Optimal Price and Quantity Determination for Retail Electric Power Contracts

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***Abstract*—Considering the viewpoint of a retailer, this paper analyzes the problem of setting up contracts on both the supplier and end-user sides to maximize profits while maintaining an acceptable level of settlement risk. The proposed stochastic optimization model can assist retailers with these efforts and guide them in their contractual arrangements. A realistic example illustrates the capabilities of the methodology proposed.**

***Index Terms*—Contract design, electricity market, mixed-integer nonlinear programming, retailer viewpoint, stochastic optimization.**

I. Introduction

The U.S. electric power market has lately undergone significant changes due to deregulation and restructuring of the industry. One result of this new marketplace is the emergence of third-party entities known as retailers or marketers. These entities purchase power from suppliers and resell to end-user customers. These retailers typically set up bilateral contracts with suppliers to procure a substantial portion of the power required to meet the demands of their customers. The remaining portion of the load is bought in the balancing energy market and settled at the marginal clearing price or the spot price.

Retailers enter into supply contracts with suppliers. These contracts range from a simple fixed-price, fixed-quantity contract for a fixed time period—where the retailer takes all the risk-or to a more collaborative arrangement where the supplier and the retailer work on a profit and risk-sharing arrangement. Retailers are also turning to other market instruments to manage their portfolio and their risks. They do this by purchasing and selling options, hedging portions of their needs especially during the volatile months, and using cross-commodity hedges (natural gas as a hedge for electricity). One could also consider sophisticated retailers purchasing weather derivatives to protect against extreme weather. However, not all retailers use such hedging options, sometimes leaving this to their suppliers to handle. In this paper, only the spot market-clearing price is considered so that the problem statement is kept simple. One could also consider many other forms of risk, such as credit risk between counterparties, market behavior risk regarding customer response and customer acquisition costs, operational risks due to human or system operational errors, and regulatory risk, to name just a few [1]. These risk factors are also not included in this paper.

In a retailer's contract with end-users, there can be a "bandwidth" constraint. Bandwidth here refers to the allowed deviation around a certain level of end-user load (i.e., the "benchmark"). If the end-user consumes a substantially different amount of electricity than this given benchmark, an appropriate charge or credit is given to these end-users.

In addition to the risk associated with uncertain loads and the obligation to provide forward load estimates to the supplier, the retailer must also consider uncertain market-clearing prices. Indeed, given the recent volatility in various regional hourly market prices, just a few hours with exceptionally high marginal clearing prices can make a difference and greatly reduce the retailer's profit. For example, in February 2004, the marginal clearing prices in ERCOT reached the maximum allowable of $999 per megawatthour, causing one retailer—Texas Commercial Energy (TCE)—to lose substantial amounts of money and declare bankruptcy. While the case is very complex, one of the reasons for the unfortunate occurrence is that TCE was purchasing the majority of their power requirements from the balancing energy or spot market and were adversely impacted when the price of balancing energy reached the cap.

Consequently, the typical retailer faces the problem of setting up contracts on both the supplier and end-user sides in such a way as to protect themselves from settlement risk. In this paper, a stochastic optimization model is proposed that can assist retailers with these efforts and guide them in their contractual arrangements. Besides considering the forward load estimates as variables, optimal end-user contract prices are also considered as part of this analysis. [1]

The retailer operates in a fine zone between profit and loss. If the price to the end-user is too high, then no customers sign on. If the price from the supplier is higher than prices in contracts with end-users, then the retailer will lose money and be forced out of business. With properly priced bandwidths, retailers may be able to customize products to end-users by offering the counterparties a choice of risk versus savings/profit that fits their appetite. Therefore, it is essential that the retailer have the right

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[1]Supplier prices could also be considered as variables but for computational reasons are treated as fixed parameters in this paper.

strategy and processes to price products to end-users while entering into favorable contracts with suppliers.

A key point regarding the uncertain loads and spot market prices is that they are temporally linked (e.g., the spot market price in hour h is related to the price in hour $h-1$) but that spot market prices and end-users loads are not necessarily strongly correlated, as demonstrated in [2]–[4]. This is in contrast to the system-wide load and market price that are generally strongly correlated. The difference is that the load from small retailers may not be large enough to sway market prices. If one wanted to consider an extension to the model, we propose (where prices and retail load are assumed to be uncorrelated based on actual observations) that this could be accomplished by changing the appropriate probability distributions for load and market price to make them joint distributions. These joint distributions would then be used in the resulting stochastic optimization model presented below.

While many recent optimization models in electricity have concentrated on the wholesale side, there have been relatively fewer models devoted to retail electric power operations. Previous efforts that also consider stochasticity in some of the elements in an optimization problem include [5]–[9], to name just a few. See [10] for a more comprehensive list of stochastic programming studies in energy as well as [11] for an overview of the various market participants. Additional information on electricity markets can be found in [12]–[14].

Three recent studies also consider stochasticity with or without optimization but directed to retail electric power markets. In [2], a Monte Carlo simulation for 8760 hours was used to analyze a set of heuristic strategies for balancing settlement risks and profits for retailers. In this paper, based on data for the PJM market, intentionally overestimating the load to the supplier as much as possible stochastically dominated all of the other strategies. The analysis for the PJM dataset was extended in [3] in which a stochastic program with recourse [15] was used to optimally determine retailer load estimates. In [4], a stochastic dynamic programming [16] extension of [3] was used and included double bandwidth constraints (one on the supplier side, one on the end-user side) and applied to the ERCOT market in Texas.

