscillating fashion. This is a consequence of the non-differentiability of the dual function. Furthermore, the oscillating behavior makes it very difficult to devise an appropriate stopping criterion. It is typically stopped after a pre-specified number of iterations.

An elaborated subgradient procedure which is more efficient than the original subgradient method is reported in [20]. However, it retains the same drawbacks as the original subgradient procedure.

3.2 Cutting plane method (CP)

The updated multiplier vector is obtained solving the linear programming problem below.

$$\begin{aligned} \text{Maximize}_{z,\theta \in \mathcal{C}} \quad & z && (20) \\ \text{subject to} \quad & z \leq \phi^{(k)} + s^{(k)\mathsf{T}}(\theta - \theta^{(k)}) && (21) \\ & k = 1, \cdots, \nu \end{aligned}$$

where $\mathcal{C}$ is a convex and compact set. It is made up of the ranges of variation of the multipliers, i.e. $\mathcal{C} = \{\theta, \underline{\theta} \leq \theta \leq \overline{\theta}\}$.

It should be noted that equation (21) represents a half-space (hyperplane) on the multiplier space.

It should also be noted that the number of constraints of the problem above grows with the number of iterations.

The above problem (20)-(21) is a relaxed dual problem which gets closer to the actual dual problem as the number of iterations grows.

The CP method achieves a dual optimum by "reconstructing" the dual function. It reconstructs the region of interest as well as other regions of no interest. This reconstruction is computationally expensive and therefore the CP method computational burden is high.

This algorithm is typically stopped when the multiplier vector difference between two consecutive iterations is below a pre-specified threshold.

3.3 Bundle method (BD)

The updated multiplier vector is obtained solving the relaxed dual quadratic programming problem below.

$$\begin{aligned} \text{Maximize}_{z,\theta \in \mathcal{C}} \quad & z - \alpha^{(k)} |\theta - \Theta^{(k)}|^2 && (22) \\ \text{subject to} \quad & z \leq \phi^{(k)} + s^{(k)\mathsf{T}}(\theta - \theta^{(k)}) && (23) \\ & k = 1, \cdots, \nu \end{aligned}$$

where α is a penalty parameter and Θ, the "center of gravity", is a vector of multipliers centered in the feasibility region so that oscillations are avoided [15].

It should be noted that the number of constraints of the problem above grows with the number of iterations.

The BD method is a CP method in which the ascent procedure is constrained by an objective function penalty. The target is to center the CP method in the region of interest. However, in order to do this, it is necessary to carefully tune-up the penalty and other parameters [15, 16]. This tune-up is problem dependent and hard to achieve.

This algorithm is typically stopped when the multiplier vector difference between two consecutive iterations is below a pre-specified threshold.

3.4 Dynamically constrained cutting plane method (DC-CP)

The updated multiplier vector is obtained solving the relaxed dual linear programming problem below.

$$\text{Maximize}_{z,\theta\in\mathcal{C}^{(\nu)}} \quad z \tag{24}$$

$$\text{subject to} \quad z \le \phi^{(k)} + s^{(k)\mathsf{T}}(\theta - \theta^{(k)}) \tag{25}$$
$$k = 1,\cdots,n; n \le \overline{n}$$

where $\overline{n}$ is the maximum number of constraints considered when solving the problem above, and $\mathcal{C}^{(\nu)}$ is a dynamically updated set defining the feasibility region for the multipliers.

When the number of iterations is larger than the specified maximum number of constraints, the excess constraints are eliminated as stated below.

At iteration ν the difference between every hyperplane evaluated at the current multiplier vector and the actual value of the objective function for the current multiplier vector (residual) is computed as

$$\epsilon_i = \phi^{(i)} + s^{(i)\mathsf{T}}(\theta^{(\nu)} - \theta^{(i)}) - \phi^{(\nu)} \quad \forall i = 1,\cdots,n. \tag{26}$$

As soon as n is larger than $\overline{n}$, the "most distant" hyperplanes are not considered, so that the number of hyperplanes is kept constant and equal to $\overline{n}$. It should be noted that the residual ϵ_i is always positive because the cutting plane reconstruction of the dual function overestimates the actual dual function. This technique to limit the number of hyperplanes considered has proved to be computationally effective.

The dynamic updating of the set $\mathcal{C}^{(\nu)}$, the feasibility region of the multipliers, is performed as stated below. Let $\theta_i^{(\nu)}$ be the i component of the multiplier vector at iteration ν.

