Economic Inefficiencies and Cross-Subsidies in an Auction-Based Electricity Pool

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***Abstract*—This paper compares two contrasting yet often used electricity market-clearing procedures: i) an auction-based algorithm including congestion management and transmission-loss cost allocation, and ii) an optimal power flow. The auction procedure produces a single-period unit commitment, and hence, can be compared directly to an optimal power flow solution. These algorithms are compared in terms of the economic efficiency of the solution attained, and in terms of cross-subsidies among generators and demands. The purpose of this comparison is to quantify the actual cost to market participants of using a simple, seemingly transparent procedure, such as an auction-based algorithm, versus an integrated but computationally intensive one, such as an optimal power flow.**

***Index Terms*—Auctions, cross-subsidies, inefficiencies, optimal power flow, pool-based electricity market.**

I. Introduction

QUITE A FEW pool-based electricity markets use a simple auction procedure to clear the market every hour. The auction-based dispatch [1]–[3] neglects the network in two respects: the limited transmission line capacities and the network losses. Therefore, the auction results must often be modified *ex post* to take the network into account. To that end, two successive procedures are normally applied. One checks for line congestion and redispatches to meet transmission capacity limits [4]–[9]. The other uses a power-flow algorithm to determine transmission losses, the costs of which are then allocated to producers and consumers using an appropriate loss allocation algorithm [10]–[17]. Redispatching should be minimal so that the auction results are preserved as much as possible, but both redispatching to alleviate congestion and loss allocation may result in significant deviation from optimality, measured in terms of social welfare. They also may result in cross-subsidies involving both generators and demands. This procedure is similar to the one used in mainland Spain [5].

An alternative market-clearing procedure, once the on/off scheduling of generators is known, is an optimal power flow (OPF) algorithm, under which both transmission constraint limits and transmission losses are optimally accounted for [18]–[20].

As a byproduct, the OPF algorithm produces pricing information that can differ by location, reflecting local market conditions as affected by network limitations. That has been seen both as a blessing and a curse by proponents and opponents, respectively.

This paper compares these two market-clearing procedures in terms of the economic efficiency of the solution attained and in terms of cross-subsidies among generators and demands. In the comparison, the single-period unit commitment is determined by an auction procedure; therefore, the comparison highlights specifically the differences between i) the OPF results and ii) the added costs of congestion management and transmission-loss allocation procedures. The purpose of this comparison is to quantify the differences between the simple, transparent but multisequenced procedure of the auction described previously and an integrated but computationally intensive OPF.

What remains of this paper is organized as follows. Section II provides the notation used throughout the paper. Section III describes i) the auction-based dispatching procedure, ii) the congestion-management algorithm, and iii) the method to allocate the cost of losses. Section IV briefly reviews the OPF used for comparison. Section V presents a realistic case study. Finally, in Section VI conclusions are presented.

II. Notation

Optimization Variables:

P_{Dik}	power block k that demand i is willing to buy at price λ_{Dik} up to a maximum of $P_{Dik}^{\max}$;
P_{Di}	power consumed by demand i;
$P_{Gj\ell}$	power block ℓ that generator j is willing to sell at price $\lambda_{Gj\ell}$ up to a maximum of $P_{Gj\ell}^{\max}$;
P_{Gj}	power produced by generator j;
ΔP_{Gj}^{up}	increment (due to congestion management) in the pool power schedule of generator j;
ΔP_{Gj}^{down}	decrement (due to congestion management) in the pool power schedule of generator j;
ΔP_{Gn}^{up}	increase (due to congestion management) in the pool power schedule of generation in bus n. It is the sum of the active power increments by all generators in bus n;
ΔP_{Gn}^{down}	decrease (due to congestion management) in the pool power schedule of generation in bus n. It is the

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sum of the active power decrements by all generators in bus n;

Q_{Gn} reactive power generated at bus n;

$S_{nm}(\cdot)$ apparent power magnitude going from bus n to bus m through line nm;

u_j 0/1 variable which is equal to 1 if generator j is running and 0 otherwise;

V_n voltage magnitude of bus n;

δ_n voltage angle of bus n;

λ_n energy locational marginal price at bus n (dual variable);

Other Variables:

C^L total cost of losses;

E_i final payment of demand i;

MS merchandising surplus;

P_{loss} represents total network transmission losses;

P_{Gj}^A active power produced by generator j as determined by the auction dispatch;

