IEEE TRANSACTIONS ON POWER SYSTEMS, VOL. 19, NO. 3, AUGUST 2004

Modeling of Start-Up and Shut-Down Power Trajectories of Thermal Units

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***Abstract*—This paper presents a detailed formulation to model the power trajectories followed by a thermal unit during the start-up and shut-down processes, as well as the ramping limitations when increasing or decreasing power. This model is formulated using mixed-integer linear constraints, thereby overcoming some drawbacks of those approaches based on heuristics, dynamic programming or Lagrangian relaxation. The formulation presented can be used in different scheduling problems arising in centralized and deregulated frameworks. The self-scheduling problem faced by a thermal generator in a pool-based electric energy market is used to illustrate the proposed model. Finally, a realistic case study is analyzed to show the computational behavior of the proposed model.**

***Index Terms*—Ramping limits, start-up and shut-down power trajectories, thermal units.**

NOMENCLATURE

The notation used throughout the paper is stated below. For unit consistency, note that hourly intervals are considered.

Functions:

$\mathrm{d}(\mathrm{p}^{\mathrm{avg}}(k))$	Production cost in period k [$/h], which is a nonlinear function of the average power output in period k, $\mathrm{p}^{\mathrm{avg}}(\mathrm{k})$.

Constants:

A	Fixed cost [$/h].
B	Start-up cost [$].
C	Shut-down cost [$].
DD	Duration of the shut-down process [h].
F	Slope of the linear variable production cost [$/MWh].
$\bar{\mathrm{P}}$	Capacity [MW].
$\underline{\mathrm{P}}$	Minimum power output [MW].
$\mathrm{P}_{\mathrm{D}}(\mathrm{i})$	Power output corresponding to the ith interval of the shut-down process [MW].
$\mathrm{P}_{\mathrm{U}}(\mathrm{i})$	Power output corresponding to the ith interval of the start-up process [MW].
RD	Ramp-down limit [MW/h].
RU	Ramp-up limit [MW/h].
UD	Duration of the start-up process [h].
$\lambda(\mathrm{k})$	Forecasted price of energy in period k [$/MWh].

Variables:

$\mathrm{d}(\mathrm{k})$	Linear production cost in period k [$/h].
$\mathrm{p}(\mathrm{k})$	Power output at the end of period k [MW].
$\mathrm{p}^{\mathrm{avg}}(\mathrm{k})$	Average power output in period k [MW].
$\mathrm{v}(\mathrm{k})$	0/1 variable which is equal to 1 if the unit is on-line in period k.
$\mathrm{y}(\mathrm{k})$	0/1 variable which is equal to 1 if the unit is started-up at the beginning of period k.
$\mathrm{z}(\mathrm{k})$	0/1 variable which is equal to 1 if the unit is shut-down at the beginning of period k.

Sets:

K	Set of indices of the intervals (hours) of the scheduling horizon.

I. INTRODUCTION

TO RIGOROUSLY model the generation scheduling of thermal units, the actual operating process of these units should be considered in detail. Moreover, with the introduction of competition, modeling accuracy is even more necessary to achieve the desired goals of efficiency and feasibility in energy production [1].

Ramping constraints, which limit the capability of units to change production over short periods of time, can have an important impact on the short-term generation scheduling. Large, efficient thermal units frequently have significant ramp limits. If the difference in system load in successive periods exceeds the ramp limits of efficient units, those which are not significantly ramp-limited gain additional value because of their ability to match the rapidly changing load. Such load-following generation units are smaller (and less efficient) coal-fired and oil-fired thermal units, gas turbines, and especially hydro units. Moreover, ramping limits may also constrain the contributions of some units to spinning reserve and operating reserves. Therefore, an accurate modeling of ramping limitations is fundamental to achieve feasible as well as efficient schedules.

According to the actual energy production process, the power output of an electric power unit is restricted by three kinds of ramp constraints.

1) Operating ramp constraints, also known as ramp-up and ramp-down rate limits.
2) Start-up ramp constraint, which involves an increasing power trajectory.
3) Shut-down ramp constraint, which involves a decreasing power trajectory.

These ramp constraints are described below.