If $\theta_i^{(\nu)} = \overline{\theta}_i^{(\nu)}$ then

$$\overline{\theta}_i^{(\nu+1)} = \overline{\theta}_i^{(\nu)}(1+a)\ , \text{ and}$$
$$\underline{\theta}_i^{(\nu+1)} = \overline{\theta}_i^{(\nu)}(1-b)\ .$$

Else if $\theta_i^{(\nu)} = \underline{\theta}_i^{(\nu)}$ then

$$\overline{\theta}_i^{(\nu+1)} = \underline{\theta}_i^{(\nu)}(1+c)\ , \text{ and}$$
$$\underline{\theta}_i^{(\nu+1)} = \underline{\theta}_i^{(\nu)}(1-d)\ .$$

Overlining indicates upper bound and underlining stands for lower bound. The parameters a, b, c and d allow to enlarge and shrink the feasibility region of the multiplier vector, i.e. the convex and compact set $\mathcal{C}$. This is efficiently accomplished because the above updating procedure is simple.

The DC-CP method is a CP method in which the ascent procedure is dynamically constrained by enlarging and shrinking the feasibility region on a coordinate bases. This is possible because the feasibility region is simple: bounds on every multiplier. Through this enlarging/shrinking procedure it is possible to center the algorithm in the area of interest which results in high efficiency. The enlarging/shrinking procedure is not problem-dependent and involves straightforward heuristics (see the simple updating procedure above).

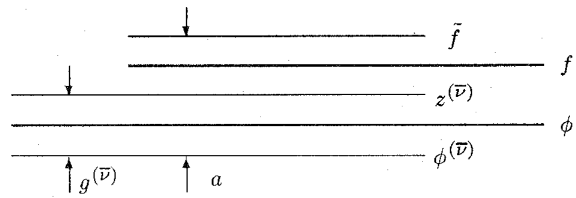

Figure 1: Upper and lower bounds for the primal and dual optima, and upper bound for the duality gap.

This algorithm is typically stopped when the multiplier vector difference between two consecutive iterations is below a pre-specified threshold. A small enough difference between an upper bound and a lower bound of the dual optimum is also an appropriate stopping criterion (see Subsection 3.5).

3.5 Duality gap

When using the CP method or the DC-CP method, at every iteration, the objective function value of the relaxed dual problem (problem (20)-(21) or problem (24)-(25)) constitutes an upper bound of the optimal dual cost value. This is so because the piecewise linear reconstruction of the dual function overestimates the actual dual function. On the other hand, the objective function value of the dual problem (evaluated through the decomposed primal problem using equation (9)) provides at every iteration a lower bound of the optimal dual cost value. This can be mathematically stated as follows:

$$z^{(\nu)} \ge \phi^* \ge \phi^{(\nu)} \tag{27}$$

where $z^{(\nu)}$ is the objective function value of the relaxed dual problem at iteration ν, ϕ^* is the optimal dual cost value, and $\phi^{(\nu)} = \lambda^{(\nu)\mathsf{T}}H + \mu^{(\nu)^{\mathsf{T}}}G + d(\lambda^{(\nu)}, \mu^{(\nu)})$ is the objective function value of the dual problem at iteration ν.

The size of the per unit gap $g^{(\nu)} = (z^{(\nu)} - \phi^{(\nu)})/\phi^{(\nu)}$ is an appropriate per unit cost criterion to stop the search for the dual optimum. *Phase 2* converts heuristically the typically primal infeasible solution provided by the dual problem to a primal reserve feasible solution. This typically originates a decrement in the objective function value, so that

$$\tilde{\phi} \le \phi^{(\overline{\nu})} \tag{28}$$

where $\tilde{\phi}$ is the objective function value for the primal reserve feasible solution (*Phase 2*), and $\phi^{(\overline{\nu})}$ the objective function value of the dual problem at the last iteration

