Self-Scheduling of a Hydro Producer in a Pool-Based Electricity Market

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***Abstract*—This paper addresses the self-scheduling of a hydro generating company in a pool-based electricity market. This company comprises several cascaded plants along a river basin. The objective is to maximize the profit of the company from selling energy in the day-ahead market. This paper proposes a 0/1 mixed-integer linear programming model to account, in every plant, for the nonlinear and nonconcave three-dimensional (3-D) relationship between the power produced, the water discharged, and the head of the associated reservoir. Additionally, start-up costs due mainly to the wear and tear are considered. Finally, different realistic case studies are analyzed in detail.**

***Index Terms*—Hydroelectric producer, mixed-integer LP, pool-based electricity market.**

NOMENCLATURE

The notation used throughout the paper is stated as follows:

Constants:

M	Conversion factor equal to 3.6×10^{-3} [Hm^{3}s/m^{3}h].
$\bar{P}_i$	Capacity of plant i [MW].
$P0_1(i)$	Minimum power output of plant i for performance curve 1 (lower interval of water content) [MW].
$P0_2(i)$	Minimum power output of plant i for performance curve 2 (intermediate interval of water content) [MW].
$P0_3(i)$	Minimum power output of plant i for performance curve 3 (higher interval of water content) [MW].
Q_i	Future value of the stored water in the reservoir associated with plant i at the end of the market time horizon [\$/Hm^3].
SU_i	Start-up cost of plant i [\$].
T	Number of periods of the market time horizon.
$\underline{U}_i$	Minimum water discharge of plant i [m^3/s].
$\bar{U}_i$	Maximum water discharge of plant i [m^3/s].
$\bar{U}_\ell(i)$	Maximum water discharge of block ℓ of plant i [m^3/s].
$W_i(k)$	Forecasted natural water inflow of the reservoir associated to plant i in period k [Hm^3/h].
$X_i(0)$	Initial water content of the reservoir associated to plant i [Hm^3].
$\underline{X}_i$	Minimum content of the reservoir associated to plant i [Hm^3].
$\bar{X}_i$	Maximum content of the reservoir associated to plant i [Hm^3].
XL_i	Lower level of the content of the reservoir associated with plant i used in the discretization of the performance curves [Hm^3].
XU_i	upper level of the content of the reservoir associated to plant i, used in the discretization of the performance curves [Hm^3],
$\lambda(k)$	Forecasted price of energy in period k [\$/MWh].
$\rho 1_\ell(i)$	Slope of the block ℓ of the performance curve 1 of plant i [MW/m^3/s].
$\rho 2_\ell(i)$	Slope of the block ℓ of the performance curve 2 of plant i [MW/m^3/s].
$\rho 3_\ell(i)$	Slope of the block ℓ of the performance curve 3 of plant i [MW/m^3/s].
τ_{ji}	Time delay between reservoir of plant j and reservoir of plant i [h].

Variables:

$d_1(i,k)$	0/1 variable used for the discretization of the performance curves.
$d_2(i,k)$	0/1 variable used for the discretization of the performance curves.
$p_i(k)$	Power output of plant i in period k [MW].
$s_i(k)$	Spillage of the reservoir associated to plant i in period k [m^3/s].
$u_i(k)$	Water discharge of plant i in period k [m^3/s].
$u_\ell(i,k)$	Water discharge of block ℓ of plant i in period k [m^3/s].
$v_i(k)$	0/1 variable which is equal to 1 if plant i is on-line in period k.
$w_\ell(i,k)$	0/1 variable which is equal to 1 if water discharged by plant i has exceeded block ℓ in period k.
$x_i(k)$	Water content of the reservoir associated to plant i in period k [Hm^3].
$y_i(k)$	0/1 variable which is equal to 1 if plant i is started-up at the beginning of period k.
$z_i(k)$	0/1 variable which is equal to 1 if unit i is shut-down at the beginning of period k.

Sets:

I	Set of indices of the plants belonging to the hydro company.
K	Set of indices of the periods of the market time horizon.
L	Set of indices of the blocks of the piecewise linearization of the unit performance curve.
Ω_i	Set of upstream reservoirs of plant i.

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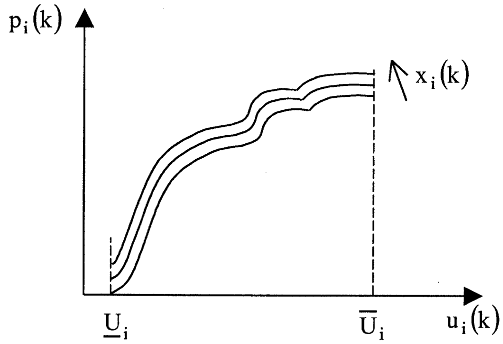

Fig. 1. Hill chart.