Operating ramp constraints: The increment or decrement of the generation level of a unit over any two successive on-line periods (excluding start-up and shut-down periods) is bounded by the ramp-up (RU) and ramp-down (RD) limits, respectively (see Fig. 1).

Manuscript received September 18, 2003. This work was supported by the Ministerio de Ciencia y Tecnología of Spain and the Junta de Comunidades de Castilla—La Mancha, projects CICYT DPI2000-0654 and GC-02-006.

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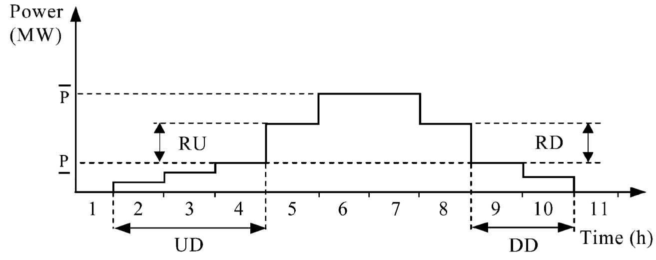
Digital Object Identifier 10.1109/TPWRS.2004.831654



k	1	2	3	4	5	6	7	8	9	10	11
v(k)	0	1	1	1	1	1	1	1	1	1	0
y(k)	0	1	0	0	0	0	0	0	0	0	0
z(k)	0	0	0	0	0	0	0	0	0	0	1

Fig. 1. Ramping model.

Start-up ramp constraint: When a unit (that is off-line) is started-up, it should follow an increasing power trajectory from 0 to the minimum power output, $\underline{P}$, during UD time periods, being $P_U(i)$ the pre-specified power output (data) in start-up period i (see Fig. 1, where UD is equal to 3 h).

Shut-down ramp constraint. When a unit (that is on-line) is shut-down, its generation level should be reduced to the minimum power output, $\underline{P}$, subject to the operating ramp constraints defined above. Then, it should follow a shut-down decreasing power trajectory during DD periods from $\underline{P}$ to 0, being $P_D(i)$ the pre-specified power output (data) in shut-down period i (see Fig. 1, where DD is equal to 2 h).

Note that both times, UD and DD, include the period in which the power output is equal to $\underline{P}$.

This paper presents a rigorous formulation of start-up and shut-down power trajectories using a mixed-integer linear programming (MILP) framework.

Several authors have modeled the ramping constraints through different approaches as described below.

Once the commitment status has been determined by an artificial intelligence algorithm, [2] deals with the start-up and shut-down ramping processes through a dynamic heuristic adjustment, thus yielding suboptimal solutions.

In [3], a price-based ramp rate model is proposed. Ramp constraints are dualized and the marginal ramp rate values are determined by an iterative algorithm; however, no modeling of start-up and shut-down ramping trajectories is provided.

In [4]–[8], ramping constraints are enforced in the framework of a Lagrangian relaxation algorithm to solve the unit commitment problem. In [4], a network model is used for the unit subproblems resulting from the application of Lagrangian relaxation to unit commitment. In [5], the previous work was expanded by adding ramping costs to the objective function in order to consider the cost associated to ramping wear and tear. In [6], ramp rate constraints are incorporated by introducing an additional ramp multiplier per period. In [7], two multipliers per generator and per period are introduced to model RU and RD constraints. In [8], the difference in demand between successive periods is incorporated in the objective function through a set of ramp multipliers. Unlike in [4] and [5], unit subproblems were solved by dynamic programming in [6]–[8].

It is remarkable that the aforementioned works either rely on heuristics or include approximations to model the shut-down and start-up processes.

The main contributions of this paper are listed as follows.