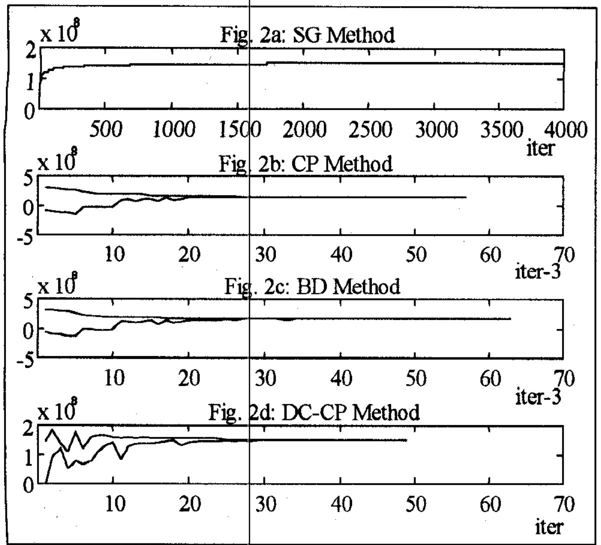

Figure 2: *Phase 1* for the testing example

of *Phase 1* which is iteration $\overline{\nu}$. Once $\tilde{\phi}$ is known, the unit commitment variables are set to fixed values, and a demand adjusted primal feasible solution can be obtained through an economic dispatch procedure (*Phase 3*). If $\tilde{f}$ is the objective function value of the demand adjusted primal feasible solution, a per unit upper bound of the duality gap value is given by $a = (\tilde{f} - \phi^{(\overline{\nu})})/\phi^{(\overline{\nu})}$, because $\tilde{f}$ is an upper bound for the primal optimum and $\phi^{(\overline{\nu})}$ is a lower bound for the dual optimum. In most practical cases the duality gap is below 0.1%. Figure 1 shows upper and lower bounds for the primal and dual optima, the actual duality gap, and an upper bound for it.

Model	*DPC* [MPta]	$g^{(\overline{\nu})}$ [%]	*Number iterations Phase 1*	*Total CPU time* [p.u.]
SG	151.002	-	4000	49.609
CP	150.853	0.087	59	1.378
BD	150.880	0.085	65	1.378
DC-CP	150.882	0.079	49	1.000

Table 1: Cost, gap, number of iterations and CPU time in per unit (normalizing factor: 6.322 seconds) for the testing example

4 Case Studies

Two case studies are analyzed. The first one is a six period testing example whose purpose is to compare the convergence properties of the four methods of solution for the dual problem described in Section 3. The planning horizon for the second case is 48 periods. This is a realistic large-scale case study.

In both case studies the generating system is based on the electric energy system of mainland Spain. It consists of 60 thermal plants and 30 hydraulic plants in one complex hydroelectric system.

Total cost is the sum of production cost, start-up cost and shut-down cost for all thermal units in all periods. The production cost of every thermal plant is considered a quadratic function of the output power. The start-up and shut-down costs of every thermal plant are considered constant. Ramp rate limits, minimum up time and minimum down time constraints are enforced. Hydro power of each hydroelectric unit is modeled as a piecewise linear function of water discharge.

Although the number of constraints of the relaxed dual problem for the CP and BD methods reported in the literature grows with the number of iterations, for the sake of comparison the CP and BD methods implemented maintain a pre-specified maximum number of constraints. The procedure used to keep limited the number of constraints is explained in Subsection 3.4.

The reported CPU time refers to a SUN ULTRASPARC 2 workstation with 128 MB of RAM.

Testing Example

Model	*DPC* [MPta]	*PPC* [MPta]	$g^{(\overline{\nu})}$ [%]
CP	1215.851	1215.374	0.105
BD	1215.809	1214.454	0.112
DC-CP	1215.973	1214.671	0.094

Table 2: Costs and gaps for the large-scale case study

Figure 2 shows the results of *Phase 1*. Figure 2a is a plot of the cost of the dual problem when the SG method is used to solve *Phase 1*. Figures 2b, 2c and 2d include a plot of the cost of the dual problem and a plot of the variable z used in the relaxed dual problem for the CP, BD and DC-CP methods respectively. It should be noted that vertical axis scales are different for the three plots of Figure 2, and that the horizontal axis scale of plot of Figure 2a is different from the horizontal axis scales of the other plots of Figure 2. The above axis scale differences are introduced to enhance clarity. For CP and BD methods the solution oscillates largely in the first few iterations. In order to better appreciate the evolution of these methods, figures 1b and 1c include the cost of the dual problem and the value of z starting from iteration number 3. The solutions of the dual problem for all the methods implemented are reserve feasible primal solutions and therefore, *Phase 2* is skipped. Table 1 shows, for the four methods of solution of *Phase 1* described in Section 3, the cost of the dual problem (*DPC*) in millions of pesetas (Pta, 1 US\$ $\simeq$ 145 Pta), the size of the gap $g^{(\overline{\nu})}$ (except for the SG method where it is not defined) where $\overline{\nu}$ is the last iteration of *Phase 1* (Subsection 3.5), the number of iterations needed to solve

Phase 1 and the required CPU time normalized with respect to the total CPU time in seconds required by the DC-CP method. Total CPU time for the DC-CP method is 6.322 seconds.

