TABLE I
SUMMARY OF STATE TRANSITION PROBABILITIES FOR A HYPOTHETICAL BULK POWER SYSTEM (BASE CASE)

From	To	p_{ij}	STP	Basis
Normal	Normal	p_{11}	0.99699	Row sums to 1
	Local Load Interrupted (LLI)	p_{13}	0.00274	1 time per year
	Alert	p_{15}	0.00027	1 time per 10 years
Off-Economic Dispatch (OED)	OED	p_{22}	0.00046	Stay in this state on average for 4 hours
	Normal	p_{21}	0.99954	Row sums to 1
LLI	LLI	p_{33}	0.00068	Stay in this state on average for 6 hours
	Normal	p_{31}	0.99932	Row sums to 1
Controlled Load Curtailment (CLC)	CLC	p_{44}	0.00046	Stay in this state on average for 4 hours
	Normal	p_{41}	0.99954	Row sums to 1
Alert	Alert	p_{55}	0.00046	Stay in this state on average for 4 hours
	Normal	p_{51}	0.39982	Of the probability of leaving this state, 40% to Normal, 20% to OED, 20% to CLC, and 20% to Emergency
	OED	p_{52}	0.19991	
	CLC	p_{54}	0.19991	
	Emergency	p_{56}	0.19991	
Emergency	Emergency	p_{66}	0.00046	Stay in this state on average for 4 hours
	CLC	p_{64}	0.49977	Of the probability of leaving this state, 50% to CLC, 40% to Alert, and 10% to EE
	Alert	p_{65}	0.39982	
	Extreme Emergency	p_{68}	0.09995	
Extreme Emergency	Extreme Emergency	p_{88}	0.6575	Stay in this state on average for 24 hours
	Restorative	p_{87}	0.93425	Row sums to 1
Restorative	Restorative	p_{77}	0.06575	Stay in this state on average for 24 hours
	Normal	p_{71}	0.93425	Row sums to 1

TABLE II
RESULTS OF BASE CASE AND TRADEOFF ANALYSIS OF CURTAILING LOAD VERSUS OPERATING IN A NONSECURE STATE (BASE CASE REPORTS NUMBER OF HOURS OUT OF TEN YEARS)

State	20% CLC (Base Case)	Sensitivity Cases (% Change from Base Case) 0% CLC	10% CLC	30% CLC
Normal	87,315.3	0.0%	0.0%	0.0%
OED	5.2	9.5%	4.5%	-4.2%
LLI	239.4	0.0%	0.0%	0.0%
CLC	7.8	-27.0%	-12.9%	11.8%
Alert	26.0	9.5%	4.5%	-4.2%
Emergency	5.2	119.0%	56.8%	-52.1%
Extreme Emergency	0.6	119.0%	56.8%	-52.1%
Restorative	0.6	119.0%	56.8%	-52.1%

Table II indicates that an aggressive policy to curtail load when the system is in the alert state (30% CLC) reduces the numbers of days in the extreme emergency and restorative states by a little more than 50% but at a cost of increasing the frequency of the CLC state by almost 12% compared to the base case.

III. FUTURE RESEARCH

Additional research is needed to develop an empirical foundation for the determination of the STPs and to integrate and compare this approach to existing methodologies. Solution strategies when the Markov assumption is relaxed should also be investigated, as in [6]. Associating costs with various states would enable the optimization of design and policy tradeoffs.

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State Estimation Observability Based on the Null Space of the Measurement Jacobian Matrix

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***Abstract*—This letter describes a new technique for state estimation observability analysis. This technique is computationally efficient and based on calculating the null space of the measurement Jacobian matrix. The technique is illustrated through a clarifying example. Conclusions are duly drawn.**

***Index Terms*—Null space, observability, state estimation.**

I. TECHNIQUE

The observability problem consists in identifying if a set of available measurements is enough to be able to estimate the state of an electric energy system. Note that observability is related not only to the number of measurements but also to their types and locations. An appropriate literature review on state estimation observability, including algebraic and topological techniques, can be found in [1].

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Consider the measurement vector equation

$$\mathbf{h}(\mathbf{x}) = \mathbf{z} \tag{1}$$

where

$\mathbf{h}(\cdot)$ m-dimensional measurement nonlinear function;
$\mathbf{x}$ n-dimensional state vector;
$\mathbf{z}$ m-dimensional constant vector of actual measurements.