1) A $0/1$ mixed-integer linear formulation is provided, which includes explicitly all the aforementioned ramping constraints, reflecting rigorously the actual operation of thermal units. Compared with the theoretical analysis described in [1], the model proposed in this paper provides appropriate accuracy for moderate computational effort.
2) The formulation presented overcomes some of the disadvantages of previous approaches. First, mixed-integer linear programming guarantees the convergence to the optimal solution in a finite number of steps [9]. Unlike heuristic approaches, unit commitment decisions are not decoupled from economic dispatch. In addition, it does not require any heuristic manipulation of the solution attained as Lagrangian-based methods do. Finally, it does not present problems with time-dependent constraints as dynamic programming algorithms do [8].
3) If compared to other traditional MILP formulations of scheduling problems, the proposed formulation does not need additional binary variables to accurately model ramping constraints. Although the total number of constraints is increased, its impact on the computational complexity is low. Therefore, an improvement in terms of accuracy modeling is achieved without significantly altering the computing burden.
4) Finally, this formulation is suitable for any short-term generation scheduling problem, either belonging to a centralized or a deregulated framework.

In order to illustrate the effectiveness of the model presented, the self-scheduling problem faced by a thermal generator in a pool-based electricity market is used [10]. This problem consists in maximizing the profit of a thermal generator from selling energy in the day-ahead market [11], [12], while meeting all the technical constraints over a time span of one day on an hourly basis.

The remainder of this paper is organized as follows. In Section II, the formulation of the ramping constraints is presented in detail. This section also provides the formulation of the problem where this ramping model is embedded. In Section III, results from a realistic case study are provided and discussed. In Section IV, some relevant conclusions are drawn.

II. FORMULATION

This section provides a detailed mathematical description of the start-up and shut-down power trajectories of thermal units. With the purpose of illustrating how this mathematical description works, at the end of the section, the self-scheduling problem faced by a price-taker thermal producer is considered.

It should be noted that hourly time intervals are considered in this paper. A finer time discretization can be easily handled by the proposed formulation, thereby increasing the number of variables required.

A. Start-Up, Shut-Down, and Operating Ramp Constraints

The following set of linear constraints precisely model the start-up and shut-down power trajectories, as well as the RU and RD limits.

For a better comprehension of the following constraints, it should be noted that the term $\sum_{i=1}^{DD} z(k+i)$ is equal to 1 for all hours k during the shut-down process, being 0 otherwise. Similarly, the term $\sum_{i=1}^{UD} y(k-i+1)$ is equal to 1 for all hours k during the start-up process, and 0 otherwise.

$$p(k) \geq \underline{P}\left[v(k) - \sum_{i=1}^{DD} z(k+i) - \sum_{i=1}^{UD} y(k-i+1)\right] + \sum_{i=1}^{UD} P_U(i) y(k-i+1) \quad \forall k \in K \tag{1}$$

$$p(k) \geq \underline{P}\left[v(k) - \sum_{i=1}^{DD} z(k+i) - \sum_{i=1}^{UD} y(k-i+1)\right] + \sum_{i=1}^{DD} P_D(i) z(k+DD-i+1) \quad \forall k \in K. \tag{2}$$

Constraints (1) and (2) above set the lower limit of the power output in each period. If the unit is on-line and is involved neither in a start-up nor in a shut-down process the first term of the right-hand side of (1) and the first term of the right-hand side of (2) (both identical) force the power output to be greater than or equal to the minimum power output. Note that during the start-up or shut-down processes this common first term vanishes in (1) and (2).

If the unit is being started-up, the second term of the right-hand side of constraints (1) forces the power output to be greater than or equal to the corresponding limit of the start-up power trajectory.

In a similar fashion, if the unit is being shut-down, the second term of the right-hand side of constraints (2) is used to ensure that the power output is greater than or equal to the corresponding limit of the shut-down power trajectory.

$$p(k) \leq \sum_{i=1}^{UD} P_U(i) y(k-i+1) + \bar{P}\left[v(k) - \sum_{i=1}^{UD} y(k-i+1)\right] \quad \forall k \in K \tag{3}$$

$$p(k) \leq \sum_{i=1}^{DD} P_D(i) z(k+DD-i+1) + \bar{P}\left[v(k) - \sum_{i=1}^{DD} z(k+i)\right] \quad \forall k \in K. \tag{4}$$

Constraints (3) and (4) above set, respectively, the upper limit of the power output to the corresponding value of the start-up or shut-down processes, provided that the unit is going through any of these processes. Otherwise, if the unit is on-line, both constraints impose the capacity of the unit as the maximum possible power output.