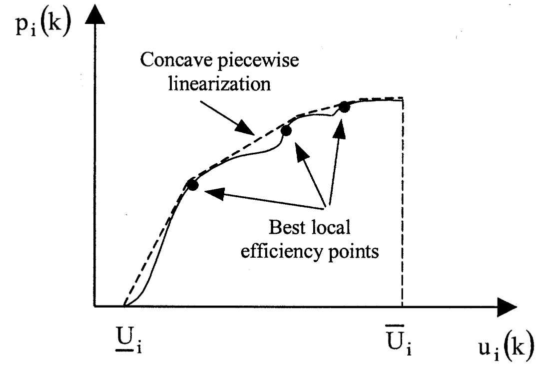

Fig. 2. Traditional approximation of the unit performance curve.

I. INTRODUCTION

THIS paper considers a day-ahead electricity market for energy based on a pool. In this framework, generating companies (GENCOs) and consumer or energy service companies submit bids for selling and buying electric energy for the next 24 hr. An independent agent, namely, the market operator (PX), is in charge of clearing the market, determining the accepted and rejected buying and selling bids. The market clearing procedure also provides the market clearing price for every period. These electricity markets based on a pool are spread worldwide (Spain [1], New England [2], Scandinavia [3]).

In this new environment, GENCOs face new challenging problems with the ultimate goal of maximizing their profits. One of those problems consists of determining, in the short term, the optimal self-schedule of the units belonging to the GENCO so that the profit from selling energy is maximized without regard to the power balance in the system. This problem is referred to as self-scheduling, and it is the problem addressed in this paper.

Another relevant problem is how to translate this information into a bidding strategy to ensure that in the day-ahead market, the GENCO achieves the maximum benefit. Bidding strategy development is outside the scope of this paper.

Recently, several contributions dealing with the problem of self-scheduling have been available in the technical literature [4], [5]. In [4], this problem is solved for a price-taker GENCO comprising one thermal unit. The same problem is solved in [5] for an oligopolistic GENCO owning several thermal units.

This paper addresses the self-scheduling problem for a hydro GENCO (H-GENCO) owning several plants cascaded along a river basin [6]. Specific features of hydro plants include i) spatial-temporal coupling among reservoirs and ii) for every plant, the nonlinear dependence between the power output, the water discharged, and the head of the associated reservoir are precisely accounted for through a 0/1 mixed-integer linear formulation. Additionally, start-up costs are also considered in this work.

In order to solve the self-scheduling problem, it is essential to use accurate models. These models should include the hydro generation characteristic describing the relationship between the head of the associated reservoir, the water discharged, and the power generated. This is a nonlinear and nonconcave 3-D relationship: the so-called Hill chart [7]. This relationship can be represented as a family of nonlinear and nonconcave curves, which are also known as unit performance curves, each for a specified value of the head (see Fig. 1).

In most optimization methods, the effect of the variation of the head has been neglected to avoid nonlinearities, which allows using a single unit performance curve. Furthermore, this single curve has been approximated either through a concave piecewise linear approximation (see Fig. 2) [8], [9] or by modeling just the so-called best local efficiency points (see Fig. 2) [10], [11]. All these simplifications may lead to inaccuracies. On the contrary, dealing with nonconcavities in a nonlinear optimization framework is difficult and may lead to multiple local optima and to unreliable solution procedures.

On the other hand, it is important to consider start-up costs for hydro units [12]. The start-up costs are due to

i) the loss of water during maintenance;
ii) wear and tear of the windings;
iii) wear and tear of the mechanical equipment;
iv) malfunctions in the control equipment;
v) loss of water during the start-up.

From the above causes, the main ones are ii) and iv). As a result, a start-up cost per megawatt unit nominal output that depends on the type of unit [12] can be derived.

The H-GENCO analyzed in this paper is considered to be a price-taker. In other words, it does not have market power, i.e., its schedule does not alter the hourly market clearing prices. Therefore, prices are assumed known. Several forecasting procedures are available to predict the day-ahead hourly market clearing prices, such as auto regressive models [3], [13], linear programming [14], or neural networks [15]. If the H-GENCO is a price-maker, a new source of complexity arises: It is necessary to model how the market clearing price changes with the H-GENCO total production. For every hour, the function modeling this variation is the so-called price-quota curve of the H-GENCO [5]. These curves have to be estimated and incorporated in the self-scheduling problem formulation of the H-GENCO.