The state vector includes voltage magnitudes for all buses and voltage angles for all buses but the reference one. The measurement function may include active and reactive power injections, active and reactive power flows, and eventually line currents. Usually, $m \gg n$, but some subareas of the system may lack sufficient measurements to guarantee observability while others may include more measurements than those needed to achieve observability.

For observability purposes and without loss of generality [1], (1) can be linearized; thus

$$\mathbf{H}\mathbf{x} = \mathbf{z} \tag{2}$$

where $\mathbf{H}$ is the $m \times n$ Jacobian measurement matrix.

The general solution of the linear system (2) is a vector space that can be expressed using a particular solution of the linear system (2), $\mathbf{x}^{\mathbf{par}}$, and the null space of matrix $\mathbf{H}$ [2].

In turn, the null space of matrix $\mathbf{H}$ can be expressed as

$$\mathbf{x}^{\mathbf{null}} = \mathbf{N}\boldsymbol{\rho} \tag{3}$$

where $\mathbf{N}$ is the $n \times k$ null-space matrix, and $\boldsymbol{\rho}$ is the arbitrary k-dimensional vector. Therefore, the general solution of linear system (2) is

$$\mathbf{x} = \mathbf{x}^{\mathbf{par}} + \mathbf{N}\boldsymbol{\rho}. \tag{4}$$

From (4), any component i of $\mathbf{x}$ has the form

$$x_i = x_i^{\mathrm{par}} + \sum_{j=1}^{k} N_{ij}\rho_j. \tag{5}$$

A relevant observation is straightforwardly obtained from (5): Any variable x_i is observable, i.e., uniquely determined, if and only if its general solution does not depend on ρ_j's, that is, if all $N_{ij}, j = 1, \ldots, k$ are null.

The algorithm below relies on the above observation.

II. Algorithm

The proposed observability algorithm works as follows.

1) Obtain the measurement Jacobian $\mathbf{H}$.
2) Compute the null-space matrix $\mathbf{N}$ of matrix $\mathbf{H}$.
3) Check columnwise matrix $\mathbf{N}^{\mathbf{T}}$ (or row-wise matrix $\mathbf{N}$). Columns (rows) containing only zeroes identify observable state variables.

The computational complexity of the proposed procedure reduces to just one null space computation. The computational burden involved is, therefore, that pertaining to the null space calculation, which is similar to the burden of solving a linear homogeneous system $(\mathrm{O}(N^3))$ [3]. Therefore, the computational burden of the proposed procedure is moderate and appropriate for observability analysis in power system state estimation.

If the Jacobian measurement matrix is ill conditioned, there are methods to avoid the numerical problems arising in the implementation of the algorithm for obtaining the null space (see, for example, [4]).

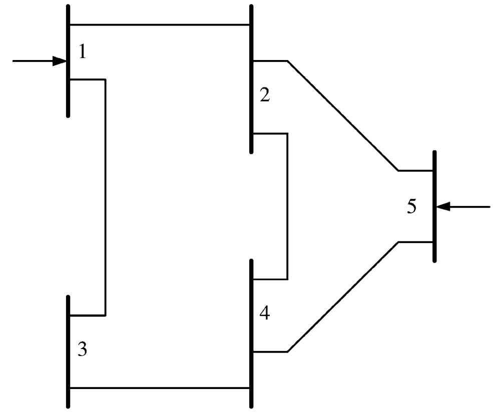

Fig. 1. Measurement configuration of a five-bus system.

It should be noted that the algorithm proposed is related to the QR factorization technique [3] to solve the normal equations of a state estimation problem, as stated, for instance, in [5]–[7].

III. Example

The example considered in [1, p. 78] is used in this section to illustrate the proposed technique. Fig. 1 depicts the considered system.

For the sake of simplicity, only active power measurements are considered. However, note that the functioning of the algorithm is identical if reactive power and current measurements are also included.

Consider that available measurements include active power injections in buses 1 and 5 and active power flows 1–2 and 1–3. Matrix $\mathbf{H}$ is

$$\mathbf{H} = \begin{bmatrix} 2 & -1 & -1 & \\ & -1 & & -1 \\ 1 & -1 & & \\ 1 & & -1 & \end{bmatrix}.$$

In the matrix above, rows 1 to 4 express power injection in bus 1, power injection in bus 5, power flow 1–2, and power flow 1–3, respectively. Columns 1–4 correspond to voltage angles 1–4 (state variables). Bus 5 is the reference bus, and its angle value is equal to 0.