In the case when the unit is off-line, constraints (1)–(4) force the power output to be equal to 0.

$$p(k) - p(k-1) \leq \bar{P} \sum_{i=1}^{UD} y(k-i+1) + RU\left[v(k) - \sum_{i=1}^{UD} y(k-i+1)\right] \quad \forall k \in K. \tag{5}$$

Constraints (5) above model the RU limit. Note that the effect of this ramp rate is ignored when the unit is being started-up through the inclusion of the capacity, $\bar{P}$, in the first term of the right-hand side. In this case, the power output must be equal to the corresponding fixed value of the start-up power trajectory, $P_U(i)$, which is enforced by (1) and (3).

$$p(k-1) - p(k) \leq \bar{P} \sum_{i=1}^{DD} z(k+i-1) + RD\left[v(k-1) - \sum_{i=1}^{DD} z(k+i-1)\right] \quad \forall k \in K. \tag{6}$$

Analogously, constraints (6) above model the RD limitation. Note that if the unit is being shut-down, the RD is replaced by the capacity of the unit, so as to deactivate its effect. In this case, the power output must be equal to the corresponding fixed value of the shut-down power trajectory, $P_D(i)$, which is enforced by (2) and (4) .

$$y(k) - z(k) = v(k) - v(k-1) \quad \forall k \in K \tag{7}$$

$$v(k) \geq \sum_{i=1}^{UD} y(k-i+1) \quad \forall k \in K \tag{8}$$

$$v(k) \geq \sum_{i=1}^{DD} z(k+i) \quad \forall k \in K. \tag{9}$$

The logic of the shut-down and start-up status change is modeled through constraints (7) above [13]. Constraints (8) and (9) above are also needed for the commitment logic. Both constraints impose that the unit is on-line when starting-up or shutting-down, respectively.

$$y(k) + \sum_{i=1}^{UD+DD-1} z(k+i-1) \leq 1 \quad \forall k \in K \tag{10}$$

$$p(k) \geq P_U(UD)\left[\sum_{i=1}^{DD} z(k+i) + \sum_{i=1}^{UD} y(k-i+1) - 1\right] \quad \forall k \in K \tag{11}$$

$$p(k) \geq P_D(1)\left[\sum_{i=1}^{DD} z(k+i) + \sum_{i=1}^{UD} y(k-i+1) - 1\right] \quad \forall k \in K. \tag{12}$$

Finally, constraints (10)–(12) above model the only possibility of overlapping of the start-up and shut-down processes. This unusual situation occurs at the last interval of the start-up power trajectory and the first interval of the shut-down process, i.e., when the unit has been brought to the minimum power output and it is scheduled to shut down. For this overlapped period, the power output should be equal to the minimum power output.

It should be noted that this new formulation, if compared with previous ones (as given, e.g., in [10]), does not require additional binary variables to model the ramping trajectories. Therefore, the computational burden is not increased. Furthermore, compared with [13], the proposed approach explicitly allows the power output to be less than the minimum power output during the start-up and shut-down processes, thus enabling a more accurate modeling of these processes.

This formulation may be adapted for the case of a unit that does not require any start-up power trajectory, but it is necessary to impose a start-up ramp rate (as in [10]). In other words, UD is equal to 1 and the power output lies in the range between 0 and the start-up ramp rate $\mathrm{P_U(1)}$ in the period in which it is started-up. In this case, constraints (1) should be replaced by

$$\mathrm{p(k) \geq \underline{P}\left[v(k) - \sum_{i=1}^{DD} z(k+i) - \sum_{i=1}^{UD} y(k-i+1)\right]} \quad \forall \mathrm{k} \in \mathrm{K}. \quad (13)$$

Additionally, constraints (11) are omitted. Note that if $\mathrm{P_U}$ (1) is equal to the capacity of the unit, the start-up ramp rate does not limit the production of the unit, thereby allowing the modeling of cycling or hydro units that are available at full power very rapidly.

On the other hand, if a shut-down power trajectory is not required, analogous modifications should be included. In this case, DD is equal to 1, and the power output in the period preceding a shut-down is restricted to be between 0 and the shut-down ramp rate, $\mathrm{P_D(1)}$. Consequently, constraints (2) are replaced by (13) and constraints (12) are removed from the model.