The self-scheduling problem for an H-GENCO can be formulated as a 0/1 mixed-integer nonlinear nonconcave and moderate scale problem.

The main contributions of this paper are the following:

1) a discretization of the Hill chart into a set of nonconcave curves depending on the reservoir content so that the traditionally neglected effect of the head is modeled;

2) a precise piecewise linear approximation of each nonconcave unit performance curve that overcomes the inaccuracies of previous approximations (see Fig. 3);
3) the modeling of start-up costs to avoid unnecessary start-ups.

This paper provides a 0/1 mixed-integer linear model that allows an accurate representation of the variation of the performance curves with the reservoir head. The number of performance curves needed to represent accurately the head variation is usually small (below 5), and therefore, the associated computational burden is moderate (minutes on a PC). Furthermore, this paper also provides a 0/1 mixed-integer linear model to represent the nonconcavities of each performance curve, which can be made as accurate as needed while keeping the computational burden low. Finally, start-up costs are also modeled using binary variables.

The above contributions are included in the model, resulting in a 0/1 mixed-integer linear programming problem that can be solved efficiently by available branch and cut solvers [16].

The self-scheduling problem addressed in this paper is similar to any of the hydro subproblems of a short-term hydro-thermal coordination problem solved by Lagrangian relaxation. In this framework, the maximum profit problem faced by the hydro producer has been traditionally solved using network programming [17] or a combination of network programming and dynamic programming [6], without taking into account the three improvements explained above.

The proposed approach allows i) the determination of the self-scheduling of the H-GENCO in the day-ahead market and ii) the efficient and accurate solution of the subproblems that result from addressing a (centralized) short-term hydro-thermal coordination problem using Lagrangian relaxation.

The ultimate motivation of this paper is to provide the H-GENCO with a short-term self-scheduling tool to achieve maximum profit from selling energy in the day-ahead market while considering all its operating constraints.

The remaining of this paper is organized as follows. In Section II, the problem is formulated in detail. This section presents a discretization of the Hill chart. In addition, this section includes a precise modeling of the piecewise linearization of the nonconcave and nonlinear relationship for every plant between the water discharged and the power output. In Section III, results from realistic case studies are provided and discussed. Finally, some relevant conclusions are drawn in Section IV.

II. Formulation

The formulation of the self-scheduling problem of an H-GENCO is presented in Sections II-A–C. This problem is formulated as a 0/1 mixed-integer linear programming problem.

A. Objective Function

The goal of any participant in an electricity market is to maximize its own profit, which is computed as the difference between the revenues and the total operating costs. The operating costs comprise production costs and start-up costs. In the case of a hydro company, the production costs are negligible. As it is reported in [10] and [12], the start-up costs have real impact on the short-term scheduling of hydro generation. Start-up costs are mainly caused by the increased maintenance of windings and mechanical equipment and by malfunctions of the control equipment. Thus, the objective function to be maximized can be expressed as

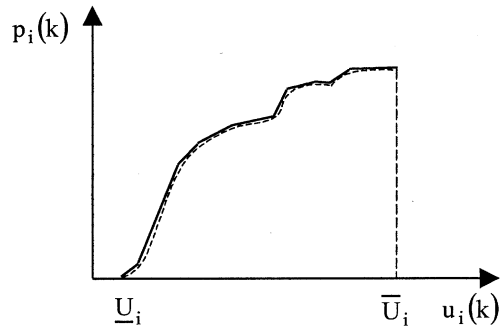

Fig. 3. Nonconcave piecewise linearization.

$$\sum_{k\in K}\sum_{i\in I}\left[\lambda(k)\,p_i(k)-SU_i y_i(k)\right]+\sum_{i\in I}Q_i x_i(T). \qquad (1)$$

In (1), the first term is related to the revenues of each plant belonging to the H-GENCO, whereas the second term represents the start-up costs, which are defined as a constant dependent on the type of the plant [10], [12]. Finally, an extra term has been added to model in a simple way the future value of stored water in reservoirs.

B. Hydro Constraints

The following expressions represent the set of constraints related to every hydro plant over the market time horizon. In this paper, the family of curves representing, for every plant, the relationship between the reservoir head, the water discharged, and the power output has been simplified to a prespecified number of curves, each one for an interval of the head. The generalization to any number of curves is straightforward. For the sake of simplicity, three curves are considered in this paper, corresponding to a low, medium, and high level of the reservoir content. Through the use of binary variables, the model accounts for these three curves.

