Market-Clearing With Stochastic Security—Part I: Formulation

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***Abstract*—The first of this two-paper series formulates a stochastic security-constrained multi-period electricity market-clearing problem with unit commitment. The stochastic security criterion accounts for a pre-selected set of random generator and line outages with known historical failure rates and involuntary load shedding as optimization variables. Unlike the classical deterministic reserve-constrained unit commitment, here the reserve services are determined by economically penalizing the operation of the market by the expected load not served. The proposed formulation is a stochastic programming problem that optimizes, concurrently with the pre-contingency social welfare, the expected operating costs associated with the deployment of the reserves following the contingencies. This stochastic programming formulation is solved in the second companion paper using mixed-integer linear programming methods. Two cases are presented: a small transmission-constrained three-bus network scheduled over a horizon of four hours and the IEEE Reliability Test System scheduled over 24 h. The impact on the resulting generation and reserve schedules of transmission constraints and generation ramp limits, of demand-side reserve, of the value of load not served, and of the constitution of the pre-selected set of contingencies are assessed.**

***Index Terms*—Deterministic/probabilistic security criteria, electricity markets, expected load not served, failure rate, reserve, stochastic programming, transmission limits, unit commitment, value of lost load.**

NOMENCLATURE

The main symbols used in both papers are defined below. Others will be defined as required in the text.

A. Variables

$ELNS_{mt}$ Expected load not served at bus m during period t.
L_{mt} Involuntarily shed load at bus m during period t.
d_{mt} Demand at bus m during period t.
g_{it} Power output of generator i during period t.
r_{it}^{up} Spinning reserve up provided by generator i during period t.
r_{it}^{dn} Spinning reserve down provided by generator i during period t.
r_{mt}^{up} Spinning reserve up provided by demand at bus m during period t.
r_{mt}^{dn} Spinning reserve down provided by demand at bus m during period t.
$\tilde{r}_{it}^{up}$ Nonspinning reserve up provided by generator i during period t.
$\tilde{r}_{it}^{dn}$ Nonspinning reserve down provided by generator i during period t.
u_{it} Binary variable (equals 1 if generator i is online during period t; 0 otherwise).
δ_{mt} Voltage angle of bus m during period t.

B. Functions

$B_d(\cdot)$ Demand-side energy benefit function.
$C_g(\cdot)$ Generation-side energy cost function.
$C_r(\cdot)$ Reserve cost function.
$f_\ell(\cdot)$ Function expressing the power flow in line ℓ.
$h_m(\cdot)$ Function describing the nodal power balance at bus m.

C. Parameters

$f_\ell^{\max}$ Maximum power flow on line ℓ.
p_0 Probability that no contingencies occur during the scheduling horizon.
$p(k,\tau)$ Probability that no contingencies occur during the scheduling horizon except for contingency k, which occurs during interval τ.
v_{mt} Value of lost load at bus m during period t.

D. Sets

$\mathcal{A}_m$ Set of generators located at bus m.
$\mathcal{B}_m$ Set of transmission lines connected to bus m.
$\mathcal{C}_k$ Set of failed components corresponding to contingency k.
$\mathcal{D}_{mt}$ Feasible operating region of demand at bus m during period t.
$\mathcal{G}_{it}$ Feasible operating region of generator i during period t.

E. Indices

i Index of generators running from 1 to I.
k Index of contingencies running from 1 to K.
ℓ Index of transmission lines running from 1 to L.
m Index of buses running from 1 to M.
t Index of time periods running from 1 to T.
τ Index of contingency occurrence intervals running from 1 to T.

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Remark 1: When augmented with the argument (k, τ), the above parameters, variables, and functions represent their value given that contingency k has occurred within interval τ.

Remark 2: A variable, function, or parameter written in bold without one or more indices is a vector form representing the corresponding quantities. For example, the symbol $\boldsymbol{\delta}_t$ represents the vector of bus voltage angles during period t.

I. Introduction

THE required levels of reserve services in pool-based electricity markets are generally set using deterministic criteria. For example, it is common to schedule enough reserve so as to counter the loss of the largest generator or system import. As this type of preventive security measure does not take into consideration the probability of occurrence of this contingency, if its probability is low, on the average, over-scheduling of reserve may result, while if the probability of occurrence is high, the reserve may be insufficient.

Kirschen [1] suggests that power system security analysis methods should evaluate the “credibility” of failures and their “expected” consequences by means of probabilistic methods. Such probabilistic security analysis of power systems however is generally cursed by computational intractability because of the need to evaluate the probabilities and consequences of a very large number of possible failure modes. In the specific case of reserve-constrained unit commitment problems, the combinatorial aspect of probabilistic methods remains a very important limiting factor to their widespread use, as is well recognized in the literature [2]–[9]. Another drawback of probabilistic methods comes from the fact that the failure rates of generators and transmission lines are never known with certainty.