Again, it should be noted that a deactivation of the shut-down ramp constraint is achieved if $\mathrm{P_D(1)}$ is equal to the capacity of the unit, allowing the modeling of cycling or hydro units which may be de-committed very rapidly.

Besides the above described ramping model, this paper also considers minimum up and down times. For the sake of simplicity, the formulation of these constraints is not included in this paper. However, the interested reader is referred to [10], where a precise formulation for these constraints is provided.

B. Objective Function

The goal of any power producer in an electricity market is to maximize its own profit, which is computed as the difference between the revenues and the total operating costs. The operating costs related to a thermal unit include variable and fixed production costs, shut-down costs and start-up costs. Thus, the objective function to be maximized can be expressed as

$$\sum_{\mathrm{k \in K}} \{\lambda(\mathrm{k})\mathrm{p^{avg}(k)} - [\mathrm{d(p^{avg}(k)) + Cz(k) + By(k)}]\}. \quad (14)$$

In (14), the first term expresses the revenues from selling energy whereas the second term represents the total operating cost. It is necessary to draw a distinction between the electrical power and the electrical energy in the generation scheduling problem, especially if unit ramping characteristics are considered. If the unit power output is modeled as a step function and the study time interval is 1 h, then the unit power output at a certain hour is numerically equal to the unit energy output during that hour. However, if the unit ramping is considered adequately, the unit energy output during one hour will be different than the power outputs at the beginning and at the end of that hour, as shown in Fig. 2.

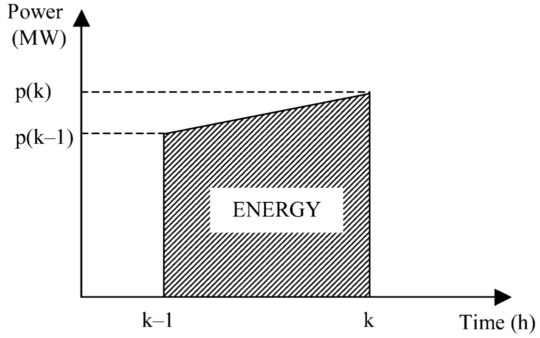


Fig. 2. Energy versus power output.

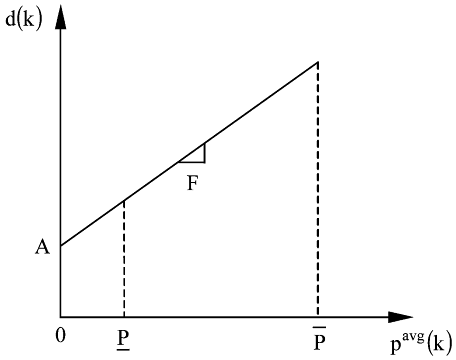


Fig. 3. Production cost.

If a linear variation of power output is assumed (as in Fig. 2), the energy is equal to the average value of the power output in two consecutive periods, as follows:

$$\mathrm{p^{avg}(k) = \frac{p(k) + p(k-1)}{2}} \quad \forall \mathrm{k} \in \mathrm{K}. \quad (15)$$

The precise model of the start-up and shut-down power trajectories presented in this paper allows the average power output to be less than the minimum power output if the unit is undergoing any of these processes, and this constitutes a more precise modeling of the operating cost involved in the start-up and shut-down processes.

For the sake of simplicity, a linear variable cost is used in this paper (see Fig. 3). The formulation is as follows:

$$\mathrm{d(k) = Av(k) + p^{avg}(k)F} \quad \forall \mathrm{k} \in \mathrm{K}. \quad (16)$$

Note that the linear production cost $\mathrm{d(k)}$ replaces $\mathrm{d(p^{avg}(k))}$ in (14). A more precise model of the production cost including nonconvexities and nonlinearities can be found in [14] and can be straightforwardly included in the formulation of the problem through the use of additional binary variables.

Finally, shut-down costs and start-up costs in (14) have been considered constant. Note that time-dependent start-up costs [10] can be easily included in the proposed formulation.