The present paper is an extension of [2]–[4] in that besides bandwidth contracts and a stochastic optimization format, optimal contract design aspects are considered as part of the proposed model. Indeed, such a model will allow retailers to optimally estimate their forward loads to suppliers and balance them with settlement on the end-user side relative to end-user forecasts of their own load (e.g., "benchmark" values). The model presented in the current paper also considers optimal end-user prices that the retailer should negotiate for as part of the contractual process; this distinguishes it from [2]–[4] and other works. Moreover, the current paper, unlike these three previous ones, has a sophisticated relationship between contract prices and eventual acceptance of contracts, price-dependent bandwidth functions, as well as a risk term that considers both peak and off-peak decisions—this is significantly different from these other works. As such, this paper is novel in that it combines stochastic optimization of retail electric power operations with contractual guidance so as to provide the most benefit to the retailer.

It is important to note that we have identified a key subset of the contractual variables available to retailers, with the bandwidths and contractual prices being the main ones. These aspects of retailer contracts were chosen since they are relatively easy to understand (hence more likely to be used by retailers) yet provide an important influence on retail net profits.

While it is assumed that one end-user and one supplier are associated with the retailer, it is not difficult to extend this analysis to multiple suppliers and end-users, as was done in [2] and [3]. Additionally, it is assumed that the retailer has already identified the supplier and end-user to contract with and does not consider a set of each of these as part of the process.

The rest of this paper is organized as follows. In Section II, notation and definitions to be used in the model are presented. Section III presents the optimization model faced by the retailer. Section IV describes numerical results. Section V presents conclusions.

II. Notation and Definitions

The following notation is used in the mathematical model presented in the next section.

A. Notation

1) Indices:

t	time period (e.g., hour, day);
T	set of time periods under consideration;
b	blocks of time periods;
$B(b)$	set of time periods in block b;
δ_{tb}	indicator function that equals 1 if $t \in B(b)$, 0 otherwise;
Ω	set of possible realizations for the random variables;
ω	realization of Ω.

Note that the set of time periods is arbitrary and can include individual hours or groups of hours (e.g., weeks, days), depending on the particular application. In addition to these time periods, there are "blocks" of time periods for which the end-user and supplier prices apply. For example, if the individual time periods are hours, then the blocks may refer to peak or off-peak hours used to distinguish different pricing. Thus, for each block, there is a mapping of which individual time periods pertain to that block.

As an example, consider a time horizon of six hours, i.e., $T = \{1,2,3,4,5,6\}$ with two blocks. Suppose that the first block of hours is $B(1) = \{1,2,6\}$ and the second block is $B(2) = \{3,4,5\}$. The indicator function can be written out as a 6×2 matrix whose transpose is

$$\left(\begin{array}{c|cccccc} t= & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline b=1 & 1 & 1 & 0 & 0 & 0 & 1 \\ b=2 & 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right).$$

2) Variables:

L_b^f	estimate provided by retailer to supplier of forward load (in megawatthours);
p_b^{eu}	contract price between retailer and end-user (in dollars per megawatthour)

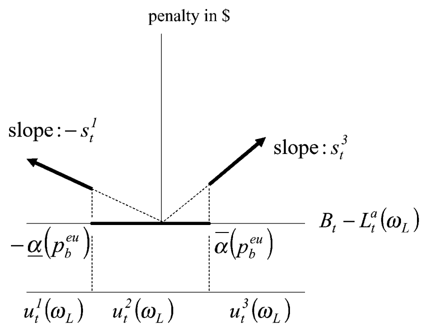

Fig. 1. End-user bandwidth functions.

$u_t^1(\omega), u_t^2(\omega), u_t^3(\omega)$ end-user segment variables whose sum is the difference between the benchmark load (made by the end-user) and actual load, i.e., the amount to be potentially penalized, in time period t. Segments 1 and 3 are penalized, and segment 2 receives no penalty (see Fig. 1). The function is discontinuous and represents a jump in the penalty from zero (within the bandwidth) to a positive amount as an inducement to the end-user to supply reasonable benchmark values. Note that both continuous as well as discontinuous penalty functions are possible given the wide variety of contracts available;

$v_t^1(\omega), v_t^2(\omega), v_t^3(\omega)$ binary variables corresponding, respectively, to the end-user segment variables.

3) Data:

$L_t^a(\omega)$ end-user actual load (in megawatthours);

$\pi(\omega)$ marginal probability mass function (pmf) for random events;

$p_b^{\rm su}$ contract price between retailer and supplier (in dollars per megawatthour);

$p_t^{\rm mcp}(\omega)$ market-clearing price (in dollars per megawatthour);

B_t end-user benchmark load (in megawatthours);

$\sigma_{t\tilde{t}}^2$ covariance between random variables $p_t^{\rm mcp}(\omega)$ and $p_{\tilde{t}}^{\rm mcp}(\omega)$. If $t = \tilde{t}$, then σ_{tt}^2 is the variance for random variable $p_t^{\rm mcp}(\omega)$;

s_t^1, s_t^3 positive penalty slopes for segments 1 and 3, respectively, for the end-user bandwidth;

$\underline{p}_b^{\rm eu}, \bar{p}_b^{\rm eu}$ positive, lower, and upper bounds for the contract end-user price, respectively;

M large positive constant;

N positive constant.