P_{Gn}^A active power generated at bus n as determined by the auction dispatch. It is the sum of the active power generated by all generators in bus n;

P_{Di}^A active power consumed by demand i as determined by the auction dispatch;

P_{Dn}^A active-power demand at bus n as determined by the auction dispatch. It is the sum of all active power demands in bus n;

R^C total generator revenues due to congestion relief;

R_j final revenue of generator j;

$\lambda_{Gs\ell*}$ offered price of the most expensive block used to supply losses by the slack generator;

λ^A market-clearing price as determined by the auction dispatch;

Constants:

B_{nm} susceptance of line nm;

G_{nm} conductance of line nm;

$P_{Dik}^{\max}$ size of the power block k that demand i is willing to buy at price λ_{Dik};

$P_{Gj\ell}^{\max}$ size of the power block ℓ offered by generator j at price $\lambda_{Gj\ell}$;

$P_{Gj}^{\max}$ maximum power output of generator j. It is assumed that $P_{Gj}^{\max} = \sum_{\ell=1}^{N_{Gj}} P_{Gj\ell}^{\max}$;

$P_{Gj}^{\min}$ minimum power output of generator j. It is assumed that $P_{Gj}^{\min} = P_{Gj1}^{\max}$;

$P_{nm}^{\max}$ transmission capacity limit (active power) of line nm;

Q_{Dn} reactive-power demand at bus n;

r_j^{up} price offered by generator j to increase its pool power schedule, for congestion-management purposes;

r_j^{down} price offered by generator j to decrease its pool power schedule (r_j^{down} is generally different than r_j^{up}), for congestion-management purposes;

$S_{nm}^{\max}$ transmission capacity limit (complex power) of line nm;

$V_n^{\max}$ maximum voltage magnitude of bus n;

$V_n^{\min}$ minimum voltage magnitude of bus n;

λ_{Dik} price offered by demand i to buy power block k;

$\lambda_{Gj\ell}$ price offered by generator j to sell power block ℓ.

Sets and Numbers:

D_n set of index of demands in bus n;

G set of index of all generators;

G^{on} set of index of on-line generators;

G_n set of index of generators in bus n;

G_n^{on} set of index of on-line generators in bus n;

N total number of buses;

N_D number of demands;

N_{Di} number of blocks requested by demand i;

N_G number of generators;

N_{Gj} number of blocks offered by generator j;

Ω_n set of index of buses connected to bus n.

Superscripts "max" and "min" indicate, respectively, maximum and minimum, superscript "A" indicates auction results, and superscript "C" denotes results after congestion management.

III. Auction-Based Procedure

The following three subsections describe an auction-based market-clearing procedure consisting of an auction dispatch, a congestion-management algorithm, and a method to allocate the cost of transmission losses to producers and consumers.

A. Auction

An auction dispatch can be formulated as

$$\text{maximize} \quad \sum_{i=1}^{N_D}\sum_{k=1}^{N_{Di}} \lambda_{Dik} P_{Dik} - \sum_{j=1}^{N_G}\sum_{\ell=1}^{N_{Gj}} \lambda_{Gj\ell} P_{Gj\ell} \tag{1}$$

subject to

$$0 \le P_{Dik} \le P_{Dik}^{\max} \qquad \forall i = 1, \ldots, N_D, \forall k = 1, \ldots, N_{Di} \tag{2}$$

$$0 \le P_{Gj\ell} \le P_{Gj\ell}^{\max} \qquad \forall j \in G, \forall \ell = 1, \ldots, N_{Gj} \tag{3}$$

$$u_j P_{Gj}^{\min} \le \sum_{\ell=1}^{N_{Gj}} P_{Gj\ell} \le u_j P_{Gj}^{\max} \qquad \forall j \in G \tag{4}$$

$$\sum_{i=1}^{N_D}\sum_{k=1}^{N_{Di}} P_{Dik} = \sum_{j=1}^{N_G}\sum_{\ell=1}^{N_{Gj}} P_{Gj\ell} \tag{5}$$

$$u_j \in \{0, 1\} \qquad \forall j \in G. \tag{6}$$

The objective function (1) represents the consumer surplus plus the producer surplus (i.e., the net social welfare). It is computed as the difference of two terms. The first term is the sum of accepted demand bids times their corresponding bid prices. The second term is the sum of accepted production bids times their corresponding bid prices. It should be noted that if producers do not bid at their respective marginal costs, the second term of the objective function is not actually the producer surplus but the "declared" producer surplus. Nevertheless, in competitive markets, rational producers tend to bid at their respective marginal costs [21]–[23]. It should also be noted that producer bids are considered convex and monotonically increasing; and consumer bids, concave and monotonically decreasing.