TABLE I
COST COEFFICIENTS

F ($/MWh)	A ($/h)	B ($)	C ($)
38	100	850	56

TABLE II
START-UP AND SHUT-DOWN POWER TRAJECTORIES

I	Start-up process $P_U(i)$ (MW)	Shut-down process $P_D(i)$ (MW)
1	37	112
2	75	70
3	112	30

III. CASE STUDY

Data for the considered thermal unit as well as price values are given in this section. Table I provides the cost coefficients.

The durations of both start-up and shut-down processes are 3 h. The power trajectories followed by the power output in each process are provided in Table II. The start-up power trajectory ends up with the minimum power output (112 MW), whereas this minimum power output corresponds to the first power output the unit must produce to begin the shut-down process.

The capacity of the unit is equal to 294 MW. Minimum up time and minimum down time for the considered unit are both 3 h. Operating RU and RD limits are 60 and 50 MW, respectively. Finally, in the period before the market horizon the unit has been running for 11 h and produces 170 MW.

Price data are provided in Table III. Energy prices are the prices obtained in the electricity market of mainland Spain [11] on May 29th, 2003. It should be emphasized that, in general, price profiles should be obtained using appropriate forecasting procedures such as the ones reported in [15], [16].

The model has been implemented on a SGI R12000, 400 MHz based processor with 500 MB of RAM using CPLEX 7.5 under GAMS [17].

Table III also shows the optimal schedule of the power output and the average power output in every period of the time span.

As can be seen in Table III, the unit undergoes a shut-down process during periods 2–4, during which the power output is equal to the corresponding value in Table II. After being off-line for 3 h (hours 5, 6, and 7) to satisfy the minimum down time requirement, the unit initiates the start-up process in period 8 and remains on-line till the end of the time span (therefore, meeting the minimum up time requirement of 3 h). Note that the power output values in periods corresponding to the start-up process (hours 8, 9, and 10) are those shown in Table II. It should be noted that RU limits are active in periods 11–13. Likewise, RD limits are active in periods 1, and 22–24.

TABLE III
PRICE DATA AND OPTIMAL GENERATING SCHEDULE

Hour k	Energy price $\lambda(k)$ ($/MWh)	Power output $p(k)$ (MW)	Average power output $p^{avg}(k)$ (MW)
1	26.950	120	145.0
2	25.817	112	116.0
3	22.209	70	91.0
4	21.219	30	50.0
5	19.668	0	15.0
6	19.800	0	0.0
7	24.167	0	0.0
8	26.950	37	18.5
9	40.700	75	56.0
10	44.110	112	93.5
11	45.177	172	142.0
12	47.025	232	202.0
13	47.773	292	262.0
14	46.200	294	293.0
15	44.033	294	294.0
16	44.550	294	294.0
17	46.200	294	294.0
18	48.950	294	294.0
19	48.840	294	294.0
20	45.100	294	294.0
21	43.252	294	294.0
22	44.000	244	269.0
23	44.000	194	219.0
24	25.850	144	169.0

TABLE IV
OPTIMAL SCHEDULE WITHOUT START-UP OR SHUT-DOWN POWER TRAJECTORIES

Hour k	Energy price $\lambda(k)$ ($/MWh)	Power output $p(k)$ (MW)	Average power output $p^{avg}(k)$ (MW)
1	26.950	120	145
2	25.817	70	95
3	22.209	0	35
4	21.219	0	0
5	19.668	0	0
6	19.800	0	0
7	24.167	0	0
8	26.950	112	56
9	40.700	172	142
10	44.110	232	202
11	45.177	292	262
12	47.025	294	293
13	47.773	294	294
14	46.200	294	294
15	44.033	294	294
16	44.550	294	294
17	46.200	294	294
18	48.950	294	294
19	48.840	294	294
20	45.100	262	278
21	43.252	212	237
22	44.000	162	187
23	44.000	112	137
24	25.850	0	56

Regarding the objective function, note that the variable production cost takes into account values of the average power below the minimum power output (hours 3–5 and 8–10) due to the shut-down and start-up processes, respectively. In this case study, the unit is shut-down in period 5, after being on-line for 15 h. In addition, the unit is started-up in period 8, after being off-line for 3 h. Therefore, the unit incurs in a shut-down cost of $56 and in a start-up cost of $850 (see Table I). The objective function value of the optimal solution is equal to $16 773.5 and was achieved in 0.66 seconds of computing time.