4) Functions:

a) End-User Bandwidth Functions: All things being equal, if the retailer receives a higher end-user price, the contractual conditions should be more favorable to the end-users. To this end, two functions $\underline{\alpha}(p_b^{\rm eu}), \bar{\alpha}(p_b^{\rm eu})$, expressed in megawatthours, are proposed. These functions are, respectively, the positive, lower, and upper bounds that the retailer will stipulate as a function of the end-user price. The higher the end-user price, the more spread out these two values will be so as to decrease the chance that the end-user is penalized (see Fig. 1). It is important to note that the x axis is the top line (coinciding with $B_t - L_t^a(\omega_L)$) with the second line below this indicating the corresponding u variables.

b) End-User and Supplier Acceptance Functions: From the retailer's point of view, as they propose larger end-user prices to the consumer, there is a smaller chance that the consumer will agree to the contract. This is because, all things being equal, the consumer will choose to do business with other retailers with more favorable prices. To this end, an "acceptance function" $A_b^{\rm eu} : R_+ \rightarrow [0,1]$ is proposed. This function can be construed as a market share in some sense. This function specifies the probability that the end-user will accept the contract given the retailer's suggested price of $p_b^{\rm eu}$. Note that $A_b^{\rm eu}(p_b^{\rm eu})$ should be decreasing. Also, $A_b^{\rm su} : R_+ \rightarrow [0,1]$ is the supplier acceptance function, where $A_b^{\rm su}(p_b^{\rm su})$ is the chance of the supplier accepting the contract given that the supplier price is $p_b^{\rm su}$ with $A_b^{\rm su}(p_b^{\rm su})$ increasing. For this paper, this price has been taken to be a constant to simplify the computational aspects. However, it is reasonable to make such a price a variable, and so we have maintained this acceptance likelihood to indicate that the results would still be applicable in this more general context. Also, note that without much change in the formulation, one could make the acceptance probability for all blocks b as a function of both the peak and off-peak prices. The complication would arise in adding complexity to the already nonconvex terms. Thus, for computational reasons, we have adopted this simpler yet somewhat realistic approach for acceptance functions. Note that since the penalty functions $\underline{\alpha}(p_b^{\rm eu}), \bar{\alpha}(p_b^{\rm eu})$ are dependent on the end-user prices, we see that as these prices change, they affect both the penalties as well as the acceptance values derived from $A_b^{\rm eu}(p_b^{\rm eu})$.

Only the case when there is both a supply and an end-user contract accepted is of interest for modeling purposes. Then, the end-user and supplier accept their contracts, and the end-user has an actual load of $L_t^a(\omega)A_b^{\rm eu}(p_b^{\rm eu})A_b^{\rm su}(p_b^{\rm su})\pi(\omega)$, where $t \in B(b)$. This computation makes the reasonable assumption that the end-user accepting the end-user price and having the specific load realized are independent events. In addition, the end-user and supplier's acceptance of the contract are reasonably taken to be independent events.

Consequently, the expected revenue to the retailer in time period t and for realization ω is given as

$$p_b^{\rm eu} L_t^a(\omega) A_b^{\rm eu}\left(p_b^{\rm eu}\right) A_b^{\rm su}\left(p_b^{\rm su}\right) \pi(\omega). \tag{1}$$

Similarly

$$p_b^{\rm su} L_b^f A_b^{\rm eu}\left(p_b^{\rm eu}\right) A_b^{\rm su}\left(p_b^{\rm su}\right) \tag{2}$$

represents the expected supply costs to the retailer. Note that one could amend the model to allow for only suppliers accepting a contract, only end users accepting a contract, or neither. In the first two cases, the spot market would be used to buy from/sell to in the case of a deficit/surplus of power. In the third case, no contracts are held on either side, and so the point would be moot. We are thus implicitly making the assumption that if only one side of the contract accepts (i.e., only suppliers or only end-users) that the penalty costs are zero.

III. Mathematical Model

The complete mathematical model that the retailer faces, including the objective function and constraints, is described in this section.

A. Objective Function

The objective function represents expected net profit that is to be maximized. This objective function is broken up into the following pieces:

1) expected end-user revenues;
2) expected supply costs;
3) expected end-user penalty charges/credits;
4) expected spot market settlement revenues/costs;
5) risk penalty.

Summing over all realizations of Ω, using (1), the expected end-user revenues for the time horizon are given as

$$\sum_t \sum_b \sum_\omega \delta_{\mathrm{tb}} p_b^{\mathrm{eu}} L_t^a(\omega) A_b^{\mathrm{eu}}\left(p_b^{\mathrm{eu}}\right) A_b^{\mathrm{su}}\left(p_b^{\mathrm{su}}\right) \pi(\omega).$$

Analogously using (2), the expected supply costs for the time horizon are given as $\sum_t \sum_b \delta_{\mathrm{tb}} p_b^{\mathrm{su}} L_b^f A_b^{\mathrm{eu}}(p_b^{\mathrm{eu}}) A_b^{\mathrm{su}}(p_b^{\mathrm{su}})$.

For each time period, there are end-user bandwidth constraints. Given the benchmark value for projected end-user load B_t, provided by the end-users, the total deviation from this amount is given by $B_t - L_t^a(\omega)$, which is partitioned into the sum $u_t^1(\omega) + u_t^2(\omega) + u_t^3(\omega)$. For each realization ω of $\Omega, (-s_t^1 u_t^1(\omega) + s_t^3 u_t^3(\omega))$ represents the end-user penalty charges/credits, as depicted in Fig. 1. Note that the coefficients s_t^1, s_t^3 should be carefully calibrated to produce answers that are deemed reasonable for each application. Also, it is important to see that this penalty curve is a nonconvex function of the variables and as such, when used in a maximization problem (shown below), needs integer variables to be realized correctly. Consequently, the expected end-user penalty value summed over the time horizon is given by

$$\sum_t \sum_b \sum_\omega \delta_{tb}\left(-s_t^1 u_t^1(\omega) + s_t^3 u_t^3(\omega)\right) A_b^{\mathrm{eu}}\left(p_b^{\mathrm{eu}}\right) A_b^{\mathrm{su}}\left(p_b^{\mathrm{su}}\right) \pi(\omega).$$

It is important to note that the penalty terms are discontinuous as a function of $B_t - L_t^a(\omega)$, as shown in Fig. 1. Both discontinuous and continuous penalty terms are possible in contracts. The next term in the objective function involves the expected settlement revenues relative to selling to or buying from the spot market. In particular

$$p_t^{\mathrm{mcp}}(\omega)\left[L_b^f - L_t^a(\omega)\right] A_b^{\mathrm{eu}}\left(p_b^{\mathrm{eu}}\right) A_b^{\mathrm{su}}\left(p_b^{\mathrm{su}}\right) \pi(\omega)$$

represents for time period t how much money is made or lost by the retailer as part of the settlement of end-user load. This amount can be either a revenue or a cost, depending on whether the forward load estimate was either above or below the actual load. For example, if the market clearing price is $30/MWh, the forward load estimate is 100 MWh, and the actual load for a particular hour is 80 MWh, then the retailer would sell the extra 20 MWh to the spot market at 30 each realizing a revenue of $600. If, instead, the actual load were 120 MWh, the retailer would need to pay $600 to purchase the residual electricity and would thus incur a cost of $600.