The block of constraints (2) specifies the sizes of the demand bids. The block of constraints (3) limits the sizes of the production bids, while (4) ensures that every generator, if running, runs between its minimum and its maximum power output. Constraint (5) states that the production should be equal to the demand, so that the market clears, and (6) is the binary variables declaration.

The above auction dispatch is similar to the heuristic procedure used to clear the electricity market in mainland Spain [24].

The solution of problem (1)–(6) provides the power produced by every generator and the power consumed by every demand, that is

$$P_{Gj}^{A} = \sum_{\ell=1}^{N_{Gj}} P_{Gj\ell}^{A} \qquad \forall j \in G \tag{7}$$

$$P_{Di}^{A} = \sum_{k=1}^{N_{Di}} P_{Dik}^{A} \qquad \forall i = 1, \ldots, N_D. \tag{8}$$

The market-clearing price is typically defined as the price of the most expensive production bid that has been accepted, and it is denoted by λ^A.

Under marginal pricing, the revenue of generator j becomes $\lambda^A P_{Gj}^A$, and the payment of demand i is $\lambda^A P_{Di}^A$. Total generator revenue is $\lambda^A \sum_{j\in G} P_{Gj}^A$ and demand total payment is $\lambda^A \sum_{i=1}^{N_D} P_{Di}^A$. It should be noted that both quantities are identical.

B. Congestion Management

The above quantities have been determined without taking into account the limited capacity of the transmission network. To manage congestion due to such limits, the problem below is solved

$$\text{minimize} \quad \sum_{j\in G} \left(r_j^{up}\, \Delta P_{Gj}^{up} + r_j^{down}\, \Delta P_{Gj}^{down}\right) \tag{9}$$

subject to

$$P_{Gn}^{A} + \Delta P_{Gn}^{up} - \Delta P_{Gn}^{down} + \sum_{m\in\Omega_n} B_{nm}(\delta_m - \delta_n) - P_{Dn}^{A} = 0 \qquad \forall n = 1, \ldots, N \tag{10}$$

$$-P_{nm}^{\max} \le B_{nm}(\delta_n - \delta_m) \le P_{nm}^{\max} \quad \forall n = 1, \ldots, N,\ \forall m \in \Omega_n \tag{11}$$

$$u_j\, P_{Gj}^{\min} \le P_{Gj}^{A} + \Delta P_{Gj}^{up} - \Delta P_{Gj}^{down} \le u_j\, P_{Gj}^{\max} \quad \forall j \in G \tag{12}$$

$$\Delta P_{Gn}^{up} = \sum_{j\in G_n} \Delta P_{Gj}^{up} \qquad \forall n = 1, \ldots, N \tag{13}$$

$$\Delta P_{Gn}^{down} = \sum_{j\in G_n} \Delta P_{Gj}^{down} \qquad \forall n = 1, \ldots, N \tag{14}$$

$$P_{Gn}^{A} = \sum_{j\in G_n} P_{Gj}^{A} \qquad \forall n = 1, \ldots, N \tag{15}$$

$$P_{Dn}^{A} = \sum_{i\in D_n} P_{Di}^{A} \qquad \forall n = 1, \ldots, N. \tag{16}$$

The objective function (9) is the sum of the amounts received (or paid) by the generators for altering their output as compared to the original auction schedule. The loads could also participate in congestion management through a similar offer to curtail or increase consumption.

The set of constraints (10) enforces power balance at every bus and the set of constraints (11) enforces line capacity limits, both using the dc load flow model. The set (12) guarantees that the rescheduled generators stay within their respective maximum and minimum power outputs. Constraints (13) and (14) relate power increments in buses and generators. Constraints (15) state that the power generation in every bus is the sum of the production of all generators in that bus. Constraints (16) are similar to constraints (15) but referring to demands.

It should be noted that in the congestion management procedure, marginal pricing is not used to compensate the generators for adjusting their power output. Instead, each generator is paid the exact amount $r_j^{up}\, \Delta P_{Gj}^{up} - r_j^{down}\, \Delta P_{Gj}^{down}$ for its adjustment. If the above quantity is negative, the corresponding generator has to pay that amount.