The corresponding transpose null-space matrix $\mathbf{N}^{\mathbf{T}}$ is

$$\mathbf{N}^{\mathbf{T}} = [-1 \quad -1 \quad -1 \quad 1].$$

Since no column includes only zeroes, no state variable is observable.

On the other hand, if we consider just the active power flow measurements 2–4 and 2–5, matrix $\mathbf{H}$ becomes

$$\mathbf{H} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

The transpose null-space matrix $\mathbf{N}^{\mathbf{T}}$ becomes

$$\mathbf{N}^{\mathbf{T}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

Since columns 2 and 4 include only zeroes, the state variables θ_2 and θ_4 are observable.

On the other hand, if we consider active power flow measurements 1–2, 2–4, and 2–5, matrix $\mathbf{H}$ becomes

$$\mathbf{H} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

The transpose null-space matrix $\mathbf{N^T}$ becomes

$$\mathbf{N^T} = [0 \quad 0 \quad 1 \quad 0].$$

Since columns 1, 2, and 4 include only zeroes, the state variables θ_1, θ_2, and θ_4 are observable.

Finally, if active power injection measurements in buses 1 and 5 and active power flow measurements 1–2, 1–3, and 2–4 are considered, matrix $\mathbf{H}$ becomes

$$\mathbf{H} = \begin{bmatrix} 2 & -1 & -1 & \\ & -1 & & -1 \\ 1 & -1 & & \\ 1 & & -1 & \\ & 1 & & -1 \end{bmatrix}.$$

The transpose null-space matrix $\mathbf{N^T}$ becomes

$$\mathbf{N^T} = [0 \quad 0 \quad 0 \quad 0].$$

Since all columns include only zeroes, all state variables are observable.

Numerical tests carried out using the IEEE Reliability Test System [8], including voltage, active and reactive power injection, and active and reactive power flow measurements, show the effective behavior of the proposed algorithm.

IV. Conclusion

This letter proposes and illustrates a novel technique to state estimation observability analysis. This technique relies on computing the null space of the Jacobian measurement matrix. The computational burden involved is, therefore, that pertaining to the null space calculation, which is similar to the burden of solving a linear homogeneous system ($\mathrm{O}(N^3)$). A simple example is used to demonstrate the efficacious functioning of the technique. Extensive computational analysis involving voltage and active and reactive power measurements as well as current measurements is the subject of a future paper.

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Coherency Identification in Power Systems Through Principal Component Analysis

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***Abstract*—In this letter, a new technique to identify coherent generators in large interconnected power system using measurements of generator speed and bus angle data has been presented. This is based on the application of principal component analysis (PCA) to measurements obtained from simulation studies that represent examples of interarea events. The results of application of PCA separately to data sets of generator speeds and bus angles, respectively, are presented. The approach of PCA was able to highlight clusters of generators showing common features when compared with the conventional modal analysis technique.**

***Index Terms*—Clusters, coherency, modal analysis, principal component analysis (PCA).**

I. Introduction

Increasing interconnection of power plants in modern large electric power systems has made power system dynamic studies much more complex. Under the deregulated business environment, the interconnections are increasingly being used for trading between utilities. This stresses the interconnections through large power transfer. As a result, the system is prone to low-frequency interarea oscillations. In such an operating scenario, the system behaves coherently with groups of coherent generators separated from other groups of coherent generators linked through weak interconnections.

The analysis of an interconnected power system is generally for a specified portion, called the study system, while the rest is the external portion of the system approximated to its equivalent of lower dimension. Since the impacts of major disturbances propagate through tie lines to neighboring systems, it is important to represent not only the power system in question but also the neighboring utilities or the external system in terms of its dynamic equivalent.

Coherency-based approaches to dynamic equivalents have been adopted in reducing the size of the power system model [1]–[3]. However, coherency was identified with the help of the linearized model of the power systems. More often than not, the access to system dynamic data involving a number of utilities is incredibly difficult. The accuracy of the system model influences the accuracy of the results. Also, owing to the dimension of the interconnected systems, it is neither easy nor desirable to represent the entire system model in detail.

In this letter, coherency is obtained from measured data, obviating the need for detailed modeling information. The method is based on principal component analysis (PCA), which is computationally simple and fast. An industrial application of this method will take the wide area measurements from GPS-based time-stamped phasor measurement units (PMUs). For the purposes of demonstration in this letter, the data measurements are generated from the Simulink model of a 16-machine 68-bus test system for generator and bus coherency identification

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