Moreover, in order to consider the nonconcavities and nonlinearities of these unit performance curves [7], [10], a precise mixed-integer linear formulation has been developed.

Finally, water balance and logical status of commitment is enforced through additional constraints.

1) Simplification of the Three-Dimensional Unit Performance Curves: For each plant, the set of curves representing the relationship between the head, the power output, and the water discharge is reduced to three curves, according to two levels of the stored water in the reservoir. Fig. 4 depicts an illustrative example of this simplification. It should be noted that this figure represents a simplification of the Hill chart (see Fig. 1), where each unit performance curve has been approximated by a nonconcave piecewise linearization (see Fig. 3). If the content of the reservoir i in period k, $x_i(k)$, is below XL_i (low level), then curve 1 is used. If $x_i(k)$ is between XL_i and XU_i, i.e., intermediate level, then curve 2 is

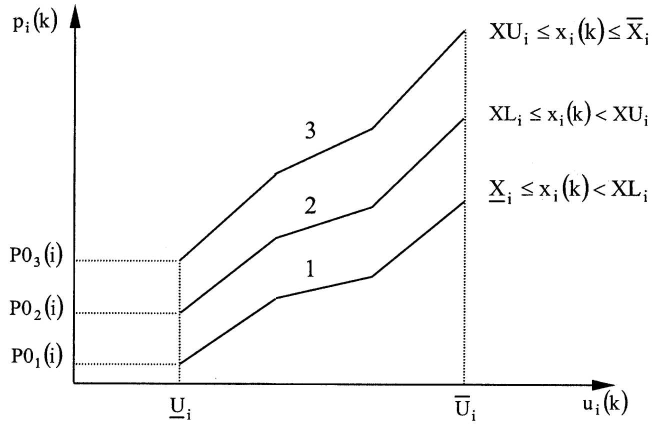

Fig. 4. Three-dimensional unit performance curves for plant i.

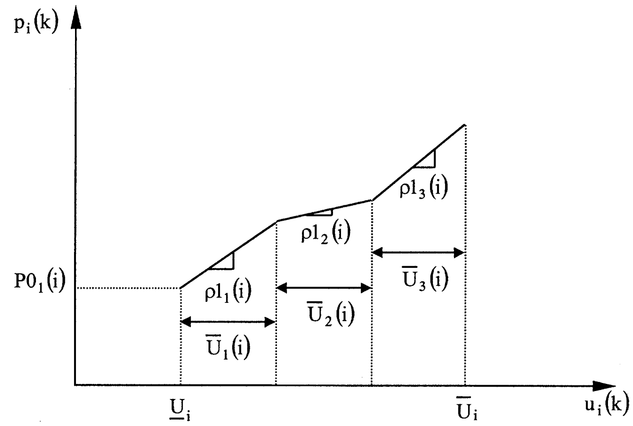

Fig. 5. Piecewise linear nonconcave unit performance curve.

used. Finally, if the stored water in period k is above XU_i (high level), then curve 3 is used. The generalization to any number of curves is straightforward.

This discretization is modeled through a 0–1 mixed-integer linear formulation, which is presented as

$$x_i(k) \geq XL_i[d_1(i,k) - d_2(i,k)] + XU_i d_2(i,k) \quad \forall i \in I, \ \forall k \in K \tag{2}$$

$$x_i(k) \leq \bar{X}_i d_2(i,k) + XL_i[1 - d_1(i,k)] + XU_i[d_1(i,k) - d_2(i,k)] \quad \forall i \in I, \ \forall k \in K \tag{3}$$

$$d_1(i,k) \geq d_2(i,k) \quad \forall i \in I, \ \forall k \in K \tag{4}$$

$$x_i(k) \geq \underline{X}_i \quad \forall i \in I, \ \forall k \in K. \tag{5}$$

In the formulation above, $d_1(i,k)$ and $d_2(i,k)$ are the binary variables used to choose the right curve according to the content level. If $d_1(i,k)$ and $d_2(i,k)$ are both equal to 0, constraints (2) and (3) force $x_i(k)$ to be in the lower interval so that curve 1 is used. If $d_1(i,k)$ is equal to 1 and $d_2(i,k)$ is equal to 0, constraints (2) and (3) force $x_i(k)$ to be in the intermediate interval, implying that the relationship between head, flow, and power output is represented by curve 2. Finally, if $d_1(i,k)$ and $d_2(i,k)$ are both equal to 1, constraints (2) and (3) force $x_i(k)$ to be in the higher level so that curve 3 is used. The formulation of each unit performance curve is presented in Section II-B2.