The resulting modified generation levels from congestion management are

$$P_{Gj}^{C} = P_{Gj}^{A} + \Delta P_{Gj}^{up} - \Delta P_{Gj}^{down} \qquad \forall j \in G. \tag{17}$$

Since it is assumed here that the demands are not modified, then

$$P_{Di}^{C} = P_{Di}^{A} \qquad \forall i = 1, \ldots, N_D. \tag{18}$$

The revenue corresponding to generator j becomes

$$\lambda^A P_{Gj}^{A} + r_j^{up} \Delta P_{Gj}^{up} - r_j^{down} \Delta P_{Gj}^{down}. \tag{19}$$

The total revenue of the generators is $\lambda^A \sum_{j\in G} P_{Gj}^A + \sum_{j\in G}(r_j^{up}\Delta P_{Gj}^{up} - r_j^{down}\Delta P_{Gj}^{down})$, and total generator revenue due to congestion relief is $R^C = \sum_{j\in G}(r_j^{up}\Delta P_{Gj}^{up} - r_j^{down}\Delta P_{Gj}^{down})$. This amount is allocated among the demands in a *pro rata* manner. Therefore, the payment of load i becomes

$$\lambda^A P_{Di}^{A} + R^C \frac{P_{Di}^{A}}{\sum_{i=1}^{N_D} P_{Di}^{A}}. \tag{20}$$

This congestion-management procedure is similar to the ones used in the markets of California [25] and mainland Spain [5].

C. Allocation of the Cost of Transmission Losses

The above quantities have been determined without taking into account transmission losses. Therefore, a procedure is needed to allocate *ex post* the cost of these losses to generators and demands. First, a load flow is run to determine the losses P_{loss} with the slack generator chosen as the cheapest offered block not saturated. If additional energy is needed to supply losses, the next cheapest block is considered and so on until enough energy is available to meet all losses.

In order to make a fair comparison, the voltage magnitude and reactive power data used to solve the above power flow are chosen to be equal to the results of the OPF presented in Section IV.

The slack generator producing the losses receives an extra payment equal to the losses times the offered price corresponding to the most expensive block used to supply losses $\lambda_{Gs\ell*}$. Total revenue of this generator then becomes

$$\lambda^A P_{Gs}^A + r_s^{up} \Delta P_{Gs}^{up} - r_s^{down} \Delta P_{Gs}^{down} + \lambda_{Gs\ell*} P_{loss} \quad (21)$$

where the cost of losses is $C^L = \lambda_{Gs\ell*} P_{loss}$. Generators and demands contribute *pro rata* (other more elaborate procedures are also available, for example, see [10]) to pay for these losses. Therefore, subtracting the generator payment for losses, final revenue for generator j (other than the slack) becomes

$$R_j = \lambda^A P_{Gj}^A + r_j^{up} \Delta P_{Gj}^{up} - r_j^{down} \Delta P_{Gj}^{down} - \frac{C^L}{2} \frac{P_{Gj}^C}{\sum\limits_{j \in G} P_{Gj}^C} \quad (22)$$

and for the slack

$$R_s = \lambda^A P_{Gs}^A + r_s^{up} \Delta P_{Gs}^{up} - r_s^{down} \Delta P_{Gs}^{down} + \lambda_{Gs\ell*} P_{loss} - \frac{C^L}{2} \frac{P_{Gs}^C}{\sum\limits_{j \in G} P_{Gj}^C}. \quad (23)$$

The final payment for demand i is

$$E_i = \lambda^A P_{Di}^A + \left(\frac{C^L}{2} + R^C \right) \frac{P_{Di}^C}{\sum\limits_{i=1}^{N_D} P_{Di}^C}. \quad (24)$$

