Energy procurement for large consumers in electricity markets

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Abstract: The paper considers a large consumer that procures its electric energy in an electricity market, involving both pool and bilateral transactions. Additionally, the consumer operates a self-production facility of limited size. In order to minimise its electricity bill, this consumer should determine the amount of energy bought from bilateral contracts, the quantity of energy purchased from the pool, and the amount of energy self-produced. The novel contribution of this paper is to provide a procedure that allows a large consumer to decide optimally its mix of purchases from different electricity sources. The bilateral contract framework used is flexible enough to accommodate many real-world bilateral electricity agreements. A medium-term decision horizon spanning from one to several months is considered. Results from a realistic case study are presented.

List of symbols

Constants

C^{FX}, C^{SD}, C^{SU}	fixed, shut-down and start-up costs for the self-production facility,
D_h	total power consumer demand during hour h,
f_ℓ	slope of the block ℓ of the piecewise linear production cost of the self-production facility,
M	large positive constant,
P_ℓ^{max}	size of the block ℓ of the self-production facility,
P^{max}, P^{min}	maximum and minimum power output of the self-production facility,
$s_{r,be}$	slope of the block r of the penalty function related to the bilateral contract b during hours type e,
x_{be}^{max}, x_{be}^{min}	upper and lower bounds of the energy to be bought from bilateral contract b during hours type e,
$\lambda_h^{P,est}$	estimate for the market-clearing price during hour h, and
λ_{bh}^{B}	bilateral contract buying/selling price during hour h for contract b.

IEE Proceedings online no. 20041252

doi:10.1049/ip-gtd:20041252

Paper first received 17th October 2003 and in revised form 25th May 2004. Originally published online: 8th April 2005

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Variables

C_b^B	total cost of buying from bilateral contract b,
C_h^G	self-production cost during hour h,
C^P	expected value of the total cost of purchasing from the pool,
C_{be}^R	penalty cost associated to bilateral contract b during hours of type e,
C^S	expected value of the total selling revenue from the pool,
E_{be}^B	energy purchased from bilateral contract b during hour of type e,
P_{bh}^B	power purchased from bilateral contract b during hour h,
P_h^C	self-produced and locally consumed power during hour h,
P_h^G	total self-produced power during hour h,
$P_{\ell h}$	self-produced power corresponding to the linear block ℓ during hour h,
P_h^P	power purchased from the pool during hour h,
P_h^S	self-produced power sold to the pool during hour h,
v_h	binary variable that is equal to 1 if the unit is on-line during hour h and 0 otherwise,
y_h	binary variable that is equal to 1 if the unit is started-up at the beginning of hour h and 0 otherwise,
z_h	binary variable that is equal to 1 if the unit is shut down at the beginning of hour h and 0 otherwise,
w_{be}^{up}, w_{be}^{down}	binary variables associated to bilateral contract b during hours of type e,

$x_{r,be}$	energy consumed from bilateral contract b during hours type e corresponding to the block r of the penalty function,
q_h	binary variable which is equal to 1 if energy is bought from the pool during hour h, and 0 if it is sold to the pool, and
λ_h^{P}	random variable representing the market-clearing price during hour h.

Sets and numbers

G	set that defines the feasible operating region of the self-producing facility,
H	set of indices of hours,
L	number of blocks of the linearization of the production cost function,
T	number of hours (cardinality of H),
Ξ	set of indices of bilateral contracts,
Ω_{be}	set of hours of type e for contract b, and
$\Omega_b = \{\Omega_{be}\ \forall e\}$	set of all sets of types of hours for contract b.

Operators

$\mathrm{Exp}_{\lambda_1,\ldots,\lambda_T}$	Expectation operator with respect to random variables $\lambda_1,\ldots,\lambda_T$.

It should be noted that 'type of hour' refers to a specific subset of hours that meet certain criterion, e. g. valley hours. Therefore, 'hours of type e' refers to any of the different subset of hours that are used for contract definition.

1 Introduction

This paper considers the perspective of a large consumer that procures its energy in an electricity market. Both pool and bilateral transactions are possible. Additionally, the consumer has available a self-production facility of limited size. A medium-term decision horizon spanning from one to several months is considered.

In order to minimise its electricity bill, the large consumer should determine (i) the amount of energy, within pre-specified bounds, bought from available (already signed) bilateral contracts, (ii) the quantity of energy purchased from the pool, and (iii) the amount of energy self-produced.