It should be noted that constraints (4) avoid the combination 0-1 for variables $d_1(i,k)$ and $d_2(i,k)$. It should also be noted that a higher number of levels can be modeled through the use of additional binary variables in a straightforward manner.

Finally, the set of constraints (5) sets the lower limit of the reservoir content in every period.

2) Piecewise Linear Formulation of the Unit Performance Curves: Unit performance curves are nonlinear and nonconcave [7], [10]. In this paper, these curves have been modeled through a piecewise linear formulation. Nonconcavities have also been modeled through the use of binary variables. Fig. 5 shows a three-piece linear nonconcave unit performance curve for a low content level. Fig. 5 is similar to Fig. 3 but simplified to illustrate the nonconcave linearization. Note that unlike in [10] and [11], the operation of the plant is not restricted to the local best efficiency points. In other words, the whole nonconcave unit performance curve is accurately modeled through the use of binary variables.

The formulation of the nonconcave unit performance curves is as follows:

$$p_i(k) - P0_1(i)v_i(k) - \sum_{\ell \in L} u_\ell(i,k)\rho 1_\ell(i) - \bar{P}_i[d_1(i,k) + d_2(i,k)] \leq 0 \quad \forall i \in I \quad \forall k \in K \tag{6}$$

$$p_i(k) - P0_1(i)v_i(k) - \sum_{\ell \in L} u_\ell(i,k)\rho 1_\ell(i) + \bar{P}_i[d_1(i,k) + d_2(i,k)] \geq 0 \quad \forall i \in I \quad \forall k \in K \tag{7}$$

$$p_i(k) - P0_2(i)v_i(k) - \sum_{\ell \in L} u_\ell(i,k)\rho 2_\ell(i) - \bar{P}_i[1 - d_1(i,k) + d_2(i,k)] \leq 0 \quad \forall i \in I \quad \forall k \in K \tag{8}$$

$$p_i(k) - P0_2(i)v_i(k) - \sum_{\ell \in L} u_\ell(i,k)\rho 2_\ell(i) + \bar{P}_i[1 - d_1(i,k) + d_2(i,k)] \geq 0 \quad \forall i \in I \quad \forall k \in K \tag{9}$$

$$p_i(k) - P0_3(i)v_i(k) - \sum_{\ell \in L} u_\ell(i,k)\rho 3_\ell(i) - \bar{P}_i[2 - d_1(i,k) - d_2(i,k)] \leq 0 \quad \forall i \in I \quad \forall k \in K \tag{10}$$

$$p_i(k) - P0_3(i)v_i(k) - \sum_{\ell \in L} u_\ell(i,k)\rho 3_\ell(i) + \bar{P}_i[2 - d_1(i,k) - d_2(i,k)] \geq 0 \quad \forall i \in I \quad \forall k \in K \tag{11}$$

$$u_i(k) = \sum_{\ell \in L} u_\ell(i,k) + \underline{U}_i v_i(k) \quad \forall i \in I \quad \forall k \in K \tag{12}$$

$$u_1(i,k) \leq \bar{U}_1(i)v_i(k) \quad \forall i \in I, \ \forall k \in K \tag{13}$$

$$u_1(i,k) \geq \bar{U}_1(i)w_1(i,k) \quad \forall i \in I, \ \forall k \in K \tag{14}$$

$$u_\ell(i,k) \leq \bar{U}_\ell(i)w_{\ell-1}(i,k) \quad \forall \ell \in L, \ \forall i \in I \quad \forall k \in K \tag{15}$$

$$u_\ell(i,k) \geq \bar{U}_\ell(i)w_\ell(i,k) \quad \forall \ell \in L, \ \forall i \in I \quad \forall k \in K. \tag{16}$$

Constraints (6) and (7) represent curve 1, i.e., the unit performance curve for the lower level (see Fig. 4). For this curve, $d_1(i,k)$ and $d_2(i,k)$ are both equal to 0, as imposed by constraints (2)–(4). Therefore, both constraints force the power output to be equal to the minimum power output plus the blocks of the lower-level piecewise linear curve. It should be noted that if $d_1(i,k)$ and $d_2(i,k)$ are not both equal to 0, constraints (6) and (7) are deactivated through the inclusion of the capacity $\bar{P}_i$.

In an analogous fashion, couples of constraints (8) and (9), as well as (10) and (11), model curves 2 and 3, respectively (see Fig. 4), through the appropriate values of $d_1(i,k)$ and $d_2(i,k)$ stated in Section II-B1.