IV. OPF

An OPF [18]—[20] is solved with inelastic demands $P_{Di}^A, \forall i = 1, \ldots, N_D$. All on-line generators (in G^{on}) are considered for production. On-line generators are known once the auction and congestion-management procedures have been performed. The production bids are considered piecewise linear cost curves and the objective of the OPF is to minimize the summation of these monotonically increasing piecewise cost curves. The formulation is as follows:

$$\text{minimize} \quad \sum_{j \in G^{on}} \sum_{\ell=1}^{N_{Gj}} \lambda_{Gj\ell} P_{Gj\ell} \quad (25)$$

subject to

$$P_{Gn} = \sum_{j \in G_n^{on}} \sum_{\ell=1}^{N_{Gj}} P_{Gj\ell} \qquad \forall n = 1, \ldots, N \quad (26)$$

$$P_{Gn} - P_{Dn}^A = V_n \sum_{m=1}^{N} V_m [G_{nm} \cos(\delta_n - \delta_m) + B_{nm} \sin(\delta_n - \delta_m)] \colon \lambda_n \quad \forall n = 1, \ldots, N \quad (27)$$

$$Q_{Gn} - Q_{Dn} = V_n \sum_{m=1}^{N} V_m [G_{nm} \sin(\delta_n - \delta_m) - B_{nm} \cos(\delta_n - \delta_m)] \qquad \forall n = 1, \ldots, N \quad (28)$$

$$-S_{nm}^{\max} \le S_{nm}(V_n, V_m, \delta_n, \delta_m) \le S_{nm}^{\max} \quad \forall m \in \Omega_n, \forall n = 1, \ldots, N \quad (29)$$

$$V_n^{\min} \le V_n \le V_n^{\max} \qquad \forall n = 1, \ldots, N \quad (30)$$

$$0 \le P_{Gj\ell} \le P_{Gj\ell}^{\max} \qquad \forall j \in G^{on}, \forall \ell = 1, \ldots, N_{Gj} \quad (31)$$

$$P_{Dn}^A = \sum_{i \in D_n} P_{Di}^A \qquad \forall n = 1, \ldots, N. \quad (32)$$

The objective function (25) is the total production cost (bids are considered costs). Constraints (26) express the power generation of any bus as the summation of the power blocks of the generators connected to this bus. Constraints (27) and (28) are the full ac active- and reactive-power balance equations, respectively. Constraints (29) enforce transmission capacity limits for all lines. Constraints (30) and (31) impose bounds on the voltage magnitudes and on the generator power blocks, respectively. Equations (32) state that the power demanded at every bus is the sum of the demands of all consumers in that bus.

The solution of the above problem provides the production of every generator, namely $P_{Gj}^* = \sum_{\ell=1}^{N_{Gj}} P_{Gj\ell}^*, \ j \in G^{on}$, and the energy locational marginal price at every bus (i.e., $\lambda_n^*, \ \forall n = 1, \ldots, N$), in which congestion and losses are accounted for.

Under marginal pricing, the revenue of generator j becomes

$$R_j^* = \lambda_j^* P_{Gj}^* \quad (33)$$

where λ_j^* is the energy locational marginal price of the bus where generator j is located.

The payment of demand i becomes

$$E_i^* = \lambda_i^* P_{Di}^A. \quad (34)$$

Due to the locational nature of locational marginal prices, there exists a merchandising surplus that can be computed as

$$MS = \sum_{n=1}^{N} \sum_{i \in D_n} \lambda_n^* P_{Di}^A - \sum_{n=1}^{N} \sum_{j \in G_n} \lambda_n^* P_{Gj}^*. \quad (35)$$

We recall that the auction schedule does not contain any merchandising surplus or any explicit payment to the transmission provider. Transmission company revenues under the auction approach are derived solely from fixed rate payment such as postage stamps. On the other hand, under OPF, during congested periods, a sizable portion of the consumer payment goes to the transmission provider in the form of a merchandising surplus. Whatever extra revenue is required by the transmission provider will be paid through a fixed rate payment. In either the OPF or auction scheme, the total payment to the transmission provider is identical.

To fairly compare the OPF dispatch with that of the auction, the merchandising surplus is subtracted *pro rata* from the demand payments. Therefore, the final payment of demand i becomes

$$E_i = \lambda_i^* P_{Di}^A - MS \frac{P_{Di}^A}{\sum\limits_{i=1}^{N_D} P_{Di}^A}. \quad (36)$$

V. CASE STUDY

A case study based on the IEEE RTS system is presented in this section. Topology, line, generator, and demand data can be