The expected settlement amount over the time horizon is thus given as

$$\sum_t \sum_b \sum_\omega \delta_{tb} p_t^{\mathrm{mcp}}(\omega)\left[L_b^f - L_t^a(\omega)\right] A_b^{\mathrm{eu}}\left(p_b^{\mathrm{eu}}\right) A_b^{\mathrm{su}}\left(p_b^{\mathrm{su}}\right) \pi(\omega)$$

The sum of these four terms constitutes the part of the objective function that the retailer is trying to maximize by choosing values for the decision variables: p_b^{eu}, L_b^f.

Additionally, the last term below limits the risk of participating in the spot market

$$\begin{aligned} -\sum_t \sum_b \sum_{\tilde{t}} \sum_{\tilde{b}} \sum_\omega \Big\{ & \Big[\delta_{\mathrm{tb}}\left[L_b^f - L_t^a(\omega)\right] \\ & \times A_b^{\mathrm{eu}}\left(p_b^{\mathrm{eu}}\right) A_b^{\mathrm{su}}\left(p_b^{\mathrm{su}}\right)\Big] \sigma_{t\tilde{t}}^2 \left[\delta_{\tilde{t}\tilde{b}}\left[L_{\tilde{b}}^f - L_{\tilde{t}}^a(\omega)\right] \right. \\ & \left. \times A_{\tilde{b}}^{\mathrm{eu}}\left(p_{\tilde{b}}^{\mathrm{eu}}\right) A_{\tilde{b}}^{\mathrm{su}}\left(p_{\tilde{b}}^{\mathrm{su}}\right)\right] \pi(\omega) \Big\}. \end{aligned}$$

It is added to the objective function using an appropriate constant N. This risk aversion term prevents the retailer from selling/buying large quantities to/from the (potentially) highly volatile spot market, thereby reducing its risk exposure. Its consideration might decrease the retailer's profit but also its risk exposure from volatile prices in the spot market. The constant N is chosen to attain the desired level of risk exposure by the retailer.

It is important to note that cost and risk are conflicting objectives. To properly combine them reflecting the tradeoff involved, the technique presented in [17] for portfolio selection is used. This technique uses a single objective function with the help of a risk tolerance parameter (or weighting factor) N.

The solution of the above optimization problem for different values of the weighting factor provides the efficient frontier [17]–[22], that is, the set of solutions for which expected profit cannot be increased without increasing profit variance, i.e., risk.

As stated in [23], the selection of the weighting factor depends on the financial situation of the company, its willingness to take risks, and the characteristics of the particular market, to name just a few items. A detailed discussion on how to obtain appropriate values for the weighting factor is outside the scope of this paper.

B. Constraints

There are three sets of constraints that the retailer faces, which are described next.

1) End-User Bandwidth Constraints: As noted above, the total deviation between the end-user supplied benchmark and the actual load is broken up for each realization ω of Ω into three variables as

$$B_t - L_t^a(\omega) = u_t^1(\omega) + u_t^2(\omega) + u_t^3(\omega) \tag{3a}$$

where $u_t^1(\omega) \leq 0, u_t^3(\omega) \geq 0$, and $u_t^2(\omega)$ can be either positive, negative, or zero. At most, one of these u variables is nonzero, referring to where on the horizontal axis of Fig. 1 the deviation in the load occurs. To that end, the associated binary variables $v_t^1(\omega), v_t^2(\omega), v_t^3(\omega)$ are used. There are three mutually exclusive and collectively exhaustive cases to consider (see the Appendix for details).

Case 1) $B_t - L_t^a(\omega) < -\underline{\alpha}(p_b^{\mathrm{eu}})$, outside the bandwidth and the benchmark is below the actual load estimate.

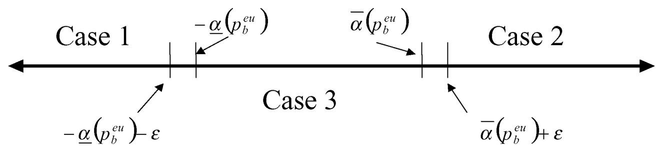

Fig. 2. Depiction of cases.

Case 2) $B_t - L_t^a(\omega) > \bar{\alpha}(p_b^{\mathrm{eu}})$, outside the bandwidth and the benchmark is above the actual load estimate.

Case 3) $\underline{\alpha}(p_b^{\mathrm{eu}}) \le B_t - L_t^a(\omega) \le \bar{\alpha}(p_b^{\mathrm{eu}})$, the benchmark is within the bandwidth of the actual load.

With $\varepsilon > 0$ chosen to be suitably small, the ranges $[-\underline{\alpha}(p_b^{\mathrm{eu}})-\varepsilon, -\underline{\alpha}(p_b^{\mathrm{eu}})]$ and $[\bar{\alpha}(p_b^{\mathrm{eu}}), \bar{\alpha}(p_b^{\mathrm{eu}})+\varepsilon]$ are arbitrarily small, and thus for practical reasons, Cases 1 and 3 can be represented, respectively, as $B_t - L_t^a(\omega_L) \le \underline{\alpha}(p_b^{\mathrm{eu}}) - \varepsilon$ and $B_t - L_t^a(\omega_L) \ge \bar{\alpha}(p_b^{\mathrm{eu}}) + \varepsilon$, as indicated in Fig. 2.