Constraints (12) state that the water discharge of plant i in period k is the sum of the water discharged in each block plus the minimum water discharge. Constraints (13)–(16) set the limits of the water discharged in each block. This discharge must be greater than 0 and smaller than the size of each block. This is ensured through a binary variable $w_\ell(i,k)$, which is equal to 1 if the water discharge of plant i in period k has exceeded block ℓ.

3) Water Balance: The continuity equation of the hydro reservoirs is formulated as

$$\begin{aligned} x_i(k) =& x_i(k-1) + W_i(k) \\ &+ M \sum_{j\in\Omega_i} [u_j(k-\tau_{ji}) + s_j(k-\tau_{ji})] \\ &- M u_i(k) - M\, s_i(k) \quad \forall\, i \in I \\ \forall\, k \in& K \end{aligned} \tag{17}$$

where M is a factor to convert water discharge units (m^3/s) into stored water units (Hm^3). For unit consistency, it should be noted that time periods of 1 hr are considered.

4) Logical Status of Commitment: The following constraints

$$y_i(k) - z_i(k) = v_i(k) - v_i(k-1) \quad \forall\, i \in I,\ \forall\, k \in K \tag{18}$$

are necessary to model the start-up and shut-down status of the plants [18]

Although variables $z_i(k)$ may seem superfluous since they only appear in (18), extensive numerical simulations have proven their ability in considerably reducing the computing time.

C. Types of Variables

Variables used in the formulation are

$$v_i(k), y_i(k), d_1(i,k), d_2(i,k) \in \{0,1\} \quad \forall\, i \in I \quad \forall\, k \in K \tag{19}$$

$$w_\ell(i,k) \in \{0,1\} \quad \forall\, \ell \in L \quad \forall\, i \in I,\ \forall\, k \in K \tag{20}$$

$$z_i(k) \in [0,\ 1] \quad \forall\, i \in I \quad \forall\, k \in K \tag{21}$$

$$p_i(k), s_i(k), u_i(k), x_i(k) \geq 0 \quad \forall\, i \in I \quad \forall\, k \in K \tag{22}$$

$$u_\ell(i,k) \geq 0 \quad \forall\, \ell \in L,\ \forall\, i \in I \quad \forall\, k \in K. \tag{23}$$

TABLE I
HYDRO DATA

#	$\underline{U}_i$	$\bar{U}_i$	$X_i(0)$	XL_i	XU_i	$\underline{X}_i$	$\bar{X}_i$	$W_i(k)$	SU_i
1	2	62	100	152	200	6	225	0.051	110
2	5	163	80	100	150	6	162	0.058	150
3	14	464	790	500	1000	6	1200	0.603	200
4	19	662	33	50	60	6	66	0.051	250
5	18	628	13	8	20	6	26	0.051	350
6	14	479	1200	1000	2000	6	2586	0.199	1500
7	29	985	50	40	100	6	115	0.500	2000
8	30	1028	90	100	150	6	181	0.048	1000

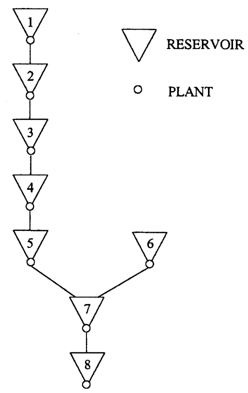

Fig. 6. Hydraulic topology of the river basin.

Variables $v_i(k)$, $y_i(k)$, $d_1(i,k)$, $d_2(i,k)$, and $w_\ell(i,k)$ are defined as binary in (19) and (20). Variables $z_i(k)$ can be defined as real variables belonging to the interval $[0,1]$, as stated in (21). Finally, constraints (22) and (23) state that the power output, spillage, discharge, reservoir content, and water discharged in each block in every period are all positive variables.

III. Case Study

Results from a realistic case study are reported in this section. The H-GENCO analyzed owns eight cascaded plants along a river basin. Table I shows the data of these plants. The spatial coupling among reservoirs is depicted in Fig. 6. For the sake of simplicity, 1-hr delays between any connected reservoirs have been considered. Additionally, forecasted water inflows are considered constant over the whole market time horizon, which is one day divided into 24 hourly periods. This case study considers that the final water content of each reservoir is identical to its initial value. Initial and final reservoir contents can be obtained by a medium-term planning procedure. Consequently, constants Q_i representing the future value of stored water are all equal to 0.