The bilateral contract framework considered is flexible enough to accommodate many real-world bilateral electricity agreements. This framework allows different contractual arrangements for different blocks of hours throughout the day and throughout the seasons of the year.

The self-production facility is intended to minimise the risk of high prices either in the pool or in contracts. Additionally, the large consumer may sell part of its self-produced energy to the pool.

Data required to properly address this problem include:

1.Consumer energy demand forecasts for all hours of the medium-term time horizon under consideration.

2. Pool price forecasts for all hours of the medium-term time horizon under consideration.

3. Characteristics in terms of energy quantity and price of all available bilateral contracts.

4. Technical and economical characteristics of the self-production facility.

The decision time framework for the large consumer spans the duration of the longer available contract, e.g. from one to several weeks, and decisions are made at the beginning of this time horizon. However, decisions can be refined re-running the model as time progresses in a window moving manner. Data requirements include price and demand forecasts for the entire time horizon. The remaining data is not uncertain and it is readily available for the large consumer.

Although the technical literature is rich in papers addressing the perspective of the producer within an electricity market, i.e. addressing the self-scheduling problem [1–3], few references are available concerning the large consumer perspective. The pioneering work of Daryanian [4], and the recent work of Gabriel *et al.* [5] and Kirschen [6] deserve special attention. In [4], the optimal response of a large consumer to electricity spot prices is derived in a centralised setting; in [5], the medium-term risk-constrained profit maximisation problem faced by a retailer is analysed; and in [6], an analysis of the tools that consumers and retailers need to participate in electricity markets is presented. As opposed to [4], a competitive environment is considered in this paper and as opposed to [5], a large consumer instead of a retailer is considered in the work reported in this paper.

Other related works are briefly analysed below. In [7], the price properties and characteristics of forward contracts with interruptible delivery are described. The coexistence of forward contracts with the spot market is discussed in [8] and [9]. The work reported in [10] addresses the optimisation of the consumer response (modelling different load types) considering a centralised scheme based on spot pricing. In [11], the authors analyse the available options for retail customers in a competitive electric energy market and show the dependency of the customer location on its costs. References [12] and [13] present criteria on how to allocate purchases in different markets (day-ahead, balance and reserve) to minimise the total electricity bill. The work reported in [14] helps consumers to evaluate and select their electricity supply contracts, modelling their own preferences through fuzzy logic theory. In [15], also using a fuzzy logic framework, the authors model contracts and decision maker preferences to help market players construct, evaluate and rank their portfolio of contracts. Finally, reference [16] addresses the problem of estimating the load faced by a retailer.

The diversification of electric energy supply generally increases reliability and decreases cost. In this context, the main contribution of this paper is to provide a method that allows a large consumer to decide its optimal mix of purchases from different electricity sources. These sources include different types of bilateral contracts, the pool, and self-production. Additionally, the method allows the large consumer to determine the optimal amount of self-produced energy to sell to the pool. Also, the paper provides a versatile framework to model different types of bilateral contract arrangements. However, this paper does not address the design of bilateral contracts but the optimal use of contracts already signed.

The method presented can be extended to address the electric energy procurement problem faced by a retailer. The retailer buys energy to sell it to its own consumers while a large consumer buys energy for self-consumption. However, the structures of these two problems are identical. Nevertheless, contract characteristics generally differ from retailers to large consumers and demand uncertainty is larger for retailers than for large consumers.

The novel contributions of this paper are twofold:

1. It provides a procedure that allows a large consumer to determine optimally its electric energy procurement from different electricity sources, namely, bilateral contracts, the pool, and self-production.

It presents a general and versatile contract framework that allows modelling diverse types of contract arrangement among suppliers and large consumers.

2 Formulation

This section provides models for bilateral contract usage, for purchasing from and selling to the pool, and for self-production. In respect of contract usage, the seller can be a producer, a marketer or a retailer. This section concludes with a mathematical formulation of the method that allows a consumer to decide its optimal mix of purchases from different electricity sources. This formulation is a mixed-integer linear programming model whose solution can be obtained using conventional solvers [17].

2.1 Bilateral contract framework and costs

A bilateral contract requires an agreement between the seller and the buyer on the contract time horizon, the hourly buying/selling price throughout this horizon, and bounds on the quantity to be traded in the different intervals (set of hours) of the contract horizon.