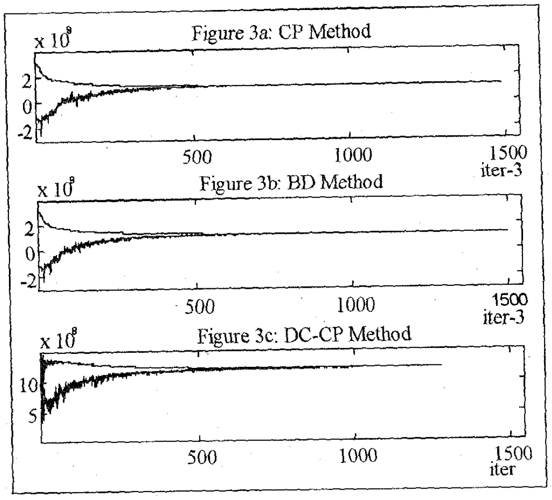

Figure 3: *Phase 1* for the large-scale case study

The SG method shows slow convergence. The number of iterations needed to solve *Phase 1* is much larger than the number of iterations required by any of the other three methods. In terms of CPU time the SG method is the most inefficient method to solve *Phase 1* and therefore it will not be used in the realistic large-scale case study presented below. If good initial values for the multipliers are available and careful tune-up of the subgradient step size is performed, the SG method may show better computational behaviour than the one shown in the previous testing example

Large-Scale Case Study

Figures 3a, 3b and 3c show respectively the results when the CP, BD and DC-CP methods are used to solve *Phase 1*. These figures provide similar information than the one provided by figures 2b, 2c and 2d respectively. It should be noted that vertical axis scales are different for the plots of Figure 3. Figures 3a and 3b include the evolution of cost starting from iteration number 3. Table 2 presents the cost of the dual problem (*DPC*) and the cost of the primal reserve feasible solution (*PPC*), both in millions of pesetas (Pta), and the the size of the gap $g^{(\overline{\nu})}$. Table 3 shows the number of iterations to solve *Phase 1* and the required normalized CPU time. Total CPU time for the DC-CP method is 11088 seconds. Most of this time (around 90 % for the CP and DC-CP methods and 78 % for the BD method) is used to solve complex hydro subproblems.

From the above results, it can be concluded that the DC-CP method is a very competive method of solution for the dual problem.

5 Conclusions

The objective of the short-term hydro-thermal coordination problem is to determine the scheduling and production of every hydro and thermal plant of an electric energy system so that the customer demand is supplied at minimum cost and a certain level of security, and all constraints related to the thermal and hydro subsystems are satisfied.

The most effective approach to solve this problem is Lagrangian Relaxation. This approach requires the solution of the dual problem of the original short-term hydro-thermal coordination problem.

This paper focuses on the updating procedure of Lagrangian multipliers which is the mechanism to solve the dual problem. A novel, non-oscillating, and computationally efficient procedure is presented. It outperforms previous approaches such as subgradient and bundle methods. Extensive computational results based on the electric energy system of mainland Spain are presented.

Model	*Number iterations Phase 1*	*CPU time* [p.u.] *Phase 1*	*CPU time* [p.u.] *Phase 2*	*Total CPU time* [p.u.]
CP	1492	1.188	0.001	1.191
BD	1503	1.431	0.002	1.436
DC-CP	1280	0.995	0.002	1.000

Table 3: Number of iterations to solve the dual problem and required CPU time for *Phase 1* and *Phase 2*, in per unit (normalizing factor: 11088 seconds), for the large-scale case study

Acknowledgments

The work reported in this paper has been partly supported by the *Ministerio de Educación y Cultura* of Spain, project DGICYT PB95-0472, by *Red Eléctrica de España, REE*, and by the *Vicerrectorado de Investigación* of the *Universidad de Málaga*. The authors are grateful to Prof. A. de la Calle for support and help.

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