Table II shows the value of the slopes of the performance curves of each plant for the low interval of their corresponding reservoir contents $\rho 1_\ell(i)$. A piecewise linear approximation with four blocks has been implemented. The performance curves for the intermediate and high water contents are obtained by adding 0.05 and 0.1, respectively, to each slope in

TABLE II
PIECEWISE LINEAR APPROXIMATION OF THE PERFORMANCE CURVES

#	$\rho l_1(i)$	$\rho l_2(i)$	$\rho l_3(i)$	$\rho l_4(i)$	$\overline{U}_\ell(i)$
1	0.80	0.30	0.20	0.10	15.00
2	0.40	0.30	0.50	0.10	39.50
3	0.20	0.10	0.30	0.20	112.50
4	0.10	0.10	0.05	0.05	160.75
5	0.10	0.40	0.20	0.10	152.50
6	1.30	3.00	1.50	0.80	116.25
7	0.75	1.50	1.20	0.90	239.00
8	0.80	0.30	0.50	0.10	249.50

TABLE III
POWER OUTPUT LIMITS

#	$P0_1(i)$	$P0_2(i)$	$P0_3(i)$	$\overline{P}_i$
1	1.440	1.530	1.620	28.62
2	1.896	2.133	2.370	69.52
3	2.700	3.375	4.050	139.05
4	1.929	2.894	3.858	116.38
5	1.830	2.745	3.660	186.66
6	18.135	18.833	19.530	833.28
7	21.510	22.944	24.378	1159.63
8	23.952	25.449	26.946	550.90

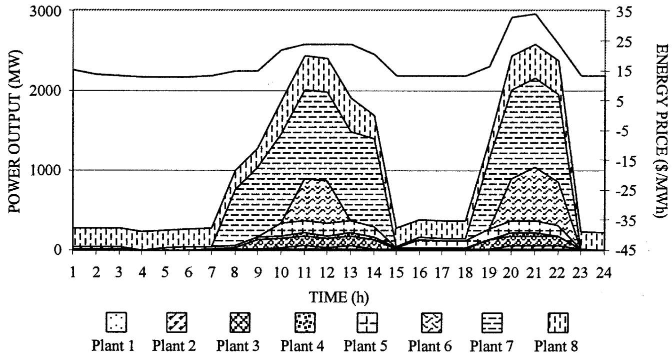

Fig. 7. Energy price profile and production schedule.

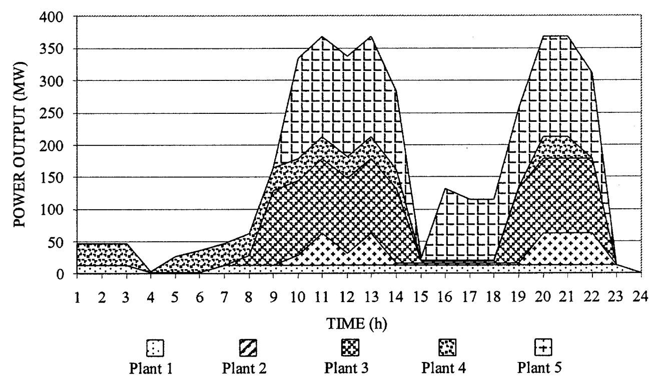

Fig. 8. Production schedule of plants 1 to 5.

Table II. For each plant, the blocks of the approximation are equally sized, as shown in the last column of the table.

Finally, Table III shows the minimum power output corresponding to each performance curve as well as the capacity of each plant.

The optimal solution is achieved in 22 min of CPU time, and the optimal value of the objective function is \$558 587.1. The model developed has been implemented on a SGI R12000, 400-MHz-based processor with 500 MB of RAM using CPLEX 7.0 under GAMS [16].

Fig. 7 shows the computed 24-hr generation schedule. It also shows the considered energy price profile corresponding to the electricity market in mainland Spain on March 3, 2001 [19]. It should be noted that the hydro production follows the shape of the price profile. For the sake of clarity, the production schedule of units 1 to 5 is also plotted in Fig. 8.

To illustrate the results obtained, plant 5 is selected. Most of the other plants behave in a similar fashion (see Figs. 7 and 8). Figs. 9–12 show the evolution of the water content, the profit, the water discharge, and the power output of plant 5 over the time span, respectively.

From Figs. 9–12, it should be noted that the plant is mainly operated on the hours with the highest prices.

As it can be seen in Fig. 9, the associated reservoir stores water in the periods 1–9 preceding the hours with the highest prices. Note that the water content is over 20 Hm^3 in periods 8–15 in order to use the high-level performance curve (see Table I), thereby yielding large profits. On the other hand, note that in hours 19–22, which also correspond to high prices, the water content decreases below 20 Hm^3 (medium level) in order to achieve the medium-term target of 13 Hm^3 in the last period of the time span. It should also be noted that only in hours 20 and 21, the production corresponds to the last block of the linearization of the performance curve, which is the most inefficient one, with a slope of 0.2 $MW/m^3/s$. This is due to the need to discharge water to reach the desired final water content.