Any given contract specifies a price for buying energy every hour of the considered time horizon. Additionally, for every set of hours, the contract defines a minimum and a maximum of energy to be bought. If the amount of energy bought is not within these bounds, the consumer incurs a penalty. Therefore, from the perspective of the consumer, two costs are involved in any bilateral contract: (i) the cost of buying energy and (ii) the possible penalties associated with consuming lower or higher amounts of energy than those bounds specified in the contract for different sets of hours.

It should be noted that the contract framework above reduces to a standard price/quantity contract if penalties are eliminated (making penalty slopes nil), and the indexation with the spot market is suppressed (eliminating appropriate terms in (1) below). It should also be noted that indexation with the spot market (contract for differences) is common practice in Europe and the USA. The proposed penalty framework in similar to the one proposed in [5] that is related to the PJM market in the USA [18]."

A possible set of sets of hours for contract definition is:

$$\{w-p, w-s, w-v, w-we,$$
$$sp-p, sp-s, sp-v, sp-we,$$
$$sm-p, sm-s, sm-v, sm-we,$$
$$a-p, a-s, a-v, a-we\}$$

This set of sets is illustrated in Tables 1 and 2. For instance, the set 'a–p' is the set of hours 11–13 and 19–21 of working days in September–November.

It should be noted that any other hour classification is possible to fit specific buyer/seller conditions. In the case study analysed in Section 4, a three-month study period is considered, classifying hours as peak, shoulder, valley and weekend, and treating months separately for contract usage.

Table 1: Season definition for hour classification

w: winter	December–February
sp: spring	March–May
sm: summer	June–August
a: autumn	September–November

Table 2: Hour types within the week

p: peak of working days	11–13, 19–21
s: shoulder of working days	1,8–10,14–18,22–24
v: valley of working days	2–7
we: weekend	1–24

On the other hand, and for the sake of simplicity, it is considered that for every hour the difference between the contract price and the pool price is equally split between the buyer and the seller. However, it should be noted that other splitting arrangements can be used. Therefore, the cost from buying from bilateral contract b is computed as

$$\begin{aligned} C_b^B &= \mathrm{Exp}_{\lambda_1^P,\ldots,\lambda_T^P}\left\{\sum_{h=1}^{T}\left(\lambda_{bh}^B + \frac{\lambda_h^P - \lambda_{bh}^B}{2}\right)P_{bh}^B\right\} \\ &= \sum_{h=1}^{T}\left(\frac{1}{2}\lambda_{bh}^B P_{bh}^B + \frac{1}{2}\mathrm{Exp}_{\lambda_h^P}\{\lambda_h^P\}P_{bh}^B\right) \\ &= \sum_{h=1}^{T}\left(\frac{1}{2}\lambda_{bh}^B P_{bh}^B + \frac{1}{2}\lambda_h^{\mathrm{P,est}} P_{bh}^B\right) \end{aligned} \quad (1)$$

Note in (1) that the expectation and summation operators can be switched as a result of linearity even though prices are dependent random variables.

Current research is directed to incorporate risk-limiting mechanisms for total profit in the energy procurement problem considered in this paper. Using these mechanisms, expected profit can be maximised ensuring an acceptable level of risk in achieving that profit.

On the other hand, energy consumed from bilateral contract b during hours of type e can be expressed as (Fig. 1)

$$E_{be}^B = \sum_{h\in\Omega_{be}} P_{bh}^B = \sum_{i=1}^{3} x_{i,be} \quad (2)$$

where $x_{i,be}$ are auxiliary real variables to compute cost penalties associated to under- or over-consumption (see Fig. 1 and constraints (3)–(6) below).