The remaining end-user bandwidth constraints are as follows:

$$-Mv_t^1(\omega) \le u_t^1(\omega) \le 0, \quad \forall t, \omega \tag{3b}$$

$$u_t^1(\omega) + \underline{\alpha}\left(p_b^{\mathrm{eu}}\right) \le M\left(1 - v_t^1(\omega)\right) - \varepsilon, \quad \forall t, \omega \tag{3c}$$

$$-\underline{\alpha}\left(p_b^{\mathrm{eu}}\right) \le u_t^2(\omega) \le \bar{\alpha}\left(p_b^{\mathrm{eu}}\right), \quad \forall t, \omega \tag{3d}$$

$$-Mv_t^2(\omega) \le u_t^2(\omega) \le Mv_t^2(\omega), \quad \forall t, \omega \tag{3e}$$

$$0 \le u_t^3(\omega) \le Mv_t^3(\omega), \quad \forall t, \omega \tag{3f}$$

$$-u_t^3(\omega) + \bar{\alpha}\left(p_b^{\mathrm{eu}}\right) \le M\left(1 - v_t^3(\omega)\right) - \varepsilon, \quad \forall t, \omega \tag{3g}$$

$$v_t^1(\omega) + v_t^2(\omega) + v_t^3(\omega) = 1, \quad \forall t, \omega \tag{3h}$$

$$v_t^1(\omega), v_t^2(\omega), v_t^3(\omega) \in \{0, 1\}, \quad \forall t, \omega. \tag{3i}$$

2) Bounds on Prices: To make the model realistic, positive lower and upper bounds on the end-user price variables and loads are also considered

$$\underline{p}_b^{\mathrm{eu}} \le p_b^{\mathrm{eu}} \le \bar{p}_b^{\mathrm{eu}}, \quad \forall b \tag{4a}$$

$$0 \le L_b^f \le \bar{L}_b^f, \quad \forall b. \tag{4b}$$

3) Complete Problem: Consequently, we see that the complete optimization model that the retailer faces is described as follows:

$$\begin{aligned}
&\mathrm{Max}_{p_b^{\mathrm{eu}}, L_b^f,\ \forall b \in B, t \in T} \\
&\sum_t \sum_b \sum_\omega \delta_{\mathrm{tb}} p_b^{\mathrm{eu}} L_t^a(\omega) A_b^{\mathrm{eu}}\left(p_b^{\mathrm{eu}}\right) A_b^{\mathrm{su}}\left(p_b^{\mathrm{su}}\right) \pi(\omega) \\
&\quad - \sum_t \sum_b \delta_{\mathrm{tb}} p_b^{\mathrm{su}} L_b^f A_b^{\mathrm{eu}}\left(p_b^{\mathrm{eu}}\right) A_b^{\mathrm{su}}\left(p_b^{\mathrm{su}}\right) \\
&\quad + \sum_t \sum_b \sum_\omega \delta_{\mathrm{tb}}\left(-s_t^1 u_t^1(\omega) + s_t^3 u_t^3(\omega)\right) \\
&\quad \times A_b^{\mathrm{eu}}\left(p_b^{\mathrm{eu}}\right) A_b^{\mathrm{su}}\left(p_b^{\mathrm{su}}\right) \pi(\omega) \\
&\quad + \sum_t \sum_b \sum_\omega \delta_{\mathrm{tb}} p_t^{\mathrm{mcp}}(\omega)\left[L_b^f - L_t^a(\omega)\right] \\
&\quad \times A_b^{\mathrm{eu}}\left(p_b^{\mathrm{eu}}\right) A_b^{\mathrm{su}}\left(p_b^{\mathrm{su}}\right) \pi(\omega) \\
&\quad - N \sum_t \sum_b \sum_{\tilde{t}} \sum_{\tilde{b}} \sum_\omega \left\{\left[\delta_{\mathrm{tb}}\left[L_b^f - L_t^a(\omega)\right]\right.\right. \\
&\quad \left.\times A_b^{\mathrm{eu}}\left(p_b^{\mathrm{eu}}\right) A_b^{\mathrm{su}}\left(p_b^{\mathrm{su}}\right)\right] \\
&\quad \times \sigma_{t\tilde{t}}^2\left[\delta_{\tilde{t}\tilde{b}}\left[L_{\tilde{b}}^f - L_{\tilde{t}}^a(\omega)\right]\right. \\
&\quad \left.\left.\times A_{\tilde{b}}^{\mathrm{eu}}\left(p_{\tilde{b}}^{\mathrm{eu}}\right) A_{\tilde{b}}^{\mathrm{su}}\left(p_{\tilde{b}}^{\mathrm{su}}\right)\right] \pi(\omega)\right\}
\end{aligned} \tag{5}$$

subject to (3) and (4).

TABLE I
TRIANGULAR DISTRIBUTION FOR LOAD

	a	m	b
S,P,H	6425.6	7770	9273
S,P,M	4913.5	6634	8053.9
S,P,L	4304.5	5414	6484.1
S,OP,H	4849.6	6164.6	6954.1
S,OP,M	3818	4344	6842
S,OP,L	2952.1	3843	5001.3

S: summer; P: peak; OP: off-peak; H: high; M: medium; L: low
a: min; m: mode; b: max in MWh.