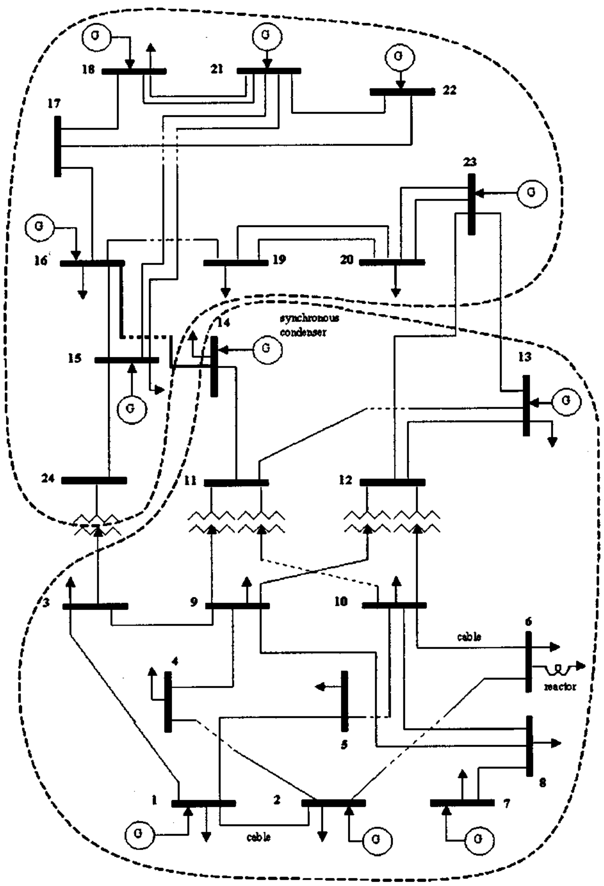

Fig. 1. IEEE reliability test system.

found in reference [26, Fig. 1 and Tables 12, 9, and 5, respectively]. Transmission capacity limits of the lines are also given in [26, Table 12]. The transmission limit of line 14–16 is reduced to 350 MVA in our study (instead of 500 MVA) so that congestion occurs. Fig. 1 depicts the IEEE Reliability Test System. Congested line 14–16 is shown in bold. The areas in which the system splits are identified.

It is considered that every generator bids at its marginal costs using as many blocks as the number of incremental heat rate blocks. This simple bidding criterion is used for simplicity, because it does not affect the comparison. The number and size of the incremental heat rate blocks are specified in [26, Table 9]. The first block is considered to be the minimum power output. For instance, the 12-MW generator offers four blocks of sizes 2.4, 3.6, 3.6, and 2.4 MW, respectively. Fuel costs have been taken from [27] and are U.S.$2.3/MBtu for #6 oil, U.S.$3.0/MBtu for #2 oil, U.S.$1.20/MBtu for coal, and U.S.$0.6/MBtu for nuclear. Using this cost data, the marginal cost values for the blocks of the 12-MW generator are 23.41, 23.78, 26.84, and 30.40 U.S.$/MWh, respectively.

The number of generators located at every bus and their maximum power output are provided in Table I.

The auction dispatch clears the market at price U.S.$20.32/MWh. Auction results are provided in Tables III–V. Solving a power flow (with OPF voltage magnitude data), it can be seen that the auction dispatch creates congestion in line 14–16 which is relieved by solving problem (9)–(16).

TABLE I
LOCATIONS, NUMBERS, AND SIZES OF GENERATORS

Bus	Generator # (Capacity, MW)
1	1(20), 2(20), 3(76), 4(76)
2	5(20), 6(20), 7(76), 8(76)
7	9(100), 10(100), 11(100)
13	12(197), 13(197), 14(197)
15	15(12), 16(12), 17(12), 18(12), 19(12), 20(155)
16	21(155)
18	22(400)
21	23(400)
22	24(50), 25(50), 26(50), 27(50), 28(50), 29(50)
23	30(155), 31(155), 32(350)

TABLE II
REDISPATCHING PRICES OFFERED BY GENERATORS

Generator #	Capacity (MW)	r_j^{up} ($/MWh)	r_j^{down} ($/MWh)
15-19	12	∞	∞
1,2,5,6	20	∞	∞
24-29	50	∞	∞
3,4,7,8	76	15.97	15.97
9-11	100	22.72	22.72
20,21,30,31	155	11.26	11.26
12-14	197	22.13	22.13
32	350	13.00	13.00
22,23	400	5.66	5.66

TABLE III
RESULT FOR THE AUCTION DISPATCH: POWER OUTPUT

Power Output (MW) λ^A = $20.32/MWh, P_{loss} = 50.75 MW		
Generator	Auction	Auction + Congestion
1,2	0.00	0.00
3,4	76.00	76.00
5,6	0.00	0.00
7,8	76.00	76.00
9	50.00	75.30
10,11	50.00	50.00
12,13	108.67	108.67
14	108.67	108.67
15,16,17,18,19	0.00	0.00
20,21	155.00	155.00
22	400.00	374.70
23	400.00	400.00
24,25,26,27,28,29	50.00	50.00
30,31	155.00	155.00
32	350.00	350.00
Total	2850.01	2850.01

Redispatching prices (consistent with OPF cost data) offered by generators are provided in Table II. Note that up and down redispatching prices are considered to be equal, although they do not have to be so. Note that the generators that can increase their respective power output are 9, 10, 11 (capacity equal to 100 MW), and 12, 13, and 14 (capacity equal to 197 MW).