The same test case has been run without considering the start-up costs. The optimal solution is achieved in 7 min of CPU time, and the optimal value of the objective function is \$567 156.3, i.e., a 1.53% increase on the total profit with respect to the original case. Aside from this difference in profit, the schedule of the plants is also affected by the omission of start-up costs. For instance, plant 1 is decommitted in hour 5, and plant 5 is decommitted in hours 15, 16, and 23.

Finally, this test case has also been run, including two simplifications that highlight how the two contributions modeled in this paper (discretization of the Hill chart and modeling of nonconcave performance curves) influence the computational burden and the objective function value. The first simplification consists of considering only one nonconcave efficiency curve for every plant. This simplification has been carried out by considering a lower level curve, a medium level curve, and a higher level curve for every plant in each case. The optimal solution was achieved for the three cases in 2 s of CPU time. The optimal values of the objective function were \$531 254.4, \$574 943.5, and \$619 109.1. In other words, the effect of not considering the nonlinear dependence between the power output, the water discharged, and the head of the associated reservoir materializes in deviations from the optimal solution of -5.1%, 2.9%,

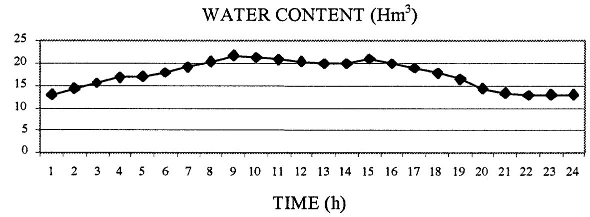

Fig. 9. Water content of reservoir associated with plant 5.

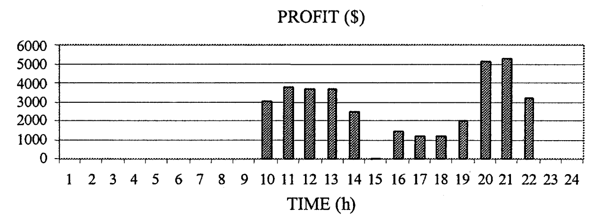

Fig. 10. Hourly profit of plant 5.

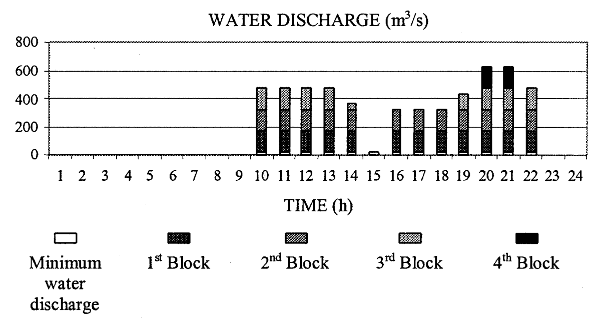

Fig. 11. Hourly water discharge of plant 5.

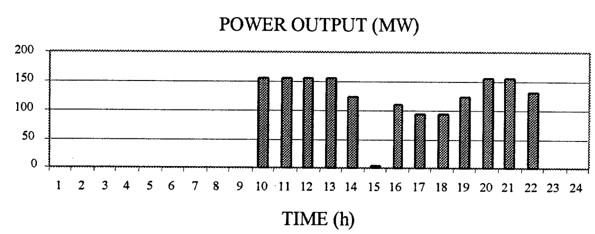

Fig. 12. Hourly power output of plant 5.

and 10.8%, respectively. Additionally, the scheduling of plants 3, 4, and 5 experiments changes with respect to the original case, mainly in hours 1–7 and 15–18, when the energy prices are low.

The second simplification considered a discretization of the Hill chart into three concave performance curves to quantify the effect of modeling several concave performance curves. The optimal solution was $642 452.7 and was attained in 4 min of CPU time. From these results, it can be concluded that the discretization of the Hill chart to model head variations takes up most of the computing time.

IV. Conclusions

This paper considers a pool-based electricity market for energy. In this framework, it provides a tool that allows a hydro generating company to optimally determine the short-term self-scheduling of its hydro plants cascaded along a river basin. The objective is to maximize the profit of the company from selling energy in the day-ahead market. This paper proposes a 0/1 mixed-integer linear programming model to account in each plant for the nonlinear nonconcave 3-D relationship between the reservoir head, the power output, and the water discharged. Additionally, start-up costs are considered. The model has been successfully tested on realistic case studies.

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