TABLE II
LOGNORMAL DISTRIBUTION OF PRICE

	μ	σ
S,P,H	97.33	141.74
S,P,M	30.48	11.39
S,P,L	11.49	5.48
S,OP,H	32.18	17.69
S,OP,M	16.26	3.97
S,OP,L	14.52	4.79

S: summer; P: peak; OP: off-peak; H: high; M: medium; L: low
Prices in $/MWh

IV. NUMERICAL RESULTS

A. Data and Parameters

For the numerical study, a contractual period comprising one week divided into two time blocks corresponding to peak and off-peak hours is considered. Peak hours include weekdays from 7 A.M. to 11 P.M. ($16 \times 5 = 80$ hours). Off-peak hours include weekdays from 11 P.M. to 7 A.M. and weekends ($8 \times 5 + 48 = 88$ hours). Twenty price/demand scenarios are considered each with an identical probability $(1/20)$ of occurring.

The stochastic model to generate demand scenarios is characterized in Table I and is described in [2].

The stochastic model to generate spot price scenarios is characterized in Table II and based on [2].

The estimated covariance matrix for the spot price has been derived from generated spot-price scenarios. It is diagonally dominant, so only diagonal elements and elements immediately below and above these diagonal elements were considered in the covariance matrix. Due to the fact that the estimated covariance matrix is diagonally dominant, no strong coupling exists between peak and off-peak hours.

The end-user bandwidth functions used are given as $\underline{\alpha}(p_b^{\mathrm{eu}}) = 10p_b^{\mathrm{eu}}$ and $\bar{\alpha}(p_b^{\mathrm{eu}}) = 10p_b^{\mathrm{eu}}$ with slopes s_t^1, s_t^3 equal to five for all cases.

The end-user benchmark loads are set to 4344 MWh for off-peak hour and to 6634 MWh for peak hours.

The acceptance functions considered are

$$A_b^{\mathrm{eu}}\left(p_b^{\mathrm{eu}}\right) = 3 - 0.10 \times p_b^{\mathrm{eu}}, \quad \text{if off-peak}$$

$$A_b^{\mathrm{eu}}\left(p_b^{\mathrm{eu}}\right) = 1.667 - 0.033 \times p_b^{\mathrm{eu}}, \quad \text{if peak}$$

$$A_b^{\mathrm{su}}\left(p_b^{\mathrm{su}}\right) = 0.1 + 0.03 \times p_b^{\mathrm{su}}, \quad \text{if off-peak}$$

$$A_b^{\mathrm{su}}\left(p_b^{\mathrm{su}}\right) = 0.2 + 0.02 \times p_b^{\mathrm{su}}, \quad \text{if peak.}$$

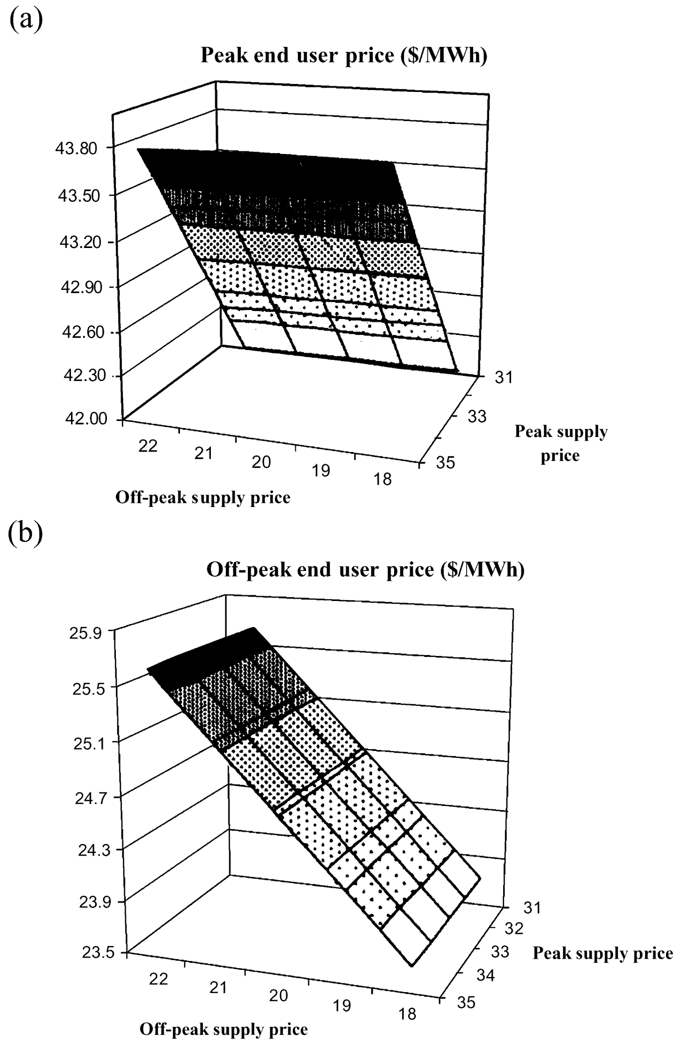

Fig. 3. Optimal end-user (a) peak and (b) off-peak price as a function of peak and off-peak supply prices.

End-user price limits are as follows:

$$\underline{p}^{\mathrm{eu}}_{\mathrm{peak}} = 20, \quad \bar{p}^{\mathrm{eu}}_{\mathrm{peak}} = 50$$
$$\underline{p}^{\mathrm{eu}}_{\mathrm{off\text{-}peak}} = 20, \quad \bar{p}^{\mathrm{eu}}_{\mathrm{off\text{-}peak}} = 30.$$

Constants M and N are set to 10^{-4}, and 10^{-5}, respectively.[2] Results are obtained for different values of supply prices that are treated as a fixed parameter. The mixed-integer nonlinear problem formulated earlier is solved using SBB/CONOPT under GAMS (http://www.gams.com) on a Dell PowerEdge 6600 server with four processors at 1.60 GHz and 2 Gb of RAM memory.