In order to compute bilateral contract penalty costs for under- or over-consumption, the following constraints allow identifying which penalty is incurred, if any

$$w_{be}^{\mathrm{down}} x_{be}^{\min} \le x_{1,be} \le x_{be}^{\min} \quad (3)$$

$$w_{be}^{\mathrm{up}}(x_{be}^{\max} - x_{be}^{\min}) \le x_{2,be} \le w_{be}^{\mathrm{down}}(x_{be}^{\max} - x_{be}^{\min}) \quad (4)$$

$$0 \le x_{3,be} \le w_{be}^{\mathrm{up}} M \quad (5)$$

$$w_{be}^{\mathrm{down}} \ge w_{be}^{\mathrm{up}};\ w_{be}^{\mathrm{down}},\ w_{be}^{\mathrm{up}} \in \{0,\ 1\} \quad (6)$$

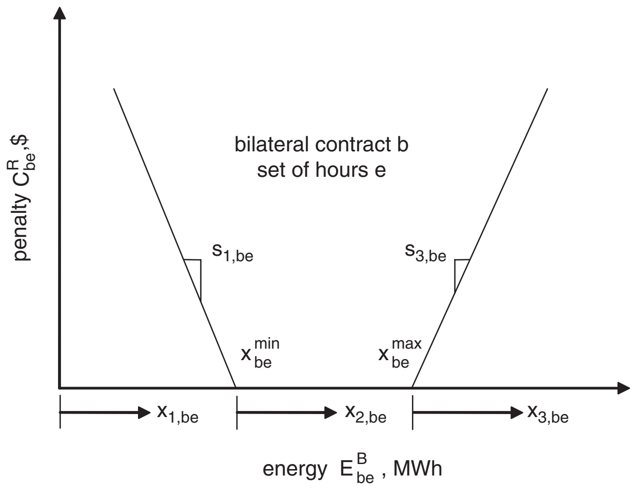

Fig. 1 *Penalty for under- and over-consumption; mathematical description*

In the constraints above, variable $x_{i,be}$ $(i=1,2,3)$ represents the energy consumed within block i. The purpose of dividing the energy consumed in cumulative blocks is to penalize under- of over-consumptions according with the corresponding contract arrangements. If for a given contract and a specified set of hours (e. g. valley hours), energy consumption falls in the first block, an under-consumption penalty is incurred. If energy consumption is above the first block but below the sum of blocks one and two, no penalty is incurred, and if the energy consumption is above the sum of blocks one and two, an over-consumption penalty is incurred. It should be noted that $x^{\min}_{be} < x^{\max}_{be}$.

Binary variables, defined through constraint (6), are used to enforce the penalty function depicted in Fig. 1. All feasible cases are analysed below. Case $w^{\text{down}}_{be} = w^{\text{up}}_{be} = 0$ implies $0 \leq x_{1,be} \leq x^{\min}_{be}$, $x_{2,be}=0$ and $x_{3,be}=0$. In this case, the energy consumed from contract b during hours of type e does not exceed the first block. Case $w^{down}_{be} = 1$ and $w^{up}_{be} = 0$ implies $x_{1,be} = x^{\min}_{be}$, $0 \leq x_{2,be} \leq x^{\max}_{be} - x^{\min}_{be}$ and $x_{3,be}=0$. In this case, the energy demand from contract b during hours of type e is within the second block. Finally, case $w^{down}_{be} = w^{up}_{be} = 1$ implies $x_{1,be} = x^{\min}_{be}$, $x_{2,be} = x^{\max}_{be} - x^{\min}_{be}$, and $0 \leq x_{3,be} \leq M$. In this case, the energy consumed from contract b during hours of type e is within the third block.

As illustrated in Fig. 1, the incurred penalty cost for hours of type e and contract b is then computed as

$$C^R_{be} = s_{1,be}\left(x^{\min}_{be} - x_{1,be}\right) + s_{3,be}x_{3,be} \qquad (7)$$

The first term on the right-hand side of the above equation is the under-consumption penalty, while the second one is the over-consumption penalty (Fig. 1).

Note that for the particular case of the convex penalty function represented in Fig. 1, binary variables $(w^{down}_{be}, w^{up}_{be})$ can be eliminated, i.e. the penalty function can be formulated without using binary variables. However, binary variables are needed to model more complex non-convex penalty functions, and this is why they are used in this paper. Note also that the number of binary variables needed to model contract penalties is small.

2.2 Purchasing from and selling to the pool

Purchasing cost from the pool is computed as

$$C^P = \text{Exp}_{\lambda^P_1,\ldots,\lambda^P_T}\left\{\sum_{h=1}^{T} \lambda^P_h P^P_h\right\} = \sum_{h=1}^{T} E_{\lambda^P_h}\{\lambda^P_h\} P^P_h$$
$$= \sum_{h=1}^{T} \lambda^{\text{P,est}}_h P^P_h \qquad (8)$$

Therefore, the purchasing cost is computed as the summation over time of hourly price times hourly purchased power.