The losses are determined by solving a load flow with slack buses chosen in succession at bus 13 (generators 12, 13, and 14) and bus 18 (generator 22). The cost of the losses is then allocated *pro rata* among the generators and demands.

Table III provides generator output results of the auction procedure, Table IV provides generator revenue results while

TABLE IV
RESULT FOR THE AUCTION DISPATCH: GENERATORS

Generator	Generator Revenues		
	Auction ($/h)	Auction + Congestion ($/h)	Auction + Congestion + Losses ($/h)
1,2	0.00	0.00	0.00
3,4	1544.32	1544.32	1530.57
5,6	0.00	0.00	0.00
7,8	1544.32	1544.32	1530.57
9	1016.00	1590.82	1577.19
10,11	1016.00	1016.00	1006.96
12,13,14	2208.11	2208.11	2360.85
15,16,17,18,19	0.00	0.00	0.00
20,21	3149.60	3149.60	3121.56
22	8128.00	7984.80	8431.10
23	8128.00	8128.00	8055.63
24,25,26,27,28,29	1016.00	1016.00	1006.95
30,31	3149.60	3149.60	3121.56
32	7112.00	7112.00	7048.67
Total	57912.00	58343.62	58859.27

TABLE V
RESULT FOR THE AUCTION DISPATCH: DEMANDS

Load	Load Payments		
	Auction ($/h)	Auction + Congestion ($/h)	Auction + Congestion + Losses ($/h)
1	2194.56	2210.91	2230.46
2	1971.04	1985.73	2003.28
3	3657.60	3684.86	3717.43
4	1503.68	1514.89	1528.28
5	1442.72	1453.47	1466.32
6	2763.52	2784.12	2808.72
7	2540.00	2558.93	2581.55
8	3474.72	3500.62	3531.56
9	3556.00	3582.50	3614.17
10	3962.40	3991.93	4027.21
13	5384.80	5424.93	5472.88
14	3942.08	3971.46	4006.56
15	6441.44	6489.45	6546.80
16	2032.00	2047.14	2065.24
18	6766.56	6816.99	6877.24
19	3677.92	3705.33	3738.08
20	2600.96	2620.34	2643.50
Total	57912.00	58343.62	58859.27

TABLE VI
LOCATIONAL MARGINAL PRICES VERSUS MARKET-CLEARING PRICES

Bus	Locational marginal prices		
	Auction ($/MWh)	OPF ($/MWh)	Difference (%)
1	20.32	20.34	-0.08
2	20.32	20.42	-0.49
3	20.32	17.72	14.64
4	20.32	21.07	-3.69
5	20.32	21.16	-4.11
6	20.32	21.52	-5.88
7	20.32	21.67	-6.64
8	20.32	22.09	-8.71
9	20.32	20.79	-2.31
10	20.32	21.39	-5.28
11	20.32	23.48	-15.54
12	20.32	20.00	1.61
13	20.32	20.32	0.00
14	20.32	28.27	-39.11
15	20.32	11.75	72.92
16	20.32	11.26	80.46
17	20.32	11.24	80.83
18	20.32	11.26	80.54
19	20.32	13.31	52.62
20	20.32	14.90	36.39
21	20.32	11.29	79.95
22	20.32	10.96	85.40
23	20.32	15.68	29.57
24	20.32	14.12	43.93

TABLE VII
GENERATORS: COMPARISON BETWEEN AUCTION (PLUS CONGESTION MANAGEMENT AND LOSS ALLOCATION) AND OPF DISPATCH PROCEDURES