These impediments have led to *hybrid* deterministic/probabilistic security analysis methods instead of purely probabilistic ones that require the full enumeration of the possible system states [2], [10], [11]. In the proposed hybrid methods, the probabilities or the expected consequences of a restricted set of *a priori*-defined events are used as security metrics instead of considering all the possible failure modes. For example, in structural design, [10] addressed the criticism related to the uncertainty of the failure probabilities and the complicating computational aspects of probabilistic metrics. In the context of power system planning, [11] proposed the use of a deterministic/probabilistic method to evaluate local and system-wide impacts of pre-defined failures of components. The combinatorics of the unit commitment with probabilistic reserve criteria provided the impetus for developing a hybrid security metric based on an upper bound on the probabilities of failure of generators taken alone or in pairs as functions of the unit commitment variables [2]. This approach can be interpreted as a probability-weighted $N-2$ criterion.

Recent major blackouts in North America and Europe [12] have rekindled interest in power system security issues that in recent years have been regretfully neglected in favor of more financial concerns. This renewed interest is motivating research into the design of market-clearing formulations that also account for a more complete set of power system security constraints. This paper and its companion [13] contribute to this research effort by developing a new market-clearing formulation with the following generalizations and improvements of the previous work in [2]: *a*) Time-coupling generation constraints like minimum up and down times as well as ramping limits are considered. *b*) A network model is included to account for load flow limits (based on a dc load flow). *c*) The hybrid security metric is the expected load not served (ELNS) due to random line and generator outages as well as load-side disturbances. *d*) Contingencies may occur randomly within the scheduling horizon. *e*) The objective function of the market-clearing problem is the expected pre-contingency social welfare plus the expected post-contingency cost of the corrective rescheduling and load shedding actions, respectively, weighted by appropriate probabilities. *f*) The hybrid security metric is calculated through a sharply reduced number of binary variables. The companion paper [13] solves the resulting stochastic optimization problem using mixed-integer linear programming for a number of test cases, including the IEEE Reliability Test System [14] over a 24-h horizon.

Finally, the importance of fast emergency procedures together with preventive and corrective security dispatch cannot be overestimated in preventing cascaded outages. Fast emergency actions of the order of seconds or less are usually local and automatic and include primary frequency regulation or load shedding based on frequency or voltage-sensitive relays. On the other hand, security-constrained dispatch is centrally computed and implemented on a longer time-scale of minutes. This is an operation-planning procedure that pre-positions the state of the system to reduce its vulnerability to outages and that pre-calculates emergency re-dispatch schemes to be implemented following an outage. These slower re-dispatch schemes presuppose that the fast emergency actions have successfully stabilized the system.

II. Market-Clearing With Deterministic Security

Before introducing the market-clearing problem with stochastic security, it is useful to recall a general formulation of the deterministic security-constrained market-clearing problem

$$\min[C_g(\mathbf{u},\mathbf{g}) + C_r(\mathbf{r}^{up},\mathbf{r}^{dn},\tilde{\mathbf{r}}^{up},\tilde{\mathbf{r}}^{dn}) - B_d(\mathbf{d})] \tag{1}$$

subject to

Pre-contingency power balance

$$\mathbf{h}(\mathbf{u},\mathbf{g},\mathbf{d},\boldsymbol{\delta}) = \mathbf{0} \tag{2}$$

Pre-contingency power flow limits

$$-\mathbf{f}^{\max} \le \mathbf{f}(\boldsymbol{\delta}) \le \mathbf{f}^{\max}. \tag{3}$$

In addition, for all the pre-selected contingencies, $k = 1,\ldots,K$, and all possible contingency occurrence times, $\tau = 1,\ldots,T$, that is for all contingency scenarios (k,τ), the minimization is also subject to

Post-contingency power balance

$$\mathbf{h}(\mathbf{u}(k,\tau),\mathbf{g}(k,\tau),\mathbf{d}(k,\tau),\boldsymbol{\delta}(k,\tau),k,\tau) = \mathbf{0} \tag{4}$$

Post-contingency power flow limits

$$-\mathbf{f}^{\max}(k,\tau) \le \mathbf{f}(\boldsymbol{\delta}(k,\tau),k,\tau) \le \mathbf{f}^{\max}(k,\tau) \quad (5)$$

Pre- and post-contingency generator constraints

$$(u_{it}, g_{it}, u_{it}(k,\tau), g_{it}(k,\tau), r_{it}^{up}, r_{it}^{dn}, \tilde{r}_{it}^{up}, \tilde{r}_{it}^{dn}) \in \mathcal{G}_{it}, \quad i=1,\ldots,I, t=1,\ldots,T \quad (6)$$

Pre- and post-contingency demand constraints

$$(d_{mt}, d_{mt}(k,\tau), r_{mt}^{up}, r_{mt}^{dn}) \in \mathcal{D}_{mt}, \quad m=1,\ldots,M, t=1,\ldots,T. \quad (7)$$

The objective of the above problem is to maximize the total social welfare, or equivalently as seen in (1), to minimize the offer-based energy and reserve production costs minus the bid-based energy consumption benefits over the scheduling horizon. The energy production cost function embeds the generators' no-load, startup, and variable costs, while the reserve cost function lumps generation and demand-side reserve costs. The consumption side of the objective function considers the net benefits from energy use in the form of bids for energy consumption.