Analogously, selling revenue from the pool is computed as

$$C^S = \text{Exp}_{\lambda^P_1,\ldots,\lambda^P_T}\left\{\sum_{h=1}^{T} \lambda^P_h P^S_h\right\} = \sum_{h=1}^{T} \lambda^{\text{P,est}}_h P^S_h \qquad (9)$$

2.3 Self-production

The energy self-produced in hour h, P^G_h, is computed as

$$P^G_h = v_h P^{\min} + \sum_{\ell=1}^{L} P_{\ell h} \qquad (10)$$

where

$$0 \leq P_{\ell h} \leq v_h P^{\max}_\ell,\ \ell = 1, \ldots, L \qquad (11)$$

The production cost is computed using a piecewise linear convex approximation as depicted in Fig. 2. A piecewise linear non-convex approximation can be implemented in a straightforward manner as described in [1].

Total self-production cost in hour h is thus computed as

$$C^G_h = v_h C^{FX} + z_h C^{SD} + y_h C^{SU} + \sum_{\ell=1}^{L} f_\ell P_{\ell h} \qquad (12)$$

Operation constraints of the self-production facility include:

1. minimum and maximum power output limits;
2. ramp-up and ramp-down limits;
3. start-up an shut-down ramping limits;
4. minimum up- and minimum down-time constraints.

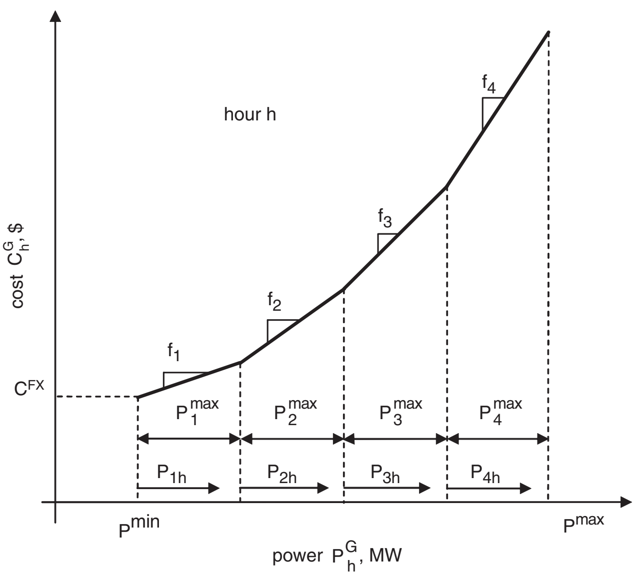

Fig. 2 *Piecewise linear convex production cost using four blocks*

The above constraints can be generally expressed as

$$P_h^G \in G \quad \forall h \in H \tag{13}$$

where G is the feasible operating region of the self-producing facility. A detailed description of the feasibility region G can be found in [1].

Binary variables used to model the on/off status of the self-production facility in period h should meet the constraint below:

$$v_h - v_{h-1} = y_h - z_h \quad y_h + z_h \leq 1 \tag{14}$$

2.4 Power balance

The power balance in hour h is expressed as

$$P_h^P + \sum_{b \in \Xi} P_{bh}^B + P_h^C = D_h \tag{15}$$

It should be noted that the demand is considered known (with no uncertainty) as the consumer has usually a good knowledge of the behaviour of its own demand. Note also that this is not usually the case for a retailer. Relaxing this reasonable assumption leads to a fully-fledged stochastic programming problem, which cannot be reduced to an equivalent deterministic one. For details on this approach, see [5].

On the other hand, self-produced energy can be locally consumed or sold to the pool in hour h, i.e.

$$P_h^G = P_h^S + P_h^C \tag{16}$$

To avoid selling and buying energy during the same hour h at an identical price (no transaction costs are considered), which gives no benefit, the constraints below can be included in the formulation

$$P_h^P \leq q_h D_h \tag{17}$$

$$P_h^S \leq (1 - q_h) P^{\max} \tag{18}$$

$$q_h \in \{0, 1\} \tag{19}$$

If the self-production facility is producing for self-consumption, constraint (17) says that power output should be less than or equal to the actual demand. On the other hand, if the self-production facility is working for selling to the pool, constraint (18) states that power output should be less than or equal to the maximum power output (capacity) of the facility. Binary variable q_h avoids simultaneous self-consumption and selling energy.