B. Results

The two plots shown in Fig. 3 provide end-user (a) peak and (b) off-peak optimal prices as a function of peak and off-peak supply prices. Note that off-peak supply prices influence mostly off-peak end-user prices, while peak supply prices affect mostly peak end-user prices.

[2]The value for M was tuned up for (3b), (3c), (3e)–(3g) to maximize solver efficiency. The value for N was determined numerically so that the penalty was reasonable large as compared to the expected profit. Retailers might want to calibrate N to accommodate their specific risk-aversion behavior.

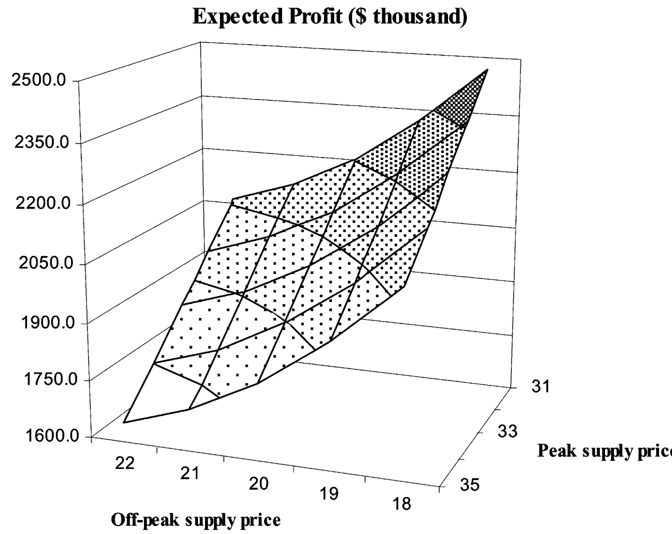

Fig. 4. Expected profit for the retailer as a function of peak and off-peak supply price.

Fig. 4 represents the expected retailer profit as a function of peak and off-peak supply prices. As anticipated, expected profits for the retailer increase as peak and off-peak supply prices (both) decrease. The increment is higher as the peak supply price decreases.

The two plots in Fig. 5 provide (a) peak and (b) off-peak forward load estimates, respectively, as a function of peak and off-peak supply prices. Note in Fig. 5(a) that peak forward load estimates depend not only on peak supply prices but also on off-peak supply price. When the off-peak supply price decreases, the off-peak forward load estimates increases, and in order to control the risk penalty term, peak forward load estimates have to decrease. The same event happens to off-peak forward load estimates, although it is less evident in Fig. 5(b).

Fig. 6 shows the expected retailer profits as a function of the number of scenarios considered in the stochastic programming framework for specified peak and off-peak supply prices. Observe that the expected profit value stabilizes with eight or more scenarios. It should be noted that a critical issue when using a stochastic programming approach based on scenarios is the actual number of scenarios to consider. Too few scenarios might result in inaccuracies, while too many could be computationally burdensome. One way to resolve this tradeoff is to increase the numbers of scenarios until the objective function value stabilizes. This is the criterion used in this paper.

C. Discussion of Value of Stochastic Solution and Tradeoff Between Profit and Risk

In Fig. 7, we show the value of the stochastic solution (VSS) defined as in [15] but adjusted for a maximization problem. VSS is defined as VSS = RP−EEV, where RP is the optimal solution of the stochastic model, and EEV is computed as the expectation (over all scenarios) of the objective function value of the solution chosen based on the decision for L_b^f and p_b^{eu} provided by a deterministic model that considers a single scenario equaling the expected value of the scenario variables. Note that VSS is a

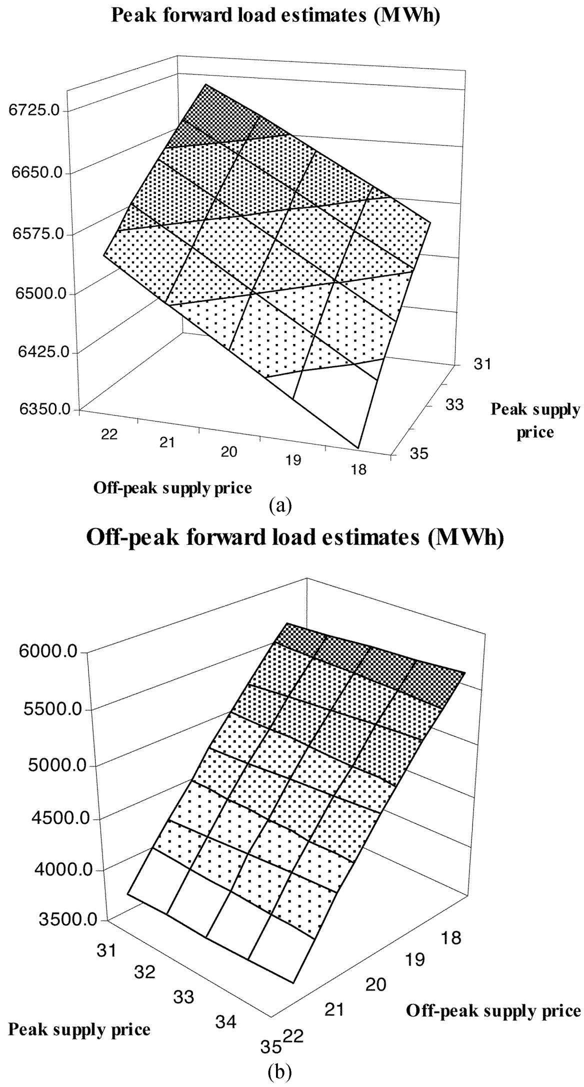

Fig. 5. Peak and off-peak forward load estimates, respectively, as a function of peak and off-peak supply prices.