Reserves are either of the *spinning* or *nonspinning* type, both of which can be up- or down-going. The assumption is that if a contingency occurs during time interval τ, then these reserves have to be deployed for all time intervals $t \ge \tau$. Generation-side spinning reserve, up or down, is provided by committed generators only, while nonspinning reserve involves changes in the scheduling status of generators. For instance, a generator that is scheduled off can provide up-going nonspinning reserve if it can be turned on to produce energy within the contingency occurrence interval τ. On the other hand, down-going nonspinning reserve is provided by a generator already online if it can be turned off within the contingency occurrence interval τ. For a consumer, providing up-going spinning reserve implies being ready to voluntarily decrease its level of consumption within the contingency occurrence interval τ [15]. In the case of down-going spinning reserve, consumers providing this service would be asked to increase their consumption level. The details of reserve calculations are addressed in Appendix A.

The pre-contingency power balance constraints at each network bus and for every period of the scheduling horizon are represented by (2), while the corresponding line flow limits are given in (3).

In this deterministic unit commitment formulation, the reserve services are scheduled with the goal of ensuring that the system can withstand any of the credible contingencies, $k = 1,\ldots,K$, occurring during any time interval of the scheduling horizon, $\tau = 1,\ldots,T$, without loss-of-load or line flow limit violations. These conditions are mathematically expressed by the post-contingency power balance relations (4) and by the post-contingency power flow constraints (5).

For the sake of simplicity and computational tractability, all of the above power flow relations use the linear dc load flow model.

Note that this approach constitutes a departure from current practice, but as justified in [16], this generalization is necessary when transmission congestion is present and when the market-clearing solution considers the expected cost of deploying post-contingency actions, as we propose in this paper. This has the advantage that all types of reserve can be explicitly defined in terms of the differences between the pre- and post-contingency states and, more importantly, that the area reserve criteria do not have to be defined *a priori*. Instead, the reserve criteria are implicit in the requirement that power must balance at every node in all post-contingency states.

The set of constraints $\mathcal{G}_{it}$ in (6) represents all generator restrictions for all pre- and post-contingent states over the time horizon, including minimum up and down times, ramping limits, as well as minimum and maximum power and reserve levels [6], [17]–[19]. Appendix B provides further details on ramping constraints. Likewise, the operating sets $\mathcal{D}_{mt}$ appearing in (7) describe demand restrictions such as elasticity limits and demand-side spinning reserve bounds.

III. Stochastic Security Metric

The deterministic security-constrained market-clearing problem summarized above has two principal drawbacks. One is that unless some load shedding is permitted, it may not be possible to meet all the required constraints, (4)–(7). Second, when the problem is feasible without load shedding, the need to cover all contingencies, irrespective of their likelihood of occurring, will decrease the overall social welfare and, particularly, may sharply increase the incremental costs of reserve and energy.

These two drawbacks motivate the formulation of a market-clearing problem with a stochastic security criterion that accounts for the likelihood of the contingencies and allows load shedding. In this paper, load shedding is taken to mean involuntary as opposed to voluntary demand reduction offered as up-spinning reserve. Thus, under this new stochastic formulation, small amounts of load shedding may be tolerated if the events leading to this loss-of-load occur with a low probability and the corresponding decrease in social welfare is small.

In the new formulation, the amount of load shed involuntarily at bus m during period t because of contingency k occurring during interval τ, denoted by $L_{mt}(k,\tau)$, is calculated from the post-contingency power balance relation

$$L_{mt}(k,\tau) = d_{mt}(k,\tau) + \sum_{\substack{\ell \in \mathcal{B}_m \\ \ell \notin \mathcal{C}_k}} f_\ell(\boldsymbol{\delta}_t(k,\tau),k,\tau) - \sum_{\substack{i \in \mathcal{A}_m \\ i \notin \mathcal{C}_k}} g_{it}(k,\tau) \quad (8)$$

where $d_{mt}(k,\tau)$ is the demand at bus m, $f_\ell(\boldsymbol{\delta}_t(k,\tau),k,\tau)$ is the power flow in line ℓ, $\boldsymbol{\delta}_t(k,\tau)$ is the vector of voltage angles, and $g_{it}(k,\tau)$ is the generation level of unit i. The above expression is applicable for single or compound outages and load disturbances. The latter can consist of the simultaneous failure of some lines and some generators, coupled with some pre-specified load disturbances.

Moreover, since load shedding cannot be negative and cannot be greater than the actual load, we impose that

$$0 \le L_{mt}(k,\tau) \le d_{mt}(k,\tau) \quad (9)$$

for $k = 1,\ldots,K$, $m = 1,\ldots,M$ and all $t \ge \tau$.