2.5 Formulation

The consumer target is to minimise cost subject to economic and technical constraints. This is formulated as

minimise

$$C^P - C^S + \sum_{h=1}^{T} C_h^G + \sum_{b \in \Xi} \left(C_b^B + \sum_{e \in \Omega_b} C_{be}^R \right) \tag{20}$$

where C^P, C^S, C_h^G, C_b^B and C_{be}^R are given in Eqns. (8), (9), (12), (1) and (7), respectively.

subject to

$$P_h^G = v_h P^{\min} + \sum_{\ell=1}^{L} P_{\ell h} \quad \forall h \in H \tag{21}$$

$$P_h^G \in G \; \forall h \in H \tag{22}$$

$$\sum_{h \in \Omega_{be}} P_{bh}^B = \sum_{i=1}^{3} x_{i,be} \quad \forall b \in \Xi, \forall e \in \Omega_b \tag{23}$$

$$\sum_{h \in \Omega_{be}} P_{bh}^B = \sum_{i=1}^{3} x_{i,be} \quad \forall b \in \Xi, \forall e \in \Omega_b \tag{24}$$

$$w_{be}^{\text{up}} (x_{be}^{\max} - x_{be}^{\min}) \leq x_{2,be} \leq w_{be}^{\text{down}} (x_{be}^{\max} - x_{be}^{\min}) \quad \forall b \in \Xi, \forall e \in \Omega_b \tag{25}$$

$$0 \leq x_{3,be} \leq w_{be}^{\text{up}} M \quad \forall b \in \Xi, \forall e \in \Omega_b \tag{26}$$

$$w_{be}^{\text{down}} \geq w_{be}^{\text{up}} \quad \forall b \in \Xi, \forall e \in \Omega_b \tag{27}$$

$$P_h^P \leq q_h D_h \quad \forall h \in H \tag{28}$$

$$P_h^S \leq (1 - q_h) P^{\max} \quad \forall h \in H \tag{29}$$

$$P_h^P + \sum_{b \in \Xi} P_{bh}^B + P_h^C = D_h \quad \forall h \in H \tag{30}$$

$$P_h^G = P_h^S + P_h^C \quad \forall h \in H \tag{31}$$

$$v_h - v_{h-1} = y_h - z_h \quad \forall h \in H \tag{32}$$

$$y_h + z_h \leq 1 \quad \forall h \in H \tag{33}$$

$$v_h, y_h, z_h, q_h \in \{0, 1\} \quad \forall h \in H \tag{34}$$

$$w_{be}^{\text{down}}, w_{be}^{\text{up}} \in \{0, 1\} \quad \forall b \in \Xi, \forall e \in \Omega_b \tag{35}$$

All constraints above have been described throughout the paper, but not their domains of definitions, which are specified above.

3 Case study

Results from a realistic case study are summarised in this section. A three-month time horizon is considered. The set of sets of hours for contract definition for the 3 months is {1p, 1s, 1v, 1we, 2p, 2s, 2v, 2we, 3p, 3s, 3v, 3we}, where 1, 2 and 3 identify the month; and p, s, v and we denote for peak, shoulder, valley and weekend, respectively. Data for the self-production facility are provided in Tables 3, 4 and 5. Data for the two bilateral contracts considered are provided in Tables 6, 7 and 8. Price forecasts and demands for a typical day are given in Table 9. The demand pattern corresponds to the first three months of the year for a large consumer. Pool prices for this time horizon are forecast using data from the electricity market of mainland Spain [19] and a transfer function algorithm is used for forecasting [20].

Table 10 summarises the numerical results of the case study. In this table, note that the net cost is calculated as the total cost of serving the demand minus the selling revenues obtained by the self-production facility from selling energy to the pool.