In order to assess the impact of start-up and shut-down processes on the operation of the unit, the same case study has been run without considering start-up or shut-down power trajectories, i.e., UD and DD are set to 1. In this case, the unit is allowed

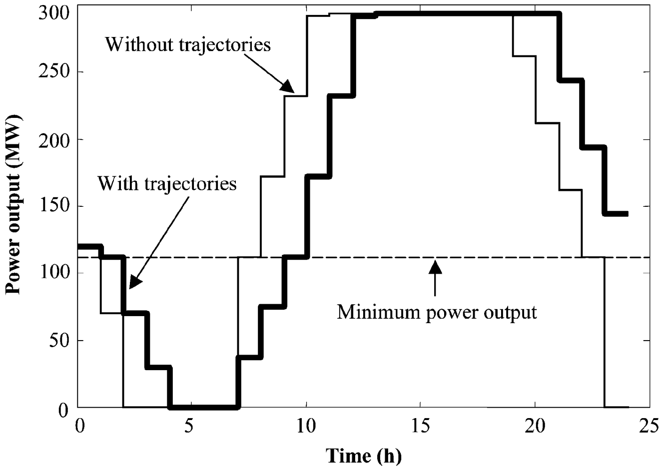


Fig. 4. Power output schedules with and without start-up and shut-down power trajectories.

TABLE V
DIMENSIONS OF THE MODELS AND SOLUTION TIMES

Case (# hours)	Optimal Solution ($)	# of Constraints	# of Real Variables	# of Binary Variables	CPU Time (s)
24	16773.5	337	48	72	0.66
48	39139.5	673	96	144	4.10
96	83871.6	1345	192	288	14.35
168	150969.7	2353	336	504	153.01

to produce between 0 and the minimum power output in the periods corresponding to the start-up or shut-down processes. The optimal generation schedule for the modified case is shown in Table IV.

As can be seen in Table IV, the commitment schedule differs considerably with respect to the original case. There are two shuts-down (periods 3 and 24), and one start-up (period 8). It should be noted that in period 2 it is more profitable for the unit to reduce its power output below the minimum power output, before being shut-down in the next period. Conversely, in period 23, preceding a shut-down, the power output is equal to the minimum power output, which also satisfies the shut-down ramp constraint. In a similar fashion, the power output in period 8 is equal to the minimum power output, thereby meeting the start-up ramp rate. Minimum up and down times are met throughout the time span. The remaining operation constraints are satisfied as well.

The objective function value of the optimal solution is equal to $21 731.8 and was achieved in 0.92 seconds of computing time. As can be noted, the profit is increased by a factor of 1.3 because the unit operation is less restricted than in the original case. However, the generation schedule does not accurately represent the actual operation of the unit.

Both schedules, with and without start-up and shut-down power trajectories, are plotted in Fig. 4. The thick line represents the generation scheme considering start-up and shut-down processes, whereas the thin line represents the schedule without taking into account both processes.

Finally, several case studies of different size have been solved to show the applicability of the proposed model. In these cases the price profile of the 24-h case has been replicated to test time horizons of up to 168 h. Table V summarizes the results and the dimensions of the problems considered. From Table V it can be concluded that the optimal solutions for larger time horizons are obtained in moderate computing times.

IV. CONCLUSION

This paper presents a rigorous formulation of the ramping constraints affecting the production of a thermal generator: 1) start-up production trajectory; 2) shut-down production trajectory; and 3) RU and RD constraints. These restrictions have been formulated as a set of mixed-integer linear constraints. Therefore, the achievement of the optimal solution to the resulting problem is guaranteed. The formulation presented has been embedded in the self-scheduling problem faced by a generator in a pool-based electricity market. Furthermore, it can be used in any thermal scheduling problem, both in centralized and competitive frameworks. Simulation results from realistic case studies show that the proposed formulation is accurate and computationally efficient. Current research includes the consideration of more complex start-up and shut-down processes, such as generator loading dependency and ramping delays.

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