We demonstrate in Appendix C that (8) and (9) are equivalent to the conditions developed in [2] that required the definition of extra binary variables in addition to the classical unit commitment variables u_{it}. This is a significant simplification of the model when the chosen criterion is the ELNS. However, if the criterion is the loss-of-load probability (LOLP), then the extra binary variables used in [2] become necessary.

For completeness, note that we should impose that $L_{mt}(k,\tau) = 0$ if $t < \tau$. This condition implies that there should not be any loss-of-load in the pre-contingency state.

To assess the impact of load shedding, we define the expected load not served [2] at bus m during period t averaged over all contingencies occurring randomly over the past intervals as

$$ELNS_{mt} = \sum_{k=1}^{K}\sum_{\tau=1}^{t} p(k,\tau)L_{mt}(k,\tau). \quad (10)$$

The quantity $p(k,\tau)$, defined as the probability of the event "*no contingencies occur during the scheduling horizon except for contingency k, which occurs during interval τ*," is calculated from historical failure rates (assumed constant) as shown in Appendix D.

There exist three ways to implement a stochastic security criterion based on ELNS. One way, as suggested in [2], is to impose an upper bound on either the local or the system-wide ELNS. A second way is to penalize ELNS inside the objective function, while the third is a combination of both an upper bound and a penalty.

There are drawbacks to imposing upper bounds on the ELNS as part of or as the sole stochastic security criterion. One is that these limits would most likely have to be defined by a regulatory body to specific values that would be difficult to set and justify. In addition, even with load shedding, it may be impossible to satisfy a specified ELNS upper bound if there are insufficient reserve resources or if these resources are very unreliable. Finally, if only a bound on ELNS were used as the stochastic security criterion, then even if sufficient reserve resources were available, involuntary load shedding would be applied until the ELNS inequalities became binding, which in general would be unacceptable to the consumers.

To avoid these shortcomings, we define a stochastic security criterion based only on the penalization of ELNS within the objective function as detailed in the next section.

IV. Market-Clearing With Stochastic Security

By explicitly considering the probability of occurrence of the various contingencies (including the probability of the event that no contingency occurs), the new market-clearing problem becomes a stochastic programming problem with uncertainty affecting only the objective function [20].

The objective function, now given by (11)

$$\begin{aligned}\min\, & p_0[C_g(\mathbf{u},\mathbf{g}) + C_r(\mathbf{r}^{up},\mathbf{r}^{dn},\tilde{\mathbf{r}}^{up},\tilde{\mathbf{r}}^{dn}) - B_d(\mathbf{d})] \\ & + \sum_{k=1}^{K}\sum_{\tau=1}^{T} p(k,\tau)[C_g(\mathbf{u}(k,\tau),\mathbf{g}(k,\tau)) - B_d(\mathbf{d}(k,\tau))] \\ & + \sum_{m=1}^{M}\sum_{t=1}^{T} v_{mt} ELNS_{mt} \end{aligned} \quad (11)$$

consists of the sum of three terms: *a*) the expected social welfare during the pre-contingency state (with probability p_0) including the expected cost of scheduling all reserve services, *b*) the total expected cost associated with the deployment of reserves in the post-contingency states, which may include turning generating units on or off and re-dispatching generation and consumption, and *c*) the last term in (11), which is the expected cost of load not served, where v_{mt} is the value of lost load at bus m during period t [21]–[25].

With respect to the contingencies, we make the following assumptions: *a*) Only one contingency can occur during the time horizon (where such a contingency could however be a simultaneous compounded failure), and *b*) Reserve levels in the post-contingency states are therefore not defined since their optimization would require the consideration of nonsimultaneous contingencies. We feel that the greatly increased complexity of considering such low probability events in the market-clearing scheme is not warranted.

Like the deterministic market-clearing problem (1)–(7), the new stochastic minimization problem is subject to the same pre-contingency power balance and line flow limits [respectively, (2) and (3)], the same post-contingency line flow limits (5), as well as the same generation and demand variables constraints [(6) and (7), respectively]. What changes, in addition to the objective function, are the post-contingency power balance relations (4), which are modified to account for involuntary load shedding

$$\mathbf{h}(\mathbf{u}(k,\tau),\mathbf{g}(k,\tau),\mathbf{d}(k,\tau),\boldsymbol{\delta}(k,\tau),\mathbf{L}(k,\tau),k,\tau) = \mathbf{0}. \quad (12)$$

The new stochastic formulation must also respect the load shedding bounds defined by (9).