Table 3: Technical characteristics of the self-production facility

$P^{\max}$, MW	$P^{\min}$, MW	Ramp-up limit, MW/h	Ramp-down limit, MW/h	Minimum up/down time, h
160	50	90	80	5

Table 4: Fixed, shut-down and start-up costs in US $

Fixed	Shut-down	Start-up
2500	1500	2000

Table 5: Piecewise linear cost for the self-production facility

Block	Block size, MW	Cost, \$/MWh
1	30	15
2	30	20
3	30	30
4	20	45

Table 6: Bilateral contracts: prices for a typical week

Hour type	Price, \$/MWh	
	Bilateral contract 1	Bilateral contract 2
Valley (2-7)	15	10
Shoulder (1, 8-10, 14-18, 22-24)	35	30
Peak (11-13, 19-21)	50	45
Weekend (Saturday & Sunday)	35	30

Table 7: Penalty data for bilateral contract 1 during any of the three months

Bilateral contract 1				
Hour type	Block	Lower, MWh	Upper, MWh	Cost, \$/MWh
Valley	1	0	6000	6
	2	6000	10000	0
	3	10000	50000	5
Shoulder	1	0	7000	6
	2	7000	10000	0
	3	10000	50000	7
Peak	1	0	8000	6
	2	8000	13000	0
	3	13000	50000	9
Weekend	1	0	9000	5
	2	9000	15000	0
	3	15000	50000	6

Fig. 3 provides the optimal mix of electricity sources to meet the demand for a representative day, while Fig. 4 depicts the optimal mix of electricity sources to meet the energy demand for the whole three-month time horizon.

From Fig. 3 it can be observed that the self-production facility operates for self-consumption during hours 1, 8–11, 14–18, 20, 23 and 24; and for selling to the pool during hours 2, 12, 13, 19, 21 and 22. During hours 2–7 (valley period), bilateral contract prices are much smaller than the corresponding prices in the pool; as a result, energy is bought exclusively from contracts during these hours. On the other hand, during hours 12, 13, 19, 21 and 22, the total energy produced by the self-production facility is sold to the pool, which is the most profitable option, and the consumer procures its energy demand from bilateral contracts. During the remaining hours (1, 8–10, 14–18, 20, 23 and 24), the pool price is higher than the corresponding contract price, and, nevertheless, part of the required energy is bought from the pool. The reason for this comes from the penalties incurred for over-consumption from bilateral contracts. In fact, in the optimal solution, and for most hours, the marginal price of supplying power is equal to the pool price,

Table 8: Penalty data for bilateral contract 2 during any of the 3 months

Bilateral contract 2				
Hour type	Block	Lower, MWh	Upper, MWh	Cost, \$/MWh
Valley	1	0	7000	8
	2	7000	9000	0
	3	9000	50000	6
Shoulder	1	0	8000	7
	2	8000	10000	0
	3	10000	50000	9
Peak	1	0	8000	9
	2	8000	10000	0
	3	10000	50000	9
Weekend	1	0	9000	7
	2	9000	10000	0
	3	10000	50000	7

Table 9: Demands and pool price forecasts for a typical working day

Hour h	Demand, MWh	Price forecast, \$/MWh
1	196.776	42.39
2	185.492	29.59
3	176.987	22.48
4	172.993	22.52
5	171.265	22.33
6	172.350	22.52
7	185.876	29.95
8	205.675	44.84
9	207.173	43.61
10	216.106	44.87
11	225.569	45.00
12	235.307	45.50
13	234.658	45.24
14	232.493	44.97
15	222.686	44.50
16	223.436	44.50
17	227.940	45.50
18	230.770	45.77
19	227.673	45.50
20	220.414	44.52
21	225.596	45.57
22	230.663	46.44
23	212.831	44.17
24	196.806	42.50

Table 10: Numerical results for the case study

Gen.	Total energy generated	177 560 MWh	
	Percentage self-consumed	39.05%	
	Percentage sold	60.95%	
Demand	Demand supplied	433 690 MWh	
	Percentage produced and self-consumed	15.99%	
	Percentage purchased from the pool	33.64%	
	Percentage bought from bilateral contracts	50.37%	
Cost	Generating cost	k$	6 008.3
	Buying from the pool	k$	4 137.7
	Buying from bilateral contracts	k$	7 617.3
	Penalties from bilateral contracts	k$	112.6
	Total costs	k$	17 875.9
	Net costs	k$	12 928.2
Selling revenues from the pool		k$	4 947.7