Generator	Generator Revenues		
	Auction ($/h)	OPF ($/h)	Difference (%)
1,2	0.00	0.00	0.00
3,4	1530.57	1545.69	-0.97
5,6	0.00	0.00	0.00
7,8	1530.57	1552.00	-1.38
9	1577.19	1217.85	29.50
10	1006.96	1083.50	-7.06
11	1006.96	1733.60	-41.91
12,13	2360.85	2401.82	-1.70
14	2360.85	2171.19	8.73
15,16,17,18,19	0.00	0.00	0.00
20	3121.56	1821.40	71.38
21	3121.56	1690.13	84.69
22	8431.10	4502.00	87.27
23	8055.63	4516.80	78.34
24,25,26,27,28,29	1006.95	548.00	83.75
30,31	3121.56	2430.71	28.42
32	7048.67	5488.70	28.42
Total	58859.27	43374.00	35.70

Table V shows the corresponding demand payments. Results are provided for the auction, the auction and congestion-management procedures, as well as for the combined auction, congestion, and cost-of-loss allocation. It should be noted that in the congestion-management procedure, the only on-line generators available to increase power are generators 9–11 and 12–14; among them, generator 9 is selected to increase power. To decrease power, the cheapest generator available is used, generator 22. Therefore, the revenues of generator 9 increase, and the revenues of generator 22 decrease. The allocation of the cost of transmission losses results in a reduction in the revenues for all of the on-line generators except for the slack Table IV. As expected, load payments increase after congestion management and after cost-of-loss allocation.

The OPF dispatch provides locational marginal prices for every bus, which are compared with the auction market-clearing price in Table VI. It should be noted that price differences are very significant due to congestion in line 14–16 (at 350 MW) that splits the system into two areas, one with excess of cheap generation and another one with expensive generation. This is illustrated in Fig. 1 that shows these two areas: the upper area with cheap generation and the lower one with expensive generation. Note that this case reproduces conditions in a predominantly radial system when a critical transmission line becomes congested, as would be observed in the California system [25]. Beyond the natural nodal price differences caused by congestion in a predominantly meshed system, as between the east/west halves of the New York system, for example [28], it can be observed that solutions obtained from approximate models exacerbate the economic inefficiencies and cross-subsidies.

TABLE VIII
LOADS: COMPARISON BETWEEN AUCTION AND OPF DISPATCH PROCEDURES

Bus	Load Payments		
	Auction ($/h)	OPF ($/h)	Difference (%)
1	2230.46	1890.99	17.95
2	2003.28	1706.44	17.39
3	3717.43	2681.31	38.64
4	1528.28	1349.85	13.21
5	1466.32	1301.30	12.68
6	2808.72	2541.32	10.52
7	2581.55	2355.15	9.61
8	3531.56	3293.66	7.22
9	3614.17	3143.38	14.97
10	4027.21	3620.01	11.24
13	5472.88	4635.16	18.07
14	4006.56	4935.39	-18.81
15	6546.80	2828.33	131.47
16	2065.24	843.12	144.95
18	6877.24	2805.92	145.09
19	3738.08	1897.82	96.96
20	2643.50	1544.85	71.11
Total	58859.27	43374.00	35.70

Table VII compares the generator revenue results of the auction and the OPF dispatch. With respect to demands, a similar comparison is provided in Table VIII.

Tables VII and VIII show that significant savings are obtained on the whole by the loads (except for load 14 located at the receiving end of the saturated line) with OPF dispatch when congestion occurs. If an auction dispatch is used, substantial cross-subsidies are apparent observing these tables, mostly benefiting the generators to the detriment of the demands.

VI. CONCLUSIONS

Auction-based dispatch methods have been adopted in several electricity markets because of their relative simplicity. They initially neglect transmission line capacities and network losses, which are accounted for by *ex post* procedures, but no effort is made to coordinate the three steps. Accepting an OPF as the yardstick, this paper quantifies the loss of economic efficiency and the cross-subsidies of an auction-based dispatch with *ex post* modifications. In a sample network, demands pay 35% more under auction than under OPF schedules. For some loads, this difference can be as high as 145%. Using other test data, an auction dispatch might have favored the demands, but irrespective of who wins the discrepancy between the results of the two procedures is disturbing. Our comparison indicates that the auction-based procedure does not determine the true value of electricity in a competitive electricity market. Regulators and authorities in charge of setting electricity market policy and rules should be aware of the social cost and loss of competitiveness of using a seemingly transparent, but overly simple procedure such as an auction (including congestion-management and transmission-loss cost allocation) versus using an integrated but more involved one such as an OPF.

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