In stochastic programming, the proposed market-clearing formulation is termed a scenario analysis problem [20], [26], [27]. Typical of such problems is the notion of *bundle constraints* that form part of the sets $\mathcal{G}_{it}$ and $\mathcal{D}_{mt}$ and model the nonanticipatory character of the optimal schedule. In other words, before the occurrence of contingency k during the interval τ, the following conditions must hold:

$$u_{it}(k,\tau) = u_{it}; \quad i=1,\ldots,I; t<\tau \quad (13)$$

$$g_{it}(k,\tau) = g_{it}; \quad i=1,\ldots,I; t<\tau \quad (14)$$

$$d_{mt}(k,\tau) = d_{mt}; \quad m=1,\ldots,M; t<\tau \quad (15)$$

$$f_\ell(\boldsymbol{\delta}_t(k,\tau),k,\tau) = f_\ell(\boldsymbol{\delta}_t); \quad \ell=1,\ldots,L; t<\tau \quad (16)$$

for $t = 1,\ldots,T$.

In addition, after contingency k occurs at time τ, all post-contingency variables must continue to satisfy the same constraints as in the pre-contingency state for $t \geq \tau$, except those variables associated with the failed elements defining the contingency. For example, if contingency k corresponds to the loss of generator $j \in \mathcal{C}_k$, the post-contingency generator variables must satisfy $(u_{it}(k,\tau), g_{it}(k,\tau)) \in \mathcal{G}_{it}$, if $i \notin \mathcal{C}_k$; otherwise, $(u_{jt}(k,\tau), g_{jt}(k,\tau)) = (0,0)$.

V. Discussion

The reader may correctly deduce that optimizing over the post-contingency on/off generator status variables $u_{it}(k,\tau)$ is computationally very costly due to the exponential growth of the number of combinations of contingencies and their time of occurrence within the scheduling horizon. Thus, when solving such large-scale problems, it might be necessary to forego optimizing over such binary variables by fixing them to their pre-contingency levels, that is, by imposing the additional constraint that $u_{it}(k,\tau) = u_{it}$ for all contingencies k and times of occurrence τ. We point out that by so doing, nonspinning reserve services can no longer be defined.

The new formulation contains a number of important generalizations compared to earlier security-constrained market-clearing schemes. For example, the optimal post-contingency corrective control actions in security-constrained optimal power flow problems pioneered in [28] did not consider unit commitment, probabilities of occurrence of contingencies, nor the costs of corrective actions. The works of [29] and [30] take into consideration expected costs of preventive and corrective security actions but again only for an optimal power flow without unit commitment. In [16], a single-period network-constrained unit commitment was proposed as a security-constrained market-clearing tool; however, this contribution did not include probabilistic features nor the cost of deploying corrective actions. Several security-constrained unit commitment formulations have been proposed and are actually in use (in PJM, New York ISO, ISO New England, and in New Zealand, for instance) [17], [31]–[39]. The schemes proposed in these references however do not consider explicitly the post-contingency states and corrective actions or their probability of occurrence, as we do in this paper.

To our knowledge, the literature has not considered the various complicating aspects jointly treated in this paper, which include multi-period unit commitment, probabilistic security criteria, transmission congestion, and the combined expected social welfare and expected cost of preventive and corrective security actions. The consideration of involuntary load shedding as a potential corrective security action is not commonplace either; however, recent events have raised awareness of the increased importance of this type of action in emergency situations [12].

Nonetheless, we feel that involuntary load shedding should be used sparingly, as it is generally much more costly to society than the scheduling and eventual deployment of reserves. Furthermore, we recognize that the exact value of involuntary lost load at a given bus and time, as specified by v_{mt}, is uncertain and may depend on the duration of the interruption. In addition, who is responsible for its specification, that is, a regulatory body or the load-serving entities, remains an open issue; see [23] and [25] for theoretical economic discussions. Currently, the regulator is probably in the best position to set a uniform value of lost load across an entire network—with the notable exception of buses feeding critical loads like hospitals and airports, for which v_{mt} should be very high. For example, in Australia, the value adopted by the regulator to estimate the value of lost load is of the order of \$10 000/MWh [23]. However, if there is insufficient transmission capability, even critical loads may have to be curtailed irrespective of the level of v_{mt}.

Since both the pre- and post-contingency load flow equations are explicitly considered, the dimensionality of the optimization problem treated here can be significant for realistic systems, as illustrated in the companion paper [13] in a case study based on the IEEE Reliability Test System. Solution techniques based on Benders' decomposition [35], [39]–[41] are promising strategies to deal with large systems by keeping the size of the constraint set to a more reasonable level and by taking advantage of the parallelism offered by this decomposition technique. Similar comments apply to the "branch-and-price" decomposition technique [42]–[44]. Moreover, several authors [45]–[47] have proposed rigorous scenario reduction algorithms for stochastic programming problems. In a pre-processing step, these algorithms aggregate failure scenarios having similar impacts on the security of the system, thus eliminating the need to consider extra associated variables and constraints lying under the umbrella of the more constraining scenarios. These aspects are currently under investigation.