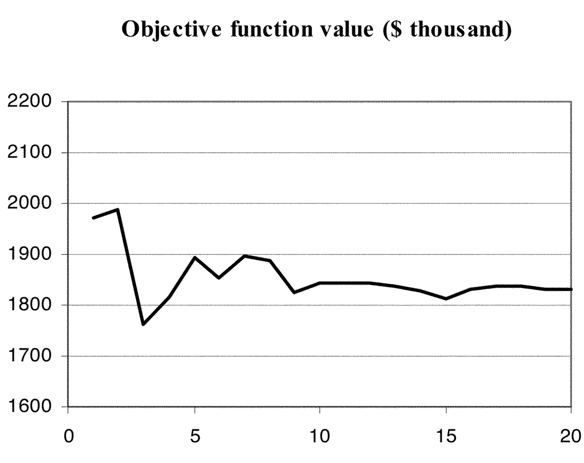

Fig. 6. Objective function value as a function of the number of scenarios considered.

measure of the advantage obtained if a stochastic model is used rather than a naïve deterministic one. Fig. 7 shows that VSS

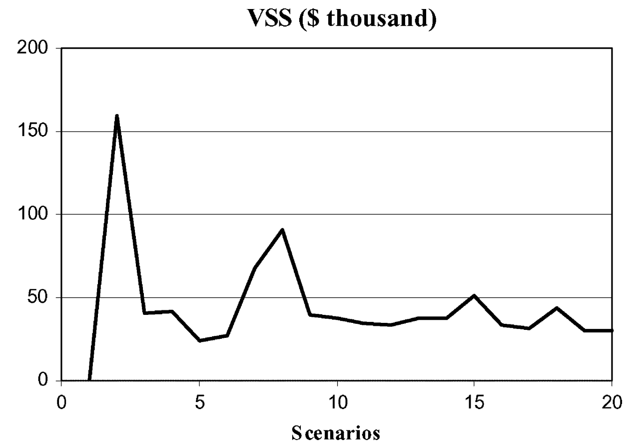

Fig. 7. VSS computed from one scenario to 20 scenarios.

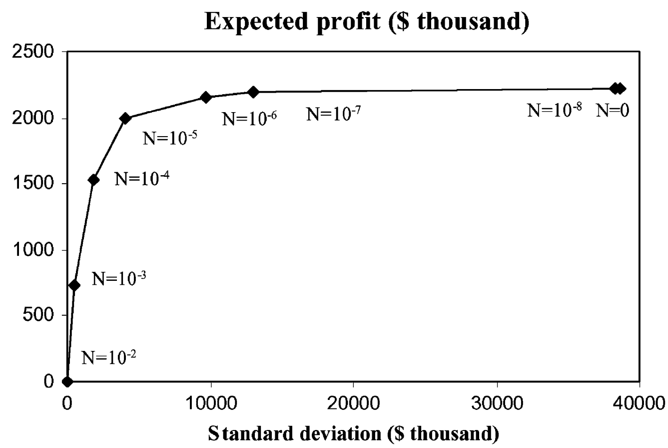

Fig. 8. Expected profit versus risk for different values of the parameter N.

also stabilizes for more than eight scenarios. The achieved result means that it would be possible to attain additional savings of \$30 000 by taking into account a stochastic solution to the problem, which justifies the additional required work to solve this type of model.

Finally, Fig. 8 provides the efficient frontier, i.e., it shows the expected profit as a function of the standard deviation of the profit (a risk measure) for various values of the parameter N.

Values in Fig. 8 have been computed for off-peak supply price $=$ 20 and peak supply price $=$ 33. Note that for $N = 1 \cdot \mathrm{e}{-02}$, the expected profit is 0, which means that when the retailer decides to minimize risk down to this level participation in retail activities is not recommended (the values of the acceptance functions in the solution are 0). As the retailer is willing to assume an acceptable level of risk (up to $N = 1 \cdot \mathrm{e}{-05}$), the end-user contract price decreases and the value of the acceptance function increases, so the expected profit grows rapidly. For a high level of risk $(N < 1 \cdot \mathrm{e}{-05})$, an incentive to buy all supplies on the spot market is provided, which means a substantial increasing risk but a insignificant increase in expected profit. For the numerical study performed in Section IV, a value of $N = 1 \cdot \mathrm{e}{-05}$ has been selected, because this is the point where risk starts to increase strongly as N decreases.

V. CONCLUSION

This paper addresses the contract design problem faced by a retailer, both at the supply and end-user levels. It provides a stochastic programming methodology that allows the retailer to make informed contractual decisions, particularly in respect to contract prices and quantities. It should be emphasized that an appropriate modeling of the risk associated with buying from the spot market is required. A realistic case study is used to show the usefulness of the technique proposed. Results are described and properly analyzed.

APPENDIX

The results here have some similarity to what was shown in [3], and the proofs are developed analogously.

Theorem 1: The following implications hold.

1) If $B_t - L_t^a(\omega_L) \leq -\underline{\alpha}(p_b^{\text{eu}}) - \varepsilon$, then

$$u_t^1(\omega_L) = B_t - L_t^a(\omega_L), \quad u_t^2(\omega_L) = u_t^3(\omega_L) = 0.$$

2) If $B_t - L_t^a(\omega_L) \geq \bar{\alpha}(p_b^{\text{eu}}) + \varepsilon$, then

$$u_t^3(\omega_L) = B_t - L_t^a(\omega_L), \quad u_t^1(\omega_L) = u_t^2(\omega_L) = 0.$$

3) If $-\underline{\alpha}(p_b^{\text{eu}}) \leq B_t - L_t^a(\omega_L) \leq \bar{\alpha}(p_b^{\text{eu}})$, then

$$u_t^2(\omega_L) = B_t - L_t^a(\omega_L), \quad u_t^1(\omega_L) = u_t^3(\omega_L) = 0.$$

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