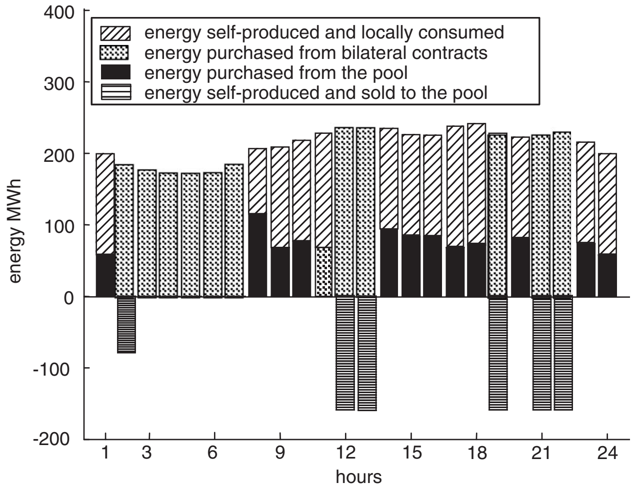

Fig. 3 *Mix of electricity sources for a typical working day*

which means that the cost of supply from contracts (including penalties) and from the pool, respectively, are equal. This is consistent with the classical microeconomics result that states that supplying sources should work at marginal cost. In other words, hourly cost sensitivities to demand changes for the typical working day considered in Fig. 3 are similar to the pool hourly prices.

Note that minimum up-time (5 h) and minimum down-time (5 h) constraints are satisfied: the self-production unit is up for more than 3 h before hour 1 in Fig. 3, remains up 2 more hours, is shut-down for 5 h (3–7), started-up in hour 8, and stays up for the remaining hours (8–24).

Note the balanced mix of electricity sources achieved by the consumer, as can be observed in Fig. 4. That is, the optimal solution contains important percentages of pool purchases, self-production and bilateral contract purchases. Therefore, the large consumer does not rely heavily on a single electric energy source. The penalties from under- or

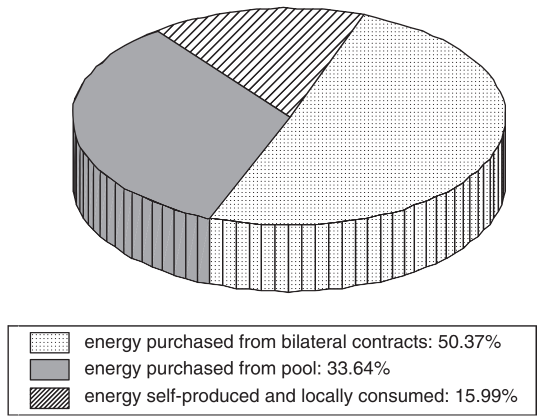

Fig. 4 *Mix of electricity sources for the whole time horizon*

over-consumption are comparatively small, as can be seen in row 11 of Table 10. This is a consequence of the flexible energy procurement framework available to the consumer.

If no self-production facility were available, the net incurred cost to serve the demand would increase by 7.6%. Therefore, the self-production facility is functioning as a hedge against the vagaries of the marketplace. If neither the self-production facility, nor bilateral agreements were available, the net incurred cost to serve the demand would increase by 21%.

The CPU time required to solve the problem considered in a Dell PowerEdge 6600 with two processors at 1.60 GHz and 2Gb of RAM memory using CPLEX 8.0 under GAMS [17] is about 15 minutes.

4 Conclusions

In the framework of an electricity market that includes bilateral contract agreements and a pool, this paper analyses the portfolio problem faced by a consumer that should determine its purchases from bilateral contracts, from the pool as well as its level of self-production. The consumer can sell back to the pool its excess self-produced energy. The target is to minimise the expected electricity bill to be paid by the consumer. The time horizon considered ranges from one to several months. The bilateral contract framework used allows the easy modelling of many real-world bilateral contract arrangements. Results from a realistic 3-month case study are summarised and discussed in the paper. The appropriate use of bilateral contracts as well as self-production facilities allows significant reduction of the electricity bill of the consumer in comparison with buying solely from the pool. It also minimises the risk associated to pool price volatility. Although the technical literature is rich in models for producers, analyses and models for consumers are not so common. This paper contributes to enhance the technical literature on models for the consumer. Current research is directed to incorporate risk-limiting mechanisms for total profit in the energy procurement problem considered in this paper. Using these mechanisms, expected profit can be maximised ensuring an acceptable level of risk in achieving that profit.

5 Acknowledgments

We thank Professor S. Gabriel from the University of Maryland and Dr S. de la Torre and Professor Arroyo from Universidad de Castilla – La Mancha for insightful comments and suggestions.

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