VI. Conclusion

The first of this two-paper series has formulated a stochastic security-constrained multi-period electricity market-clearing problem with unit commitment. The stochastic security criterion accounts for a pre-selected set of generator and line outages with known historical failure rates as well as for random demand disturbances. In addition, unlike in the classical deterministic security-constrained unit commitment, here the reserve services are determined by economically penalizing the operation of the market by the expected cost of the load not served due to involuntary load shedding.

The new formulation is a stochastic program containing significant generalizations of earlier work on the topic of security-constrained market-clearing, namely, the consideration of *a*) time-coupling generation constraints like minimum up and down times as well as ramping limits, *b*) a network model to account for line flow limits (based on a dc load flow), *c*) involuntary load shedding in addition to demand-side reserve (voluntary consumption adjustments), *d*) contingencies that may occur randomly within the scheduling horizon, *e*) a stochastic market-clearing objective function measuring the expected pre-contingency social welfare plus the expected post-contingency cost of the corrective rescheduling and involuntary load shedding actions, and *f*) a hybrid security metric defined by the ELNS due to random line and generator outages calculated without the need to define extra binary variables.

The philosophy behind the proposed security-constrained market-clearing approach recognizes the inherent complex engineering and economical couplings that make the operation of modern power systems such a challenging task. In particular, the proposed approach acknowledges the evidence that the management of randomness in power system operation is one of the core elements of this challenge. Power system operation planning based on a stochastic security criterion as proposed here has the advantage that it permits the system operators to gauge how likely contingencies are and what are the possible actions and expected costs associated with preparing for and responding to those random contingencies. As seen from the results of the companion paper, the stochastic approach leads to a more efficient utilization of energy and reserve resources.

We recognize that planning the operation of power systems under a probabilistic security criterion constitutes a significant departure from traditional deterministic approaches that are indifferent to the likelihood of occurrence of the credible contingencies and to the expected value of lost load. The potential adoption of stochastic methods in operation planning will therefore require further analysis and debate. In our opinion, stochastic methods should be given strong consideration since, as shown in this two-part paper, they offer many advantages and there are no major technical impediments to their eventual implementation. Clearly there are practical issues to be refined, such as the collection of sufficient and reliable statistical outage and value of lost load data. However, as described here, there are significant ongoing research and industry efforts addressing these issues.

Appendix A
Reserve Determination Constraints

A. Spinning Reserve

The generator up-spinning reserve are restricted by

$$0 \leq r_{it}^{up} \leq r_{it}^{up\ \max} u_{it} \tag{17}$$

for all generators $i = 1, \ldots, I$ and for all time periods $t = 1, \ldots, T$. The parameters $r_{it}^{up\ \max}$ are the upper limits on the up-spinning reserve offers imposed by each of the generators. Clearly, up-spinning reserve can be provided by generator i during period t only if it is on, that is, if $u_{it} = 1$.

In the current formulation, the up-spinning reserve provided by generator i during period t is the largest among all possible contingencies k and failure times τ of the difference between its post and pre-contingency generation levels. Mathematically, for $k = 1, \ldots, K$ and $\tau = 1, \ldots, T$

$$r_{it}^{up} \geq g_{it}(k,\tau) - g_{it} - g_i^{\max}(2 - u_{it} - u_{it}(k,\tau)) \tag{18}$$

where $g_i^{\max}$ is the maximum power output of generator i.

To see how (17) and (18) set the reserve levels in a consistent manner, we examine the possible cases that may arise.

First, we assume that $u_{it} = 0$. From (17), r_{it}^{up} should equal zero. To demonstrate that (18) is consistent with this result, consider the case where the post-contingency commitment variable $u_{it}(k,\tau)$ is equal to 1—meaning that generator i has been turned on to counter some outage. Then, (18) requires that $r_{it}^{up} \geq g_{it}(k,\tau) - g_i^{\max}$, which is less than or equal to zero, and is consistent with the lower bound in (17). A similar argument applies if $u_{it}(k,\tau) = 0$.

When $u_{it} = 1$, the up-spinning reserve provided by generator i during period t lies in the range $[0, r_{it}^{up\ \max}]$ as required by (17). The actual level of reserve is set from (18), considering all the contingencies. The case when $u_{it}(k,\tau) = 0$ does not impose any constraint on the reserve; however, if $u_{it}(k,\tau) = 1$, inequality (18) requires that $r_{it}^{up} \geq g_{it}(k,\tau) - g_{it}$ for all k and τ. In other words, the up-spinning reserve provided by generator i is the largest up-going power production deviation away from the pre-contingency dispatch g_{it}.

Likewise, the down-spinning reserve provided by generator i during period t is determined by the bounds

$$0 \leq r_{it}^{dn} \leq r_{it}^{dn\ \max} u_{it} \tag{19}$$

$$r_{it}^{dn} \geq g_{it} - g_{it}(k,\tau) - g_i^{\max}(2 - u_{it} - u_{it}(k,\tau)), \quad \tau = 1, \ldots, T; k = 1, \ldots, K \tag{20}$$

where $r_{it}^{dn\ \max}$ are the upper limits on the down-spinning reserve offers imposed by each of the generators. Like with up-spinning reserve, down-spinning reserve is set by the largest down-going post-contingency power production deviation away from the pre-contingency dispatch g_{it}.

In the case of demand at bus m, providing up-spinning reserve during period t boils down to reducing its consumption from d_{mt} to $d_{mt}(k,\tau)$, with $(d_{mt}, d_{mt}(k,\tau)) \in \mathcal{D}_{mt}$, given that $m \notin \mathcal{C}_k$—that is, if contingency k is not a demand disturbance at bus m. This reserve must also satisfy the two sets of inequalities

$$0 \leq r_{mt}^{up} \leq r_{mt}^{up\ \max} \tag{21}$$

$$r_{mt}^{up} \geq d_{mt} - d_{mt}(k,\tau); \ \tau = 1, \ldots, T; k = 1, \ldots, K \tag{22}$$

where $r_{mt}^{up\ \max}$ is the maximum amount of up-spinning reserve demand at bus m is willing to provide during period t. Similar bounds apply to down-going demand-side spinning reserves if $m \notin \mathcal{C}_k$

$$0 \leq r_{mt}^{dn} \leq r_{mt}^{dn\ \max} \tag{23}$$

$$r_{mt}^{dn} \geq d_{mt}(k,\tau) - d_{mt}; \ \tau = 1, \ldots, T; k = 1, \ldots, K \tag{24}$$

where $r_{mt}^{dn\ \max}$ is the maximum down-spinning reserve amount that can be provided at bus m during period t.

B. Nonspinning Reserve

The up-going nonspinning reserve contributions from the generators $i = 1, \ldots, I$ at time periods $t = 1, \ldots, T$ are restricted by

$$0 \leq \tilde{r}_{it}^{up} \leq \tilde{r}_{it}^{up\ \max}(1 - u_{it}) \tag{25}$$

and, for $k = 1, \ldots, K$, $\tau = 1, \ldots, T$

$$\tilde{r}_{it}^{up} \geq g_{it}(k,\tau) - g_i^{\max} u_{it} \tag{26}$$

where $\tilde{r}_{it}^{up\ \max}$ are offer-based upper limits on up-going nonspinning reserve. The inequalities (25) and (26) indicate that up nonspinning reserve can be provided only by generators not already online, that is, those with $u_{it} = 0$, which are capable of being turned on during period t.

The down-going nonspinning reserve supplies from the generators $i = 1, \ldots, I$ and for $t = 1, \ldots, T$ are restricted by

$$0 \leq \tilde{r}_{it}^{dn} \leq \tilde{r}_{it}^{dn\ \max} u_{it} \tag{27}$$

and, for $k = 1, \ldots, K$, $\tau = 1, \ldots, T$

$$\tilde{r}_{it}^{dn} \geq g_{it} - g_i^{\max} u_{it}(k,\tau) \tag{28}$$

where $\tilde{r}_{it}^{dn\ \max}$ are offer-based upper limits on down-going nonspinning reserve. The inequalities (27) and (28) indicate that down nonspinning reserve can be provided only by the generators that are already online, that is, those with $u_{it} = 1$, which are capable of being shut down during period t.

APPENDIX B
RAMPING AND OUTPUT CAPACITY CONSTRAINTS

The following parameters are required to define the generator capacity and ramping limitations:

u_{i0} commitment state of generator i during period 0;
g_{i0} power output of generator i during period 0;
$g_i^{\min}$ minimum power output of generator i;
$g_i^{\max}$ maximum power output of generator i;
R_i^{dn} ramp-down limit of generator i;
R_i^{up} ramp-up limit of generator i;
R_i^{sd} shutdown ramp limit of generator i;
R_i^{su} startup ramp limit of generator i.

The bounds on the available power are, for all the generators $i = 1, \ldots, I$ [18]

$$g_i^{\min} u_{it} \leq g_{it} \leq g_i^{\max} u_{it}; \quad t = 1, \ldots, T. \tag{29}$$

The up-ramping limitations are, for generators $i = 1, \ldots, I$

$$g_{it} \leq g_{i(t-1)} + R_i^{up} u_{i(t-1)} + R_i^{su}(u_{it} - u_{i(t-1)}) + g_i^{\max}(1 - u_{it}) \quad t = 2, \ldots, T \tag{30}$$

$$g_{it} \leq g_{i0} + R_i^{up} u_{i0} + R_i^{su}(u_{it} - u_{i0}) + g_i^{\max}(1 - u_{it}) \quad t = 1. \tag{31}$$

Furthermore, down-ramping constraints are imposed on every generator $i = 1, \ldots, I$

$$g_{it} \geq g_{i(t-1)} - R_i^{dn} u_{it} - R_i^{sd}(u_{i(t-1)} - u_{it}) - g_i^{\max}(1 - u_{i(t-1)}), \quad t = 2, \ldots, T \tag{32}$$

$$g_{it} \geq g_{i0} - R_i^{dn} u_{it} - R_i^{sd}(u_{i0} - u_{it}) - g_i^{\max}(1 - u_{i0}), \quad t = 1. \tag{33}$$

Note that the above set of constraints, (29)–(33), applies equally to the post-contingency generation variables $g_{it}(k, \tau)$ and $u_{it}(k, \tau)$.

APPENDIX C
EQUIVALENCE OF LOSS-OF-LOAD CONDITIONS

In [2], it was suggested to use binary indicator variables $\sigma_{mt}(k, \tau)$ to signal whether load would be lost at bus m during period t if the contingency scenario (k, τ) were to occur. In this approach, the indicator variable takes the value 1 if there is loss-of-load, that is, if $L_{mt}(k, \tau) > 0$, and 0 otherwise. The values of the indicator variables and of the load shedding amounts are calculated for $m = 1, \ldots, M$, $t = 1, \ldots, T$, $k = 1, \ldots, K$, and $t \geq \tau$ using the expressions

$$L'_{mt}(k, \tau) = d_{mt}(k, \tau) + \sum_{\substack{\ell \in \mathcal{B}_m \\ \ell \notin \mathcal{C}_k}} f_\ell(\boldsymbol{\delta}_t(k, \tau), k, \tau) - \sum_{\substack{i \in \mathcal{A}_m \\ i \notin \mathcal{C}_k}} g_{it}(k, \tau) \tag{34}$$

$$\frac{L'_{mt}(k, \tau)}{Z} \leq \sigma_{mt}(k, \tau) \leq 1 + \frac{L'_{mt}(k, \tau)}{Z} \tag{35}$$

$$L_{mt}(k, \tau) = \sigma_{mt}(k, \tau) L'_{mt}(k, \tau) \tag{36}$$

where $L'_{mt}(k, \tau)$ is a dummy variable, Z is a sufficiently large positive number, and $\sigma_{mt}(k, \tau) \in \{0, 1\}$.

We prove next that these conditions on the load shedding variables are equivalent to those proposed in the body of the paper, namely, (8) and (9).

Proof

The conditions (34)–(36) imply (8) and (9) by inspection.

We demonstrate now that (8) and (9) are sufficient to replace (34)–(36). Let the optimal amount of load shedding found using (34)–(36) equal $L^*_{mt}(k, \tau)$ and that found using (8) and (9) equal $L^\dagger_{mt}(k, \tau)$.

First, we shall assume that $L^*_{mt}(k, \tau) < L^\dagger_{mt}(k, \tau)$. This requires that $L^\dagger_{mt}(k, \tau) > 0$, satisfying (8) and (9). However, this contradicts the assumed optimality of $L^\dagger_{mt}(k, \tau)$, because the value $L^\dagger_{mt}(k, \tau) = L^*_{mt}(k, \tau)$ is also feasible according to (8) and (9) yet it yields a lower value of the objective function. Next, we assume that $L^*_{mt}(k, \tau) > L^\dagger_{mt}(k, \tau)$. This is a contradiction of the assumed optimality of $L^*_{mt}(k, \tau)$ because the solution $L^*_{mt}(k, \tau) = L^\dagger_{mt}(k, \tau)$ is feasible according to (34)–(36) and yields a lower value of the objective function. Therefore, $L^*_{mt}(k, \tau) = L^\dagger_{mt}(k, \tau)$ must always be satisfied. ■

APPENDIX D
CONTINGENCY PROBABILITIES

Classical reliability theory [3], [5], [48] models the time of failure τ of a given piece of equipment as an exponentially distributed random variable. Here, we illustrate some basic results based on contingencies that may include the loss of several components at the same time but exclude contingencies made up of nonsimultaneous failures. The probability p_0 that none of the pre-selected contingencies occur during the scheduling horizon is calculated as

$$p_0 = \prod_{k=1}^{K} e^{-\lambda_k T} \tag{37}$$

where the parameter λ_k represents the reciprocal of the mean time to the occurrence of contingency k, a quantity estimated from historical data.[1] In addition, since repair times are usually longer than the 24-hour scheduling horizon of most markets, in this formulation, repairs are ignored so that once some equipment fails, it is assumed to be unavailable for the remainder of the horizon.

Likewise, $p(k, \tau)$, the probability that contingency k occurs during the interval τ given that all other system components are available is

$$p(k, \tau) = e^{-\lambda_k \tau}(e^{\lambda_k} - 1) \prod_{\substack{z=1 \\ z \neq k}}^{K} e^{-\lambda_z T}. \tag{38}$$

Note that in deriving the above probabilities, we have assumed that the pre-selected contingencies are statistically independent and that since this set is not exhaustive, the probabilities p_0 and $p(k, \tau)$ sum to a number less than one.

[1]The Canadian Electricity Association, for example, has been compiling information about individual line and generator failures since 1977 [